Routs to Rosenfield first please.

V. A. Krasil'nikov

B052095

SOUND and ULTRASOUND WAVES

52095

THIRD EDITION

Revised and Augmented

TRANSLATED FROM RUSSIAN

Published for the National Science Foundation, Washington D. C. and the National Aeronautics and Space Administration by the Israel Program for Scientific Translations

V. A. Krasil'nikov

SOUND and ULTRASOUND WAVES
in Air, Water and Solid Bodies

(Zvukovye i ul'trazvukovye volny v vozdukhe, vode i tverdykh telakh)

Third Edition
Revised and Augmented

Gosudarstvennoe izdatel'stvo
fiziko-matematicheskoi literatury
Moskva 1960

Translated from Russian

Published for the National Science Foundation, Washington, D. C. , by the
Israel Program for Scientific Translations
Jerusalem 1963

NASA TT-F-96

Published Pursuant to an Agreement with
THE NATIONAL AERONAUTICS AND SPACE ADMINISTRATION, U.S.A.
and
THE NATIONAL SCIENCE FOUNDATION, WASHINGTON, D.C.
by the
ISRAEL PROGRAM FOR SCIENTIFIC TRANSLATIONS

Translated by N. Kaner and M. Segal

Edited by R. Hardin

Printed in Jerusalem by S. Monson

IPST Cat. No. 811

Price $3.50

Available from the Office of Technical Services,
U.S. Department of Commerce, Washington 25, D.C.

TABLE OF CONTENTS

Annotation

The book presents in a simple way the basic physical questions connected with the propagation of sound and ultrasound waves in air, water and solids, as well as various applications of these waves.

A large section has been allotted to ultrasound waves and their applications, as well as to the propagation of sound in the atmosphere (atmospheric acoustics), in the sea (hydroacoustics), and in the earth (seismology). The propagation of high-intensity sound and ultrasound waves in gases and, particularly, in liquids are discussed, as well as the more important problems of aerothermoacoustics (jet noise, generation of sound by turbulence). The question of the propagation of elastic waves in solids (particularly in metals) is considered, as well as the basic ultrasonic applications in examining the elastic properties of solids. Special attention is given to the physical significance of the various phenomena.

The book is intended for people with a secondary education and can be useful to instructors in secondary schools, students, technicians, engineers, marine hydroacousticians, and those working in all fields related to acoustics.

Preface to the Third Edition

A number of substantial additions have been made to the third edition, related to questions which only recently became interesting. New material on sound scattering by atmospheric turbulence and on noise produced by turbulence [aerodynamic noise] has been added. In the section on the propagation of ultrasound in liquids, material on the relaxational absorption and the dispersion of sound has been inserted, as well as on hypersound and "second sound" in liquid helium.

Nonlinear acoustics developed successfully during recent years and, since questions of nonlinear processes in liquids are now of considerable general interest, a new chapter on ultrasound waves of high intensity (waves of finite amplitude) in liquids was added.

In the chapter on the propagation of elastic waves, additions have been made dealing with the absorption of ultrasound in solids, with the propagation of waves in a granular medium, with anomalous reflection and anomalous transmission of sound at membranes and films, and on ultrasonic delay lines. Finally, a number of minor additions were made, and also mistakes and imprecise statements in the previous edition were corrected.

Because of the large amount of material describing ultrasound waves and their application, the previous title of the book "Sound Waves" was changed to "Sound and Ultrasound Waves".

In the preparation of the manuscript of the third edition I received great help from L.K. Zarembo and K.V. Goncharov. Also, discussions with S.N. Rzhevkin, A.M. Obukhov, I.L. Fabelinskii, L.M. Lyamshev, and K.N. Baranskii were of great value. To all these I express my gratitude.

The Author

From the Preface to the First Edition

During the last decades acoustics, like other fields of physics, has developed rapidly. Due to the appearance of electronics and the rapid development of radio during these years, the content and nature of acoustics have changed fundamentally. New branches of acoustics were created and extensively developed—electroacoustics, architectural acoustics, acoustical measurements, atmospheric acoustics, ultrasonics with scientific and technical applications, molecular acoustics, hydroacoustics and underwater sonic location [sonar], various military applications of acoustics, etc. The treatment of these advances in acoustics by contemporary textbooks is quite inadequate, and acoustics is still presented in just the same way as it was nearly 50 years ago.

Therefore, not only secondary-school graduates, but also students of institutes of higher learning often have an insufficiently clear idea of the content and the nature of contemporary sound theory and of its role in science and technology. As far as popular-science texts are concerned, they are either out of date or deal only with specific technical acoustic questions.

The present book is not a textbook on acoustics; it endeavors to present in a simple form only some of the questions included in the inexhaustible material of contemporary acoustics, especially questions concerned with the propagation of sound waves in air, water, and solid bodies, and with basic applications of this.

Therefore, the reader will not find in this book a discussion of such topics as vibrations of strings, membranes, and plates, or musical and physiological acoustics. These questions have been, and are being, dealt with in physics courses and in popular books on sound, and, while they play an important role in contemporary acoustics (especially in electroacoustics), it was considered possible to omit them. Also, little space has been devoted to electroacoustics; basic information in this important and vast field of acoustics deals only with the solution of certain problems on sound propagation in different media. In the same way, such important technical applications of acoustics as sound recording and reproduction, telephony, broadcasting, etc., have not been considered. Electroacoustics and similar technical applications of acoustics still need a book devoted to them.

On the other hand, since radio and electronic equipment is all-important in contemporary acoustics, sufficient space has been given to discussion of the most important applications in acoustics of electronic oscillographs, audio oscillators, amplifiers, etc.

Sound waves are elastic waves traveling in gases, liquids, and solids, and which have been produced by vibrating bodies. To refresh the memory of the reader, the first chapter presents briefly the basic laws of vibration and wave motion; and the nature of wave motion is clarified by a study of water waves.

Subsequent chapters deal with the propagation of sound, infrasound, and ultrasound waves in gases and liquids, particularly in air and in water; and basic applications of these waves are also discussed. The last two chapters deal with the propagation in solids of elastic waves of different frequencies. Also included in the book is some basic information on general and applied seismology, primarily a discussion of the propagation of elastic waves in the crust of the earth. Seismic waves are elastic waves of large wave length (infrasonic waves) and in this sense seismology represents a branch of acoustics.

The author has endeavored, as far as possible, to show the reader the basic physical content of a number of acoustical problems, some already solved and some requiring solution in the future. Thus, it is hoped that the reader will not consider the results obtained in acoustics as unchangeable and firmly established. There are still many unsolved problems in acoustics, just as in any other field of physics; and many new problems arise due to practical requirements.

Chapter I

VIBRATIONS AND WAVES

In natural phenomena, in scientific experiment, and in technology, various forms of vibratory and wave motion are very often encountered. Among such movements are the well-known swinging of a clock pendulum, the vibration of a string, the movement of waves on the surface of water, the propagation of radio waves, and many others. Sound is also a wave motion. Sound waves arise and propagate not only in air and other gases but also in liquids and solids. To understand the properties of sound phenomena it is necessary to understand what vibrations and wave motion really are. It is therefore necessary to recall the basic properties and laws which characterize these two types of phenomena.

§ 1. Free vibrations

Oscillation of a Pendulum. Let us consider one of the best-known examples of vibratory motion—the swinging of a pendulum.

The simplest pendulum is a metal ball suspended on a thread. When the pendulum does not move, it is in the position of equilibrium 1 (Figure 1). If it is moved from the position of equilibrium to position 2 and then released, it begins to swing on the thread, performing periodic movements—oscillations. Such motion is called the *free* or *natural* oscillation of a pendulum.

The free oscillation of a pendulum may be explained as follows. The pendulum, brought into position 2, descends under the action of gravity, gaining speed progressively; when position 1 is reached by the pendulum, the speed will be a maximum. Although at this point gravity no longer accelerates the pendulum, it continues to move due to inertia, and it rises again. Now, however, gravity slows down the motion; and when position 3 is reached, the pendulum will stop. But the pendulum cannot remain in this position either. Under the action of gravity it begins to descend, and once again, due to inertia, overshoots the equilibrium position.

The oscillation of a pendulum can be explained in another way. In raising the pendulum to position 2 a certain amount of energy is communicated to it. This energy is called potential energy. When the pendulum descends under the action of gravity, its potential energy decreases; however, its speed, and therefore its energy of motion, or kinetic energy, increases. In position 1 the potential energy is a minimum, but the kinetic energy reaches a maximum; all the potential energy has been transformed into kinetic energy. On the contrary, as the pendulum continues toward point 3, the kinetic energy decreases and the potential energy increases. Finally,

at point 3 the potential energy is again a maximum and the kinetic energy becomes zero. Thus, during free oscillation of a pendulum, potential and kinetic energies are cyclically transformed into one another. The total energy of the pendulum, however, which is the sum of the potential and kinetic energies, remains constant.

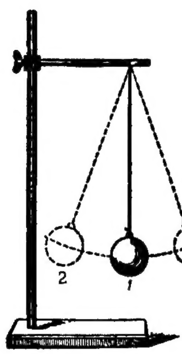


Figure 1. Positions of an oscillating pendulum

But it is easy to see that this description of pendulum motion is incomplete and approximate. In reality, a pendulum left to swing by itself must always stop at some time, since air resistance and friction at the point of suspension gradually reduce the energy of oscillation of the pendulum.

Quantities Determining Vibratory Motion. The example of the swinging pendulum makes it possible to define certain basic quantities which characterize all vibratory motion.

The distance the pendulum swings away from the equilibrium position 1—to the extreme left position 2 or extreme right position 3—is called the amplitude of oscillation.

The time during which the pendulum goes through one complete oscillation (e.g., from extreme left to extreme right and back again) is called the period of oscillation. The number of periods of oscillation per second is called the frequency of oscillation.

It is evident that the period T and the frequency f of any vibratory process are related by the equation:

$$f = \frac{1}{T}.$$

For instance, if the period is 1/50 sec, then in one second the process will be repeated 50 times, and the frequency of oscillation is 50 periods per second (this is the frequency of the [European] alternating current in a municipal power system).

One oscillation per second is taken as a unit of frequency; this unit is called a cycle per second.

On what factors do these basic quantities depend?

Experiment shows that the amplitude of free oscillation of a pendulum is determined by the energy communicated to it from outside or on the initial energy. But the period of a free, or natural, oscillation depends neither on the amplitude (provided swing is not too wide) nor on the weight of the ball, but only on the length of the thread*.

Oscillation of a Weight on a Spring. Let us consider another example of free oscillation—the vertical movement of a heavy ball suspended on a helical spring (Figure 2). If the ball is pulled downward and then released, it will move up and down, performing free oscillations. The mechanism of the motion is as follows. When the ball is pulled downward, the spring is extended; the spring opposes this tension and attempts to return to its original position. More precisely, the spring possesses elasticity, which

* The period of oscillation of the pendulum, for small amplitudes, is given by the formula $T = 2\pi\sqrt{\frac{l}{g}}$, where l is the length of the pendulum and g is the acceleration of gravity. If T and l are measured by observation then g can be determined. This method is still a basic one for determination of the acceleration due to gravity.

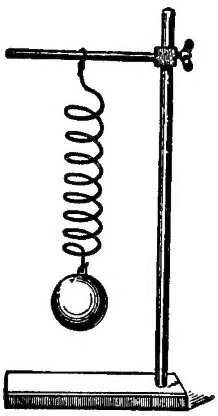


Figure 2. Weight on a spring

during tension (or compression) creates an elastic force tending to return the spring to the equilibrium state. If the ball is now released, the spring will compress, pulling the ball up with it. In the position of equilibrium the ball moves upward at its greatest speed. Therefore it cannot stop immediately, and due to inertia the ball passes the equilibrium position and begins to compress the spring. But the elastic force of the spring opposes the compression and the weight stops. Because of its elasticity the spring expands again, and the weight starts to move downward; because of inertia the ball again passes through the equilibrium position and oscillation continues.

Experiment shows that, in this case too, the amplitude of oscillation of the ball depends on the initial energy. But the frequency of free oscillation of the system depends only on the elasticity of the spring and the mass of the weight; the frequency of free oscillation is proportional to the square root of the ratio of the elasticity k of the spring to the mass m of the weight:

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

The larger the elasticity and the smaller the mass, the higher is the frequency of oscillation.

The oscillation of the ball and spring would continue perpetually, if there were no friction between the ball and the air, and no friction in the spring suspension and in the spring itself. In reality, after a certain time, oscillation stops.

Thus, free oscillation of a heavy ball on a spring is a result of the elasticity of the spring and the inertia of the ball.

A general conclusion may now be stated. In order for a body, displaced from its equilibrium position, to oscillate, it is necessary, first, for the body to possess inertia, and second, for a force to exist which tends to return the body to its original position. During oscillations of a ball on a spring this force is the force of elasticity; for oscillations of a pendulum, it is the force of gravity.

To ascertain completely the character of oscillatory motion, it is important to represent the progress of the oscillation in time. There exist many methods which make possible a visual picture of the oscillation.

Recording of Oscillation. Figure 3 shows how pendulum oscillations may be recorded. A thin stream of sand pours from an opening in a suspended cone, and this sand sticks to a moist strip of paper. If this strip is pulled uniformly in the horizontal direction, then, during oscillations of the "sand" pendulum, a curve representing the motion is "inscribed" on the strip.

Thus, as time passes, the oscillation unfolds itself on the paper strip. Such a recording provides much information on the progress of the oscillatory process. The curve drawn permits evaluation of the form of oscillation, the amplitude, and (if the speed of motion of the paper is known) determination of the period of oscillation.

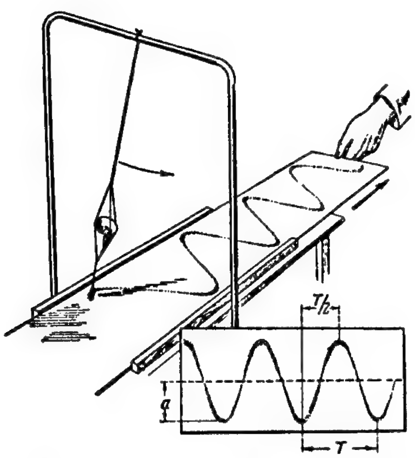


Figure 3. Recording of pendulum oscillations

T–period of oscillation, a–amplitude of oscillation

The curve in Figure 3, which represents small oscillations of the pendulum, is a sine curve. If the oscillations of a weight on a spring are recorded, a sine curve will again be obtained for small amplitudes of oscillation. Sinusoidal oscillation is the simplest type of oscillation; it is also called *harmonic* oscillation.

Oscillations are harmonic only if the restoring force is proportional to the deviation of the oscillating body from its equilibrium position. If the amplitude of oscillation of a pendulum or a weight on a spring is large, this proportionality will be disturbed and the oscillation is not sinusoidal. Later, when sound and ultrasound vibrations and waves are discussed, more will be said about sinusoidal, or harmonic, oscillation, as well as about oscillation which is not sinusoidal.

The Phase of Oscillation. Phase Displacement. Beside the above quantities describing oscillatory motion — amplitude, frequency, period of

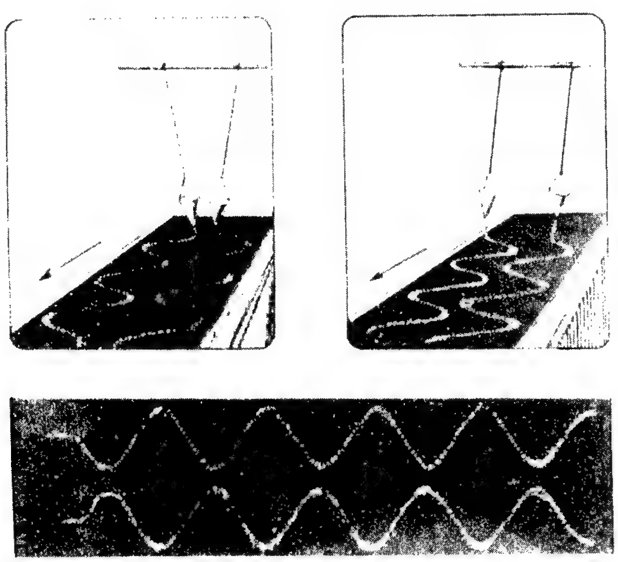


Figure 4. Phase displacement between the oscillations of two equal "sand" pendulums

On the left, the oscillation is in antiphase; on the right the oscillation is in phase; below, a recording of the antiphase oscillation

oscillation—the phase of the oscillation and the phase displacement are also of importance.

Let us compare two equal "sand pendulums (Figure 4). If the pendulums are deflected equally from their equilibrium positions, but one of them is released first and, after a short time, the second is released, then the state, or phase, of their oscillation will be different. At the moment when the first pendulum passes through the equilibrium point the second pendulum may be, for instance, at its maximum deviation, or in any other position. If the oscillation of both pendulums is recorded, two equal sine curves are obtained, but these curves are displaced with respect to each other by a certain fixed portion of the oscillation period. If the pendulums are deflected by the same amount and released simultaneously, the sine waves will correspond to each other (Figure 4, right). But if one pendulum is deflected to the right and the other to the left and they are then released simultaneously, the sine curves will be shifted by half a period (Figure 4, left). The same relative displacement will be obtained in the case when the two pendulums are deflected in the same direction, but where the second pendulum is released when the first has reached the opposite position.

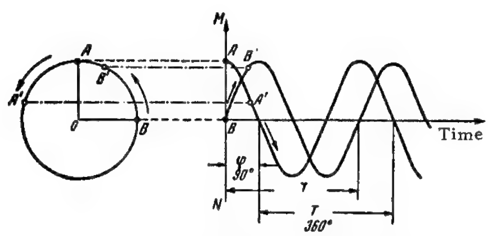


Figure 5. The phase displacement in time between two sinusoidal oscillations may be expressed as an angle

The displacement of one sine curve with respect to another is called the phase displacement. In order to obtain an exact measure of the phase or the phase displacement, let us consider two small balls, A and B, rotating uniformly and with equal speed around the point O (Figure 5). Note how the projections of the positions of the balls onto the vertical plane MN vary with time. If a screen is placed in this plane, and if the rotating balls are lighted from the left (along the plane of rotation), then it is seen that the shadows of the balls on this screen form periodic motion. We may speak about the oscillation of these shadows; the period T of oscillation is the time required for the ball to make one complete rotation. If these oscillations are recorded by moving the screen MN (made of photographic paper) at a uniform speed, two sine curves will be obtained. The amplitude will be equal to the radius of the circle of rotation; and the relative displacement of the curves will be defined by the angle $\varphi = AOB = A'OB'$, since the distance between the balls does not change and they move with a constant speed. Thus, the time-change of the projections of balls A and B onto plane MN is obtained, as shown in Figure 5.

The position of each ball on the circle depends on the time which has passed since the beginning of rotation. This position can be determined by the angles α and β through which the balls have rotated during this time

interval. During a full rotation angles α and β change by 360°, during half a rotation by 180°, and so on.

Thus, the period of oscillation of the shadow of the ball corresponds to one complete rotation of the ball around the circle, or 360°. The constant relative displacement in time of the sine curves in Figure 5, the angle φ—the phase displacement—is a measure of how much the sinusoidal movement of the first ball leads or lags the sinusoidal movement of the second ball.

If the phase displacement is zero, then the curves in Figure 5 merge into one, and the oscillations are said to be in phase. If this angle is not zero, the oscillations are out of phase. The oscillations of the two pendulums in Figure 4 (left) are in antiphase. As one pendulum moves to the left, the other moves to the right.

The concept of phase displacement (or phase difference) refers to two or more oscillations. If only one oscillation is involved, however, it may also be characterized by phase. The phase of oscillation in this case is the state of motion of an oscillating body with respect to a certain reference position, for instance, the equilibrium position of the body. The phase may be measured from this position, and then any other position of the body has a definite phase, with respect to the reference position.

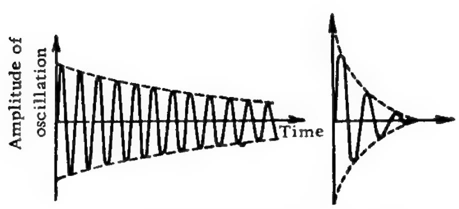


Figure 6. Damped natural oscillations

On the left–light damping, on the right–heavy damping

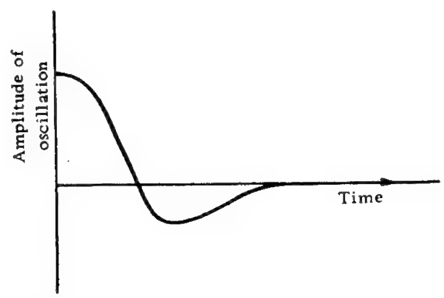


Figure 7. Damped natural oscillations in a system with heavy friction

Damped Oscillations. Free oscillations of a pendulum or a weight on a spring cannot continue perpetually. Because of friction, the energy imparted to a pendulum or a weight on a spring, as a result of deflection from the equilibrium, decreases gradually. The amplitude of oscillation diminishes and oscillation stops. Such oscillations are called damped.

On the left in Figure 6 is shown a graph of a lightly damped oscillation. If damped and undamped oscillation are compared, it is seen that damped oscillation is not strictly periodic. Indeed, during such oscillation the states of motion do not repeat themselves exactly, as they do in undamped oscillation. For instance, the maximum deflection of the pendulum does not reoccur, because the amplitude of oscillation decreases with time. Nevertheless, the same basic notions which were introduced for undamped oscillation are used for damped oscillation, since the latter has much in common with the undamped variety, particularly if damping is slight.

The larger the friction, the stronger is the damping in the system. For very high friction the body will not oscillate at all, but will perform motion which is not periodic. If the pendulum is placed in a vessel containing liquid (for instance, water or oil) and it is given a push, then, due to the

high friction between the pendulum and the liquid, the motion will differ from that occurring when the same push is given to the same pendulum in air. Depending on the force of the push, the pendulum will either return gradually to equilibrium position or will overshoot it by an insignificant amount and then stop (Figure 7). This is a case of damped oscillation — the oscillations of the pendulum stop very rapidly. Damped oscillation is used in many instruments, for instance, when an instrument pointer, attached to a hairspring, should not oscillate when power is applied, but should show a constant deviation. An oscillating body expends power not only on friction, but also in causing the surrounding medium (for instance, air) to oscillate. The body creates in the medium elastic waves; this will be considered later in detail.

§ 2. Forced oscillation. Resonance. Self-oscillation

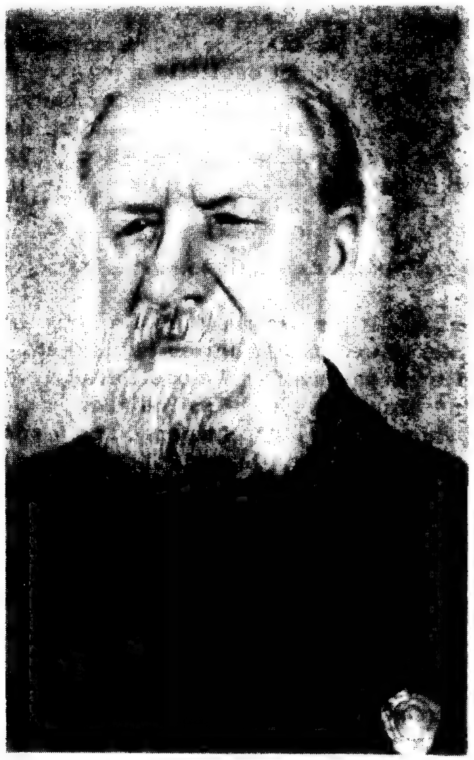
Forced Oscillation. If the oscillations of a body are not sustained, they stop after some time. For the oscillation to continue indefinitely (that is, to be undamped), the energy lost by the oscillating body must be replenished. This can be done in different ways. For instance, a periodically varying force may act somehow on a weight suspended on a spring. This force may be supplied by an electromagnet, through the coil of which alternating current flows. If the weight is made of iron, the electromagnet will attract the weight periodically; under the influence of this force the weight will perform undamped oscillation. This oscillation has the same frequency as the applied driving force, here the attraction of the electromagnet. This is not free oscillation, because a periodic external force acts on the oscillating body. The nature of the oscillation in such a system is determined not only by the properties of the system itself, but also, to a large extent, by the external force.

Oscillation occurring under the influence of external forces which act independently of the oscillations in the system itself is called *forced oscillation*.

Resonance. The exact way in which the external force acts on the oscillating body is very important. Let us consider a weight on a spring, pushed slightly several times. The system has its own rhythm, or natural period of oscillation, defined by the mass of the weight and the elasticity of the spring. If the time intervals between pushes are equal to the natural period of oscillation of the weight, then, after only a few pushes, the amplitude of oscillation increases noticeably. But if the time interval between pushes differs from the natural period, then the influence of the force upon the weight will be smaller, because, part of the time, the external force opposes the motion, rather than assists it. Anyone who tries to walk across a long board thrown across a brook can observe such a phenomenon. If someone marches along the elastic board in rhythm with its natural period, the shaking of the board becomes so great that it is dangerous to continue. But if steps are taken at some other time interval, the board will not vibrate strongly.

There are many similar examples. They all refer to cases of forced oscillation in which the frequency of forced oscillation equals the natural frequency of the system; this type of forced oscillation is called

resonance. The phenomenon of resonance is very important in nature and in technology. Very often it can lead to unexpected consequences, if its occurrence is not forseen. The great Russian mechanician and mathematician A.N. Krylov gave the following example of the appearance of resonance.



ALEKSEI NIKOLAEVICH KRYLOV
(1863-1945)

"It was, I think, during the Napoleonic Wars in Spain. An army detachment was crossing a bridge, with the soldiers marching all in step (probably an important officer was standing on or behind the bridge). It was a chain bridge, and the powerful marching cadence happened to be equal to the natural frequency of the bridge. The swaying increased so much that the chains broke and the bridge tumbled into the river. After this occurrence a rule was made for all armies, not to cross bridges "in step". But only thirty years ago, in the Petersburg of that time, there was a chain bridge, called the Egyptian bridge, crossing the Fontanka. Once it was being crossed by a company of cavalry, I do not remember of which regiment. The horses were well trained to march ceremonially and to walk in precise step; and this coincided with the natural frequency of oscillation of the bridge — the chains broke, the bridge fell into the water, and nearly 40 people perished. The phenomenon of resonance thus received additional confirmation".

Harmful effects of resonance are encountered, for instance, when unbalanced machines are operated on a base provided with insufficient shock-absorption. If the frequency of oscillation transmitted by the machine to the foundation of the building is equal to the natural frequency of oscillation of some part of the building (especially of the upper floors and ceilings), then prolonged operation of the machine leads to quite high amplitudes of oscillation. In some cases the result is a progressive destruction of the building; it sinks, and cracks appear. Many more examples can be given, in which breaking of machine crankshafts, ship screws, or airplane propellers was caused by resonance effects.

But the role of resonance in technology is not only a harmful one. The principle of resonance is made use of in numerous mechanical and electrical instruments. It will be shown later how resonance is utilized in acoustics.

The Role of Damping. If a body oscillates in the presence of very light damping, then an external force with a period equal to the natural period of oscillation of the body will, after some time, produce very large amplitudes of oscillation. The prolonged action of even a very small periodic force can thus cause a body of large mass to swing with a large amplitude. But all oscillations in nature and in technology are damped by friction. Therefore, if the pushes only replace the energy losses due to friction over one oscillation period, the amplitude of oscillation remains constant. The greater the friction, the heavier is the damping of the oscillation, and the stronger must be the pushes required for oscillation at a given amplitude.

The Resonance Curve. The Establishment of Oscillation. Let us plot on the horizontal axis of a graph the frequency of the external driving force and on the vertical axis the amplitude of oscillation of a system, e.g., a pendulum. This curve is called the resonance curve (Figure 8). When the frequency of the pushes is equal to the natural frequency of the pendulum oscillation, the amplitude will be a maximum; this frequency is called the resonance frequency. If the driving-force frequency decreases or increases, the amplitude of oscillation decreases. The quickness with which the resonance curve drops off on both sides of resonance frequency depends on the amount of damping in the oscillating system. If damping is low, the resonance curve is "sharp" (curve a), while for heavy damping the curve becomes flat (curve b).

When an external periodic force is applied to an oscillatory system (e.g., to a pendulum), this system does not swing immediately with a constant amplitude determined by the applied force and the damping of the system; the establishment of oscillation is gradual. The time for establishment of oscillation depends on the form of the resonance curve. The sharper the curve, the smaller is the damping of the system and the more time is needed to bring it into full swing; the system vibrates for a long time before the amplitude of oscillation reaches a uniform value. On the contrary, if the damping is increased, the time for establishment is shorter.

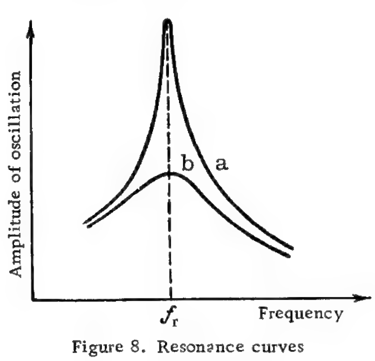


Figure 8. Resonance curves

Figure 9 shows the establishment of oscillation in a case when the external force has the same frequency as the natural frequency of the system, but acts in opposite phase.

Self-Oscillation. Let us consider whether free and forced oscillations represent all possible types of oscillation. For instance, to which type do the oscillations of a clock pendulum belong? They are clearly not free. Actually, the oscillations in a clock are not damped; the consumed energy is replenished by the elasticity of the coiled spring, and the pendulum swings for a long time at constant amplitude. But these oscillations cannot be called forced, because forced oscillations are produced by an outside force independent of the oscillation of the system itself.

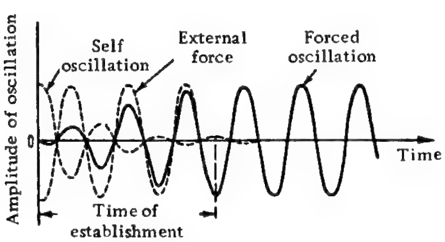


Figure 9. Establishment of forced oscillation

But in a clock the pendulum itself turns on and off the supply of energy from the coiled spring or from the raised weight. During the greater part of the period the pendulum moves freely, and only when it passes through equilibrium, at which point it has its greatest speed, does it make contact with the ratchet-wheel. The elastic force of the spring, or the gravitational force of the weight, acts upon this wheel through a gear system; and when meshing occurs the pendulum receives a short push. These pushes are very weak, but, since they are in phase with the free oscillations of the pendulum, the energy which they impart is sufficient to replace the energy lost to friction; and oscillations of constant amplitude are sustained. Thus, undamped oscillations of a clock pendulum are established possessing an amplitude whereby energy losses due to friction are replaced exactly by the potential energy of the coiled spring or the raised weight. The energy transfer is controlled by a mechanism which is a part of the system itself, and the source of energy which sustains oscillation is an integral part of the system. Such systems are called self-oscillating, and the undamped oscillation performed by them self-oscillation.

Self-oscillation is very widely used in technology, especially in radio applications. Apart from clocks, examples of self-oscillating systems are: an electric bell, an electronic generator of undamped electric oscillations, and many others. It will be seen later that an organ pipe and the apparatus of human speech are also self-oscillating systems. Thus, in a self-oscillating system the amplitude as well as the frequency of oscillation are determined by the properties of the system itself, while for forced oscillations the nature of the oscillation depends to a large degree on the properties of the external periodic force.

In the following discussion these basic and very simple facts about oscillations will be useful. The study of oscillations is at present developing very rapidly. In every field of technology and in physics oscillation is encountered to a greater or less degree. The laws of various oscillatory motions are widely used by mechanical engineers — in the construction of various machines and mechanisms, by ship builders, and by aircraft designers, in constructing new types of aircraft and engines. The study

of alternating current and radio are completely based on oscillatory processes. Scientists in geophysics, optics, mechanics, acoustics, atomic physics, and seismology constantly deal with oscillations. Even an architect who designs apparently stable, immovable buildings and bridges cannot dispense with applications of the basic laws of oscillation theory; at any rate he must take account of them.

§ 3. Wave motion. Water waves

Wave motion. In the preceding sections the oscillation of a single body has been discussed —a pendulum or a suspended weight. A single, isolated pendulum performs only natural oscillations, completely determined by its inherent properties. It is a different matter if oscillation is produced in a system of several pendulums, connected elastically to each other. In a system of coupled pendulums the oscillation of every pendulum depends on the motion of the others, and the nature of this oscillation differs basically from the nature of the oscillation of a single pendulum.

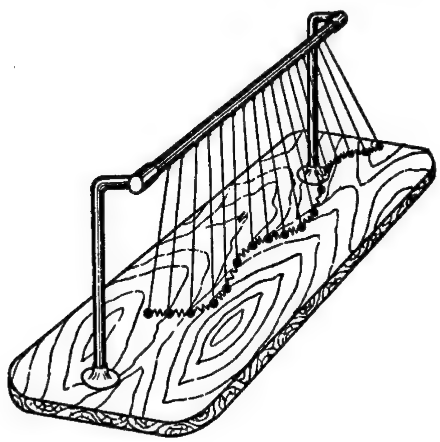


Figure 10. Pendulums coupled by springs—an example of wave motion

Consider the oscillation of several pendulums coupled by equal springs (Figure 10). If the ball at the end of the row is deflected to the side, it pulls with it the second ball, which in turn pulls the third, and so on. When the end ball is released it will begin to oscillate and, since it is connected to the other balls, will cause all of them to move. However, the oscillatory motion is not transmitted instantly from one ball to another, but with some delay. Because of inertia, each successive ball begins to move only gradually, increases its speed, and then moves the next ball out of its equilibrium position. Thus, the oscillatory motion is communicated from one ball to the other at a certain speed, which depends on the elasticity of the springs and the inertia of the balls. More precisely, the propagation speed of an oscillatory motion is higher for greater elasticity of the springs and for smaller mass of the balls. This type of oscillatory motion is called wave motion.

In reality, isolated oscillation of a single body does not exist. Whatever the medium in which the oscillating body is situated — air, water, or solid body — its oscillations will not remain isolated. Every one of these media has its own elastic properties; in each of them, because of the interaction between its particles (which behave as if connected by "springs"), an oscillation starting in one part is transmitted to other parts of the medium. Waves propagate through the medium.

A study of waves on the surface of water is the simplest and clearest way to become acquainted with the nature of oscillatory motion*.

The movement of well-defined waves as they run up onto the shore, or of waves produced by a ship moving on quiet water, gives the impression that a wave is a mass of water moving forward. However, this is not the case. A person swimming in the sea during windy weather, or in a boat among the waves, is passed by the waves, not carried with them**. First the rider rises on the wave, then the wave pushes him forward; then it lowers him and finally pushes him back. The study of wave propagation on a water surface shows that, when a wave moves, the water particles perform a nearly circular movement, with the plane of the circle in the direction of propagation of the wave (Figure 11).

It is interesting to watch a family of ducks swimming in single file in water ruffled by waves, with every duckling keeping a definite distance from the next one. If the direction of the "column" lies in the direction of wave propagation, then, by careful observation, it may be seen that the ducks do not move far away from their original positions, but just perform comparatively small displacements, rising and descending. In addition, they do not rise and descend all together, but rather one after the other.

Figure 11. Particle trajectories in waves on the surface of water

These examples make it evident that a wave is not a forward motion of a mass of water. The displacements of the water particles in the waves are comparatively small. These displacements are periodic, and they are not produced all at the same time but in a certain order; each particle follows its neighboring one in performing the motion. The succession of such motions forms what is called wave motion. Therefore, wave motion is motion which begins at some point in the medium, and propagates from point to point, while the medium as a whole remains stationary.

The Formation of Waves. It has been observed that when a system consisting of coupled pendulums is disturbed, wave motion is produced through the action of the elastic springs and the inertia of the balls. A disturbance on a water surface produces waves through the action of gravitation and inertia. Gravitation here plays the role which elastic forces play during oscillations of a weight on a spring. This force causes the water to

* When an analogy between water waves and sound waves is made, the differences between these two kinds of wave motion are often not made sufficiently clear. During the following discussion particular attention will be paid to these differences.

** The situation is different near the shore, where the waves break as surf, but this is a case of destruction of waves, not of their free propagation.

resist every attempt to change its horizontality; this is why these waves are called gravitational waves on a water surface. If a stone is thrown into water, it sinks and creates a hollow in the surface, which begins to fill up immediately with the water flowing into it from all sides. Like the weight on the spring, which does not stop at its equilibrium position, but bypasses it because of its inertia, the water, having filled up the hollow, continues to move due to inertia. As a result, where there was previously a hollow, the water gathers and forms a crest of water; this crest subsides and again a hollow is formed, which fills up again with water; thus, circular waves begin to spread from the spot into which the stone was thrown.

Thus, if the horizontality of the surface of a liquid is disturbed, waves are produced. Sea waves and waves on rivers and lakes are produced by the wind. Why the wind originally disturbs the horizontality of a water surface is not yet clear; but it is easy to understand why the wind reinforces slight waves which already exist. In this case the moving air exercises a pressure upon the inclined surface of the wave, as upon a sail, and moves it forward, driving water away from some places and gathering it together in others. In any case it has been established that, if two layers of liquid or air of different densities, or a layer of air and a layer of liquid, move with respect to each other, then the more quickly moving layer always produces waves in the second one.

Figure 12. Imprints of waves on a beach during low tide

Phenomena of this kind take place during the formation of "wave imprints". Figure 12 shows a photograph of such imprints on a beach during low tide. Their appearance is explained in the following way. When the sand is covered by water, its upper layer is saturated and behaves like a liquid. The water, moving during low tide with greater speed than the "liquid" sand layer, causes waves in the sand, which remain on it as imprints. Figure 13 shows a photograph of waves produced by the action of the wind upon a sand surface. Such waves originate in the air. If two layers of air of different density move past each other at different speeds, waves are produced. Wavy clouds, which can be observed rather often, are formed for just this reason.

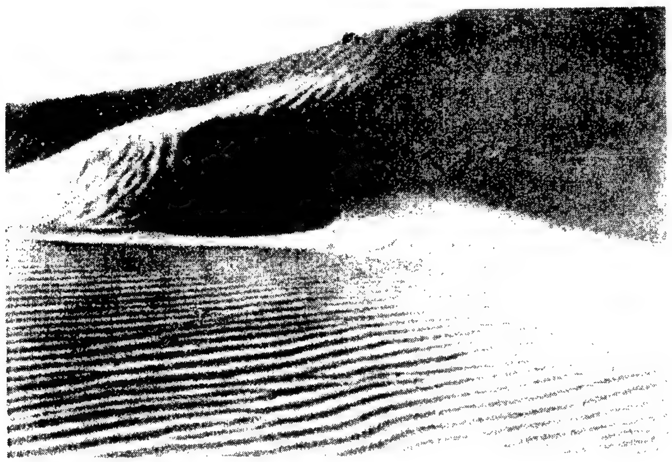


Figure 13. Waves on a sand surface, produced by the wind (Kara Kum)

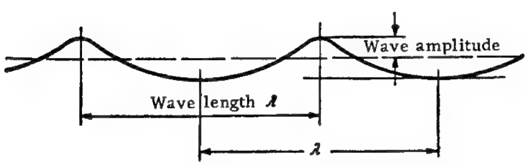


Figure 14. Wave length and wave amplitude

Wave Length. Now let us establish those quantities which determine completely a wave and wave motion in general. One such basic quantity is the wave length. The wave length is the distance between two consecutive crests or troughs (Figure 14) or, generally, between any two points of a wave which are mutually in phase. The length of waves on rivers and lakes during a brisk wind is several meters; in the shoreless expanse of an ocean the length of waves can reach several hundreds of meters.

Wave Amplitude. The amplitude of a wave is the distance over which a particle of the medium is deflected from its equilibrium position, that is, half the crest-to-trough height of a wave (see Figure 14). Sea waves during a strong wind may reach a height of 15-20 m; the amplitude of such waves is therefore 7.5-10 m.

We have already pointed out that the water particles move in circular paths during the passage of a wave; it is found that the amplitude of displacement of the particles from their equilibrium positions decreases rapidly with depth. At a depth of one wave length the amplitude has already decreased to 1/535 of that at the surface. Even when huge waves move across a stormy sea, the water a few tens of meters below the surface is almost at complete rest, a fact well-known to submarine crews.

If the number of waves which pass during a definite time interval is counted (this is easily done by observing the number of times some floating object rises (or descends)), the period of the wave and its frequency may

be determined, where the frequency is the reciprocal of the period. The period of long waves can be up to 10-15 sec; therefore, their frequency is 0.1-0.06 cps.

Velocity of Propagation. It may be observed that waves move forward with a definite velocity. But it should be remembered that the velocity of separate particles in a wave and the velocity of the wave motion itself are completely different concepts, and the two should not be confused.

In all types of wave motion there is a simple relation between the velocity of wave propagation, the wave length, and the frequency (or period). If the velocity of the wave motion is denoted by c, the wave length by λ, the frequency by f, and the period by T, then this relation is $c = \lambda f$ or, in terms of the period, $c = \frac{\lambda}{T}$. Since during a single period T the wave travels a distance equal to one wave length λ, the significance of the latter equation is clear. These formulas will be used constantly in the following.

In deep water, where the depth is equal to several wave lengths, the velocity of a wave is greater for longer waves. There is a simple rule for determination of the velocity of the wave*:

$$c \text{ (kilometers per hour)} = 4.5 \sqrt{\lambda \text{ (meters)}}$$

Thus, the velocity of waves in deep water depends on their wave length; this phenomenon is called d i s p e r s i o n. In contrast to water waves, sound waves of different lengths (or frequencies) propagate through air with the same velocity, that is, without dispersion.

Let us calculate the velocity of waves in a river, if their length is 4 m. The above rule gives

$$c = 4.5 \sqrt{4} = 9 \text{ km/hour}.$$

For sea waves 25 m in length

$$c = 4.5 \sqrt{25} = 22.5 \text{ km/hour}$$

a velocity which is rather large.

Thus, the velocity of a wave is greater, the longer its wave length. In addition, waves of the same length move more slowly in shallow water than in deep water. This partly explains the fact that, when approaching the shore, a wave becomes steeper, forms "white-caps" and, finally, will break upon the shore.

In approaching a shallow spot, the upper part of the wave, which moves over deeper water, moves more quickly than the lower part. Moreover, the lower part of the wave undergoes friction at the sea bottom, and the backward motion of the particles (since particles move in the wave around vertically oriented circles) is braked. The formation of "white caps" and the breaking of waves are also frequently observed in large rivers, near banks and shoals.

* The theory of deep-water waves gives the following formula for the velocity of wave propagation: $c = \sqrt{\frac{g\lambda}{2\pi}}$, where g is the acceleration of gravity. From this formula it is easy to obtain the given rule. Long waves, with a length greater then the depth h, move across a flat bottom with a velocity determined by the formula $c = \sqrt{gh}$.

§ 4. Propagation of waves

Refraction. When waves approach a gently sloping shore, they move in swells, oriented parallel to the shore. Even if, at a distance away from land, their orientation was oblique with respect to the shoreline, the waves turn toward the shore. This is an example of wave refraction, which takes place during the propagation of waves of all kinds.

Refraction arises when waves, moving with a certain velocity in a given medium, pass into a medium in which their velocity is different. This is the case when waves approach a shallow shore, since in deep water their velocity is greater than in shallow water.

Let the velocity of the wave be 10 km/hour up to line aa' in Figure 15; and behind this line let the velocity drop to 9 km/hour, because of the decreasing depth. The left part of the wave will reach line aa' first and will continue at a velocity of 9 km/hour, while the right part still moves with the higher velocity of 10 km/hour. In this way the wave must turn somewhat toward the shore; while the left part passes over 9 m, the right part passes over 10 m. When the whole wave has passed line aa', its motion will be directed more toward the shore. As they gradually approach the shoreline, the waves will move in continually shallower water and will turn more and more toward the shore; and if the shore is sufficiently shallow, they will reach it as parallel swells. Figure 16 shows a photograph of surf breaking on the shore, where this phenomenon is evident.

If the velocity of the wave in one part of the medium (e. g. up to line aa' in Figure 15) is equal to c_1, and in another part of the medium (beyond line aa') is equal to c_2, then the ratio $n = \frac{c_1}{c_2}$ is called the refractive index.

Capillary Waves. Reflection. So far waves have been discussed which have had unrestricted freedom of propagation. It is known that when a wave encounters an obstacle it is reflected by it like a thrown ball rebounding from a wall. The phenomenon of wave reflection can often be observed at a steep rocky coast, or at a dock near a passing ship.

In the study of wave propagation a "wave bath" is often used. This is a vessel like a dinner-plate, which has gently sloping sides in order to avoid visible reflections of arriving waves. A small ball is submerged several millimeters into the bath; and a small motor with a cam arrangement causes vertical vibrations of the ball. The frequency of vibration is arbitrary; normally a. c. power from the municipal system is used, which has a frequency [in Europe] of 50 cycles.

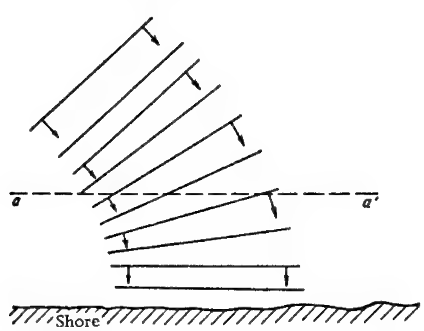


Figure 15. Diffraction of waves approaching a shallow shore

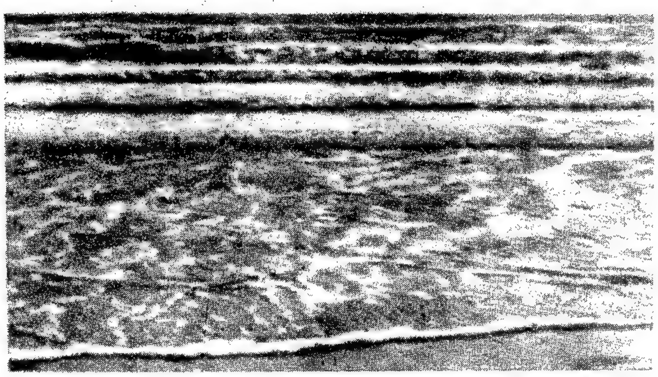


Figure 16. Surf on a shallow shore

The glass bottom of the plate is lighted from below, and the image of the waves on the surface of the plate is projected onto a large screen. In order to obtain on the screen a stationary image of the water, the light source is exposed, only at certain moments, through a slit in a disk which rotates with the frequency of vibration of the ball. (This method of obtaining a stationary image from a moving object, with the help of intermittent illumination, is called the stroboscopic method.)

The waves produced in the wave bath by the vibrations of the ball are only between several millimeters and several centimeters long. These small waves, or ripples, which are often observed during sudden gusts of wind or when rain drops fall on a smooth river, lake, or pool, have a cause different from that of the large waves which are produced by gravity and inertia. Ripples, small waves with a length 1.75 cm or less, are a result of the surface tension of the liquid and of inertia.

Almost everyone has seen light insects skating about freely upon a quiet, smooth pond; it is also known that if a needle is carefully placed on a plate of water, it does not sink, but remains on the surface. These, at first sight strange, phenomena are explained by the fact that water and all other liquids possess surface tension. The liquid surface can be compared to a film of soap stretched over a ring of wire; it always tends to shrink, and if extended by some means it opposes this extension.

If a rain drop, which has small weight and small size, falls on water, then, contrary to a stone, it only dimples the surface slightly, which then draws out flat because of surface tension. This motion does not stop when the original equilibrium position is reached. Because of inertia this position will be passed by, and as a result the falling drop produces on the water surface ripples, or capillary waves.

In addition to surface tension, the force of gravity plays a role, too; for waves of short length, however, the chief effect is that of surface tension. For longer waves gravity becomes more important, and for very

long waves the forces of surface tension can be neglected. The transition between these two types of waves — capillary and gravitational — takes place at a wave length of 1.75 cm. Waves of this length move along a water surface with a velocity of 23.5 cm/sec.

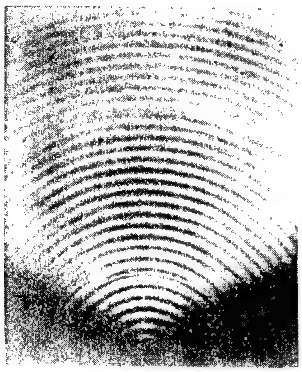


Figure 17. Circular capillary waves on a water surface, produced by the vibrations of a ball

Figure 18. Reflection of plane waves from a plate, produced by the vibrations of a small rod

It is interesting to note that, while for long waves the velocity of propagation increases with wave length, the opposite is true for ripples — the shorter the wave length, the greater the velocity.

Thus, a small ball vibrating in a wave bath can be considered to be a radiator of capillary waves or ripples. Figure 17 shows a photograph of circular capillary waves, produced by vi brations of the ball.

When they encounter an obstacle, the waves are reflected and again travel out in circles. The point from which the reflected waves seem to radiate is on the opposite side of the obstacle, at the same distance from it as the vibrating ball. In this case the reflected waves seem to originate at an imaginary source, just as in a mirror an image seems to be the same distance behind the mirror as an object is in front of it.

If a small rod is vibrated instead of a ball, then plane waves are produced. If they fall upon a plate at some angle, the fundamental law of wave reflection is observed: the angle of incidence is equal to the angle of reflection (Figure 18).

Figure 19. Interference of waves

Two balls, mounted on the armature of an electromagnet, constitute the source

For circular waves, the curves connecting the points in the medium at which the moving particles are in phase (e. g., crests or troughs of the wave) are also circular. The curve over which the phase is constant at a certain moment is called the wave front. For circular waves the fronts are circular; for plane waves they are plane-parallel.

Interference. If, instead of one ball, two balls vibrate at two different points in the bath, then circular waves originate at each ball. A study of the concentric waves, or an instantaneous photograph of the wave pattern, shows that the two wave systems move independently of one another. The same phenomenon is observed if two or more stones are thrown onto the smooth surface of a river or pond. The independence of the wave propagation from different sources is a very important property of wave motion in general — the resultant wave at any point of the medium is a simple sum of the waves arriving there from different sources. This property is called the principle of superposition of waves.

If two groups of waves, produced by vibrating balls, reach a certain point on the surface in such a way that their crests or troughs coincide at this point, then the deviation of the water particles from equilibrium will increase. This is the case when the difference in the distances traveled by the waves from their respective sources, the path difference, is equal to an integral number of wave lengths; that is, if the waves arrive at the point in phase.

If a wave crest from one source and a wave trough from the other source arrive at the same point, which is the case when the path difference is equal to an odd number of half wave lengths, then the waves will cancel each other.

Figure 19 is a photograph of this phenomenon. This type of reinforcement or cancellation of wave motion is called interference. Although the waves themselves move, the interference pattern as a whole remains stationary, since the path difference to a given point does not change with time and is defined only by its position and by the wave length.

Interference is not always observed. It can take place only if both sources produce waves of equal frequency or, more precisely, if the frequencies of the two sources may be expressed as a ratio of integers (1:1, 1:2, 2:3, etc), and if the phase difference between the two wave trains is constant*. Such sources are called coherent. Only in this case will waves having a definite, constant phase difference arrive at every point of the medium. If the phase difference between waves from two sources varies, that is, if at a certain moment one wave has a crest at the point and the other has a trough, and at another moment both waves have crests, then the positions of the maxima and minima of the disturbances on the surface will change; and no stationary amplitude distribution will exist. The photograph in Figure 19 was obtained by mounting two balls on the same armature of an electromagnet.

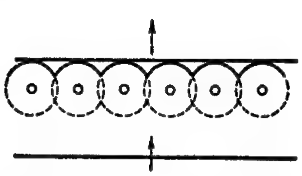


Figure 20. Passage of a plane wave through a row of nails

Huygens' Principle. If a straight row of nails (with small separations between the nails) is driven into a wooden tray containing water and then a plane wave is sent through the row, each of the

* Or, at any rate, changes slowly. In such a case, while the interference pattern will not be stationary, it is still possible to observe it.

nails appears to be a source of new waves (Figure 20). The envelope of these waves will coincide with the wave front after its passage through the row of nails. This example is a crude illustration of the principle of Huygens, which states that every point in a medium which is reached by a wave becomes a source of secondary waves; the envelope of these waves is the front of the actual propagating wave.

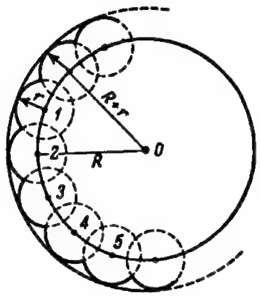


Figure 21. Construction of the front of a circular wave (according to Huygens' principle)

For instance, let us use Huygens' principle to construct a circular wave front, produced by the vibrations of a ball in a wave bath. Let the wave reach, at a certain moment, water particles 1, 2, 3, ..., at a distance R from the source O of the waves (Figure 21). According to Huygens' principle all these particles will become sources of circular waves from this moment on; and when the wave from O simultaneously reaches them, they will begin to vibrate in phase. After a certain time the secondary waves will have traveled a distance r and will

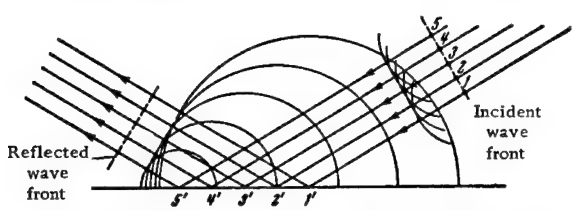


Figure 22. Reflection of a plane wave from a plane obstacle (according to Huygens' principle)

have an envelope coinciding with the front of the actual propagating wave, at a distance $R + r$ from the vibrating ball; other regions of the secondary waves are cancelled due to interference.

Huygens' principle explains simply the law of wave reflection from a plane obstacle. Let a plane wave fall on such an obstacle (Figure 22).

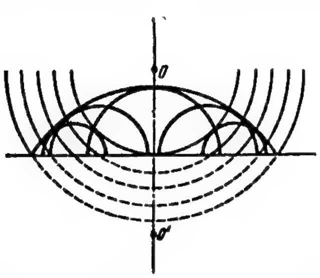


Figure 23. Reflection of a circular wave from a plane obstacle (according to Huygens' principle)

According to Huygens' principle any plane wave can be considered to consist of a very large number of circular waves, originating at points near to each other and situated along a straight line, coinciding with the wave front. Every point 1, 2, 3, ... represents a source of simultaneously originating waves. If there are many such sources, and they are near to each other, then their envelope will be a plane wave. When it encounters a plane obstacle oriented at an angle to the wave front, the wave is reflected. If at the moment t_1 the wave from point 1 reaches the obstacle at point 1', then according to Huygens' principle this point becomes a source and begins to emit secondary waves. At moment t_2 the wave from point 2 reaches the obstacle at point 2' and also

becomes a wave source. Finally, at moment t_5, the wave from point 5 will reach the obstacle. The secondary waves with centers at 1', 2', 3', 4', and 5' will propagate over a distance corresponding to a time from $t_1 + \tau$ to $t_5 + \tau$, and their envelope represents the front of the reflected wave. It is easy to see from this construction that the angle of incidence of the wave is equal to the angle of reflection.

In the same way it is easy to construct the reflection of a circular wave from a plane obstacle (Figure 23).

Diffraction. During the observation of waves on quiet water it is seen that waves which meet various obstacles in their paths surround them, and enter the region of geometrical shadow. Thus, waves from a boat moving through quiet waters enter creeks and branches of a river as an irregularly shaped shore is approached.

This bending of waves is called wave diffraction; diffraction, along with interference, is characteristic of all wave motion.

With the help of a wave bath it is easy to observe the basic properties of the diffraction of water waves. Figure 24 is a photograph of the passage of plane waves through an aperture in a wall, when the dimensions of the aperture are larger than the wave length. In Figure 25 is shown an obstacle instead of an opening, but which is the same size. In the first case the screen marks off a band of plane waves, of the same width as the width of the aperture; in the second case, a band of shadow (of the same diameter). The boundaries of the wave can be quite precisely indicated by straight lines. These lines, perpendicular to the front of the propagating wave, are called rays.

But more attentive observation shows that the waves nevertheless pass beyond the limiting rays, which were imagined to be drawn from the source to the edges of the screen; while beyond the limiting rays the wave amplitude is smaller. If the dimensions of the aperture or obstacle are reduced,

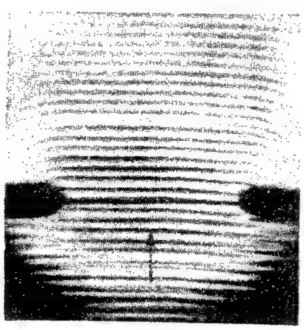


Figure 24. Passage of plane waves through an aperture in a wall

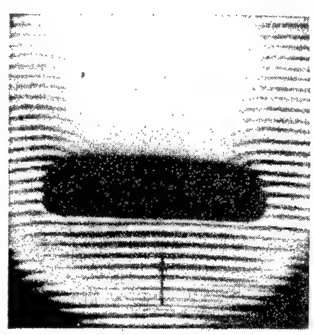


Figure 25. Passage of plane waves around an obstacle the same size as the aperture in Figure 24

and its size approaches the wave length, then bending of the wave becomes more marked, as may be seen in Figures 26 and 27. In Figures 28 through 30 are shown examples in which the dimensions of the aperture or obstacle are close to the wave length. Beyond the aperture the waves already fill up

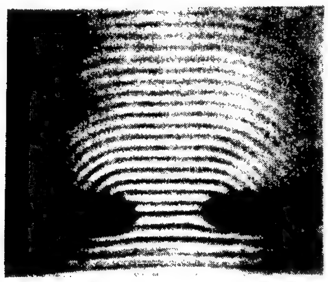


Figure 26. The aperture is reduced

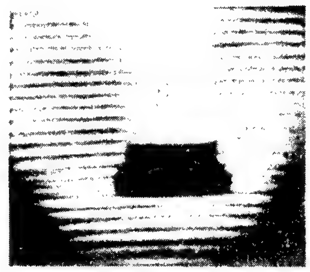


Figure 27. The obstacle is the same size as the aperture in Figure 26

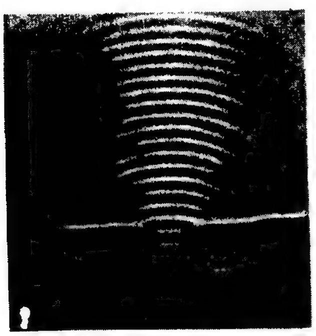


Figure 28. The aperture is further reduced

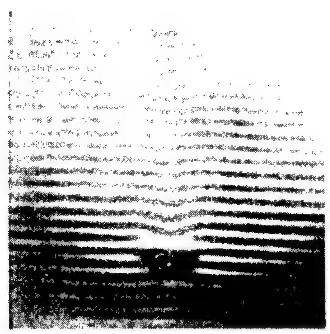


Figure 29. The obstacle is the same size as the aperture in Figure 28

Figure 30. Waves pass around a small obstacle. Slight curving of the front (diffraction)

Figure 31. Aperture smaller than the wave length. The aperture becomes a source of waves spreading in semi-circles

a total region bounded by an angle of about 90°. Beyond the obstacle there is no longer shadow; the effect of the obstacle is seen only in its immediate neighborhood.

If the aperture is small compared to the wave length it becomes a point of origin of waves spreading as semicircles (Figure 31); the amplitude of the waves decreases as the size of the aperture decreases.

How can all these phenomena of wave diffraction be explained? Here Huygens' principle is useful. Let us first consider the passage of waves through a small aperture (Figure 31). By Huygens' principle every point of the aperture which has been reached by a wave may be considered a source of secondary circular waves; the vibrations of all these sources will be in phase. Because the aperture is small in comparison with the wave length, the path differences from these sources to some point beyond the aperture will be very small, so that we can replace all these sources by a single one, without introducing large error. This source will produce circular waves, as seen in Figure 31.

However, if the aperture is comparable in size to the wave length, it cannot be considered a single source. At a point beyond the aperture separate waves from different points in the aperture will arrive, each with a definite path difference with respect to another (particularly for those coming from the sides of the aperture); and the phenomenon of wave interference appears. Because of interference the pattern of the wave propagation through the aperture will not be the same as in the preceding case.

If the aperture is large in comparison with the wave length, the transmitted wave will be as shown in Figure 24; the wave front will be only slightly bent at the edges of the band, where it enters the shadow region.

Thus, for diffraction phenomena to appear, the size of an obstacle must be comparable to the wave length or even smaller; otherwise, the waves are not bent and the region of shadow will be formed by purely geometrical laws.

Rays. A ray has been defined as a line perpendicular to the front of a propagating wave. Now that diffraction has been introduced, the concept of a ray must be made more precise, and the limits of its applicability must be defined.

Let us consider a plane wave. If such a wave propagates in a homogeneous medium, in which the velocity of the wave is the same throughout, then rays are parallel straight lines coinciding with the direction of wave propagation. When the wave falls on a screen containing an aperture, a portion the same size as the aperture is "cut out" of the wave. From the previous discussion it is known that, if the aperture is considerably larger than the wave length, the "cut out" portion of the plane wave will continue to be transmitted with no change in direction; in this case it may be considered a ray, or a beam. But it is impossible to call a transmitted wave a ray if the size of the aperture is comparable with or smaller than the wave length, because in this case the waves will propagate in all directions beyond the aperture.

The situation is the same when a wave encounters an obstacle. If the obstacle is larger than the wave length, there will be a sharply-defined shadow beyond it. For instance, during the propagation of a band of ripples, caused by a gust of wind on a quiet water surface, the water will remain

still behind an obstacle. In this case the wave propagation may be described in terms of rays, provided the wave motion remains linear. In this way a geometrical construction may be made, using rays, to describe the arrival and reflection of a wave at an obstacle.

LEONID ISAAKOVICH MANDEL'SHTAM
(1879-1944)

If the obstacle is comparable in size to the wave length, there will be no simple wave reflection; diffraction effects will appear, and the concept of a ray will not be applicable.

Thus the concept of a ray can be applied only when the ray path is much greater than the wave length and when the width of [a beam of] rays is equal to many wave lengths.

If the wave is circular, it propagates in all directions and of course cannot be described by a single ray, since rays spread from the source in all directions. But a single small portion of a circular wave, far from the source, can be considered plane, and its propagation can be described by a single ray (provided the dimensions of this part of the wave are large in comparison with the wave length).

Thus, a ray can only indicate the direction of propagation of a section of a plane wave which is considerably larger than the wave length. Therefore, the concept of a ray cannot be applied to sections of the wave where changes of phase and amplitude are noticeable, since such a wave is not plane. This is precisely the situation when waves pass near the edges of an obstacle and diffraction appears.

By means of the example of waves on water, the concept of wave motion has been introduced and the characteristic properties of wave propagation have been studied. Thus, it has been seen that a wave is a transfer of a certain state of the medium at a definite velocity, while the medium as a whole remains stationary. The phenomena of reflection, refraction, interference, and diffraction were also discussed.

Wave motion has an extremely wide application in nature and in technology; sound, light, and radio waves are wave phenomena possessing different physical natures and having different causes. The great Russian scientists A.S. Popov, P.N. Lebedev, N.A. Umov, B.B. Golitsyn, N.E. Zhukovskii, A.N. Krylov, and L.I. Mandel'shtam have made great contributions to the theory of oscillation and wave motion.

Now the important, basic questions about sound will be answered. What is sound? What are sound waves? How do they propagate in various media —air, water, and solid bodies? What are the main applications of sound?

Chapter II

SOUND WAVES IN AIR

§ 1. Formation of sound waves in air

Formation of Sound Waves. Sound waves are elastic waves, which are usually propagated in air. Let us try to understand how sound waves originate.

Air, as well as all other gases, possesses three-dimensional elasticity. When air is being pumped into the inner tube of a bicycle or automobile tire, the pumping becomes progressively more difficult as the tube is filled. This means that the air pressure in the tube is increasing and that more effort is needed to push in an additional amount of air. The property of three-dimensional elasticity of air is still more evident in the following experiment. If a piston in a closed cylinder containing air is depressed, and then released, the piston moves back as if driven by a compressed spring. Here, the air acts as a spring; its three-dimensional elasticity causes it to resist compression.

The three-dimensional elasticity of air is comparatively small; and this fact is made use of in automobile tires. If the rims of the wheels were simply covered by rubber, the car would experience strong jolting. Because the elasticity of rubber is high, jolts would be easily transmitted to the chassis while driving on a bad road. The air in the tubes, however, because of its low elasticity (and consequently much greater compliance than in the case of rubber), softens the jolts and the car runs more smoothly.

Like every substance, air has mass and therefore inertia. If motion of the air is caused, it continues, even when the force which caused the motion ceases to operate.

It was noted in the first chapter that the combined presence of elasticity and inertia can cause wave motion under certain conditions. In particular, the elasticity and inertia of the air make possible the generation of elastic waves in air.

An elastic wave in air is formed by a sudden change in air density, that is, by a condensation or rarefaction at some point. For example, when a highly inflated rubber balloon bursts, the compressed air released strikes the surrounding air, which is at normal pressure, and pushes it in all directions. Because of its inertia the air cannot expand instantly and only the nearest layer is compressed. This layer has three-dimensional elasticity and so it expands again, and consequently compresses the next layer; this layer expands in turn and compresses the next layer. Thus, a spherical elastic wave is formed in air, the conditions of compression and rarefaction being transferred from one layer to the next. In the wave every air

particle moves foreward and backward in the direction of propagation of the wave, that is, along the radiuses through the center of the burst balloon. Thus, in an elastic wave in air the air particles vibrate in the direction of propagation of the wave; such waves are called longitudinal. It should be recalled that the particle movement in water waves has a completely different character, since the water particles move in circular orbits whose planes lie in the direction of propagation of the wave.

The bursting of a balloon generates in the air a single pressure pulse. During continuous vibration of some body (e.g., the prongs of a tuning fork or a piano string) elastic waves are continuously generated in air. When the vibrating body moves forward, it compresses the air in front of it; and this compression is transmitted to the surrounding layers of air. When the body moves backward, a rarefaction begins to be propagated immediately after the compression. Then, when the body moves forward again, a new compression is transmitted, etc. Thus, a vibrating body continuously generates, or rather radiates, elastic waves which consist of consecutive compressions and rarefactions of the air.

These elastic waves of compression and rarefaction, produced in air by vibrating bodies, are called sound waves or sound. Sound may be generated and propagated not only in air and other gases but also in liquids and in solid bodies.

Photographing Sound Waves. A sound may be heard, but not seen. By certain means, however, sound waves can be made visible. The simplest of these is the method of the luminous point. A point source of light, perhaps a small aperture with an electric arc behind it, lights a screen a

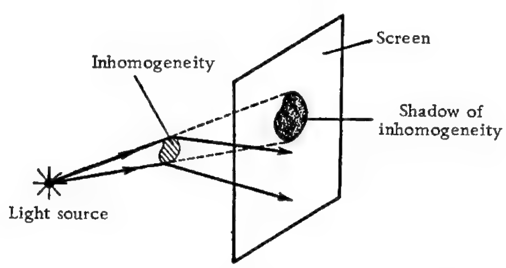


Figure 32. The method of the luminous point

certain distance away from it. If the medium between the light source and the screen is homogeneous, the screen is lighted uniformly. However, if the medium contains inhomogeneities (for example, density differences invisible to the eye), then the shadows of these inhomogeneities appear on the screen (Figure 32). The rays, diverging radially from the source, experience a slight bending if they encounter an inhomogeneity in their path and reach a different spot on the screen. Therefore, a shadow showing the contours of the inhomogeneity appears on the screen.

Let us see why the inhomogeneities in air density bend the light rays.

Compressed air has a larger, and rarefied air a smaller, coefficient of bending [refractive index] than air at normal pressure. When passing from air at normal pressure into compressed or rarefied air, light rays bend slightly and deviate from their original direction. Because of this

property, if rising streams of irregularly heated air (the density of which, therefore, is inhomogeneous) are looked through on hot summer days, the contours of objects observed seem to vibrate.

When light rays pass through air in which a sound wave is propagated, they traverse regions of compression and rarefaction caused by the sound and therefore change their direction somewhat. This makes it possible to "see" the sound.

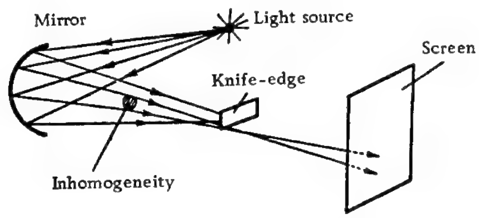


Figure 33. The schlieren method

The schlieren, or dark field, method represented in Figure 33 is more improved. Light from a slit or a small round aperture which is lighted by some strong source falls on a large concave mirror. The mirror, situated at a considerable distance from the shining slit or aperture, has a rather large focal length of several meters. A knife-edge (the Foucault knife-edge) is placed at the point where the mirror focuses the light rays from the source in such a way that the whole beam of light strikes the knife-edge and the screen remains dark.

If the light rays encounter inhomogeneities in the density of the medium during their travel, as shown in Figure 33, they bend slightly, change their direction, and bypass the knife-edge; as a result, some of the rays reach the screen. The image of the inhomogeneity then appears on the screen.

If a sharp sound pulse travels between the mirror and the light source (e.g., due to the flash of an electric spark), the image of the sound wave on

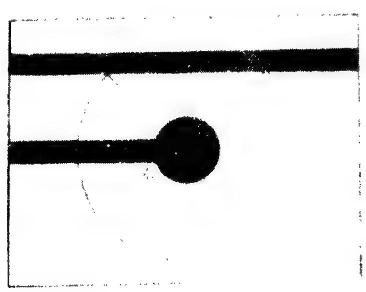


Figure 34. Reflection from a flat wall of the sound pulse from an electric spark

Photograph obtained by the schlieren method

Figure 35. Passage of a sound pulse through apertures in a cylindrical wall

Photograph obtained by the schlieren method

the screen can be photographed. The electric spark produces an extremely short flash; but the speed of light is immeasurably greater than the speed of sound (as will be discussed later). Therefore, during the flash [of a second spark, a little later] the light will travel so quickly that the image will appear stationary. Figure 34 shows the photograph of a sound pulse from an electric spark, obtained by the described method. We see the spherical sound wave which strikes the wall and is reflected from it. A comparison of this photograph with Figure 23 shows complete analogy with the reflection (from a flat partition) of a circular wave on the surface of water.

Figure 35 shows a photograph of a sound pulse passing through apertures in a cylindrical wall. This photograph again clearly shows the nature of Huygens' principle. It is evident how every aperture serves as a source of secondary spherical waves and how the envelope of these waves coincides with the front of the actual propagating wave.

Wave Length, Frequency, and Velocity of Sound. Sound waves, as well as water waves, are characterized by wave length, frequency, and velocity of propagation. By wave length is meant the distance between two successive compressions or rarefactions of the air (Figure 36) or, generally, the distance (along the direction of wave propagation) between two successive points in the air which are vibrating in the same phase. The number of waves which pass in one second through a given part of the medium is called the frequency of the sound.

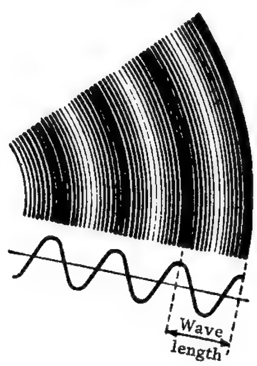


Figure 36. Compression and rarefaction in sound waves

Only a part of the spherical waves is shown schematically

Sound waves are propagated with a definite velocity c, called the sound velocity. Between the velocity of sound propagation c, the wave length λ, and the frequency f (or period T) there exists the relation which has been mentioned in the first chapter:

$$\lambda = cT = \frac{c}{f}.$$

Elastic waves in air have a very wide range of frequency and wave length. Sound waves are elastic waves with frequencies which lie within the hearing range of the human ear, e.g., approximately from 16 to 20,000 cycles per second. This corresponds to wave lengths in air from 20 m to 1.7 cm. Elastic vibrations at a frequency below 16 cycles are called i n f r a s o n i c, those above 20,000 cycles, u l t r a s o n i c, and those above 1000 megacycles, h y p e r s o n i c.

§ 2. Velocity of sound

Measuring the Velocity of Sound. A rough measurement of the velocity of sound in air can be made by anyone.

While traveling in the mountains, going in a boat on a quiet river with steep or wooded banks, or standing in front of a thick forest, it is easy to produce the well-known effect of echo. Just as waves on water are reflected when they meet an obstacle in their path, so are sound waves reflected from obstacles. If sound waves strike a surface at a right angle, the waves are reflected in exactly the opposite direction.

The echo effect, which is a reflection of sound from an obstacle in the path of the spreading sound waves, enables measurement of the velocity of sound by means of a comparatively simple method. For such a measurement a watch with a second hand or, still better, a stopwatch is needed. The moment of producing the sound (a shout, clapping of hands, or knock) and the moment of receiving the echo are noted; then the velocity of sound c can be determined, provided the distance L from the reflecting surface (woods or river bank) is known. The velocity of sound c is then determined by the formula:

$$c=\frac{2L}{t},$$

where t is the measured time interval. In this formula the number two appears because the sound traverses the distance L twice. From this formula the distance L to the obstacle can be determined, if the sound velocity c and the time interval t are known. It will be seen later that measurements of the depth of seas and rivers are based on this principle, using echo depth finders.

The velocity of sound can also be measured by making use of the fact that the velocity of light is immeasurably greater than the velocity of sound (about a million times). A white cloud of steam is seen, but only later is heard the whistle of the approaching locomotive; lightning is seen, but only after some time is thunder heard. The velocity of light is 300,000 km/sec, and a greater speed of energy transfer has never been observed.

If, at a distance L from the point of observation, a sound is produced simultaneously with a flash of light (e. g., an explosion or shot) and the time interval t between the flash and the arrival of the sound signal at the point of observation is measured, then the velocity of sound will be equal to

$$c=\frac{L}{t}.$$

In such a measurement we neglect the time for travel of the light from the source of the sound to the point of its reception. However, this time is so insignificant that its neglect does not introduce any error into the results of the measurement.

There are many other, more precise, methods for measuring the velocity of sound in air; some of them will be presented later. The resulting value for the velocity of sound at a temperature of 0°C is 331.5 m/sec, or about 1200 km/hour.

To get a clear idea of this value, note that the speed of present-day jet airplanes approaches the velocity of sound (e. g., the TU-104 passenger plane can develop a speed of nearly 1000 km/hour) and sometimes exceeds this value. The speed of antitank and antiaircraft shells is 1000 m/sec and higher, several times the velocity of sound; the flight speed of the rocket which puts a sputnik into orbit is slightly more than 8 km/sec.

Factors Determining the Velocity of Sound. What determines the velocity of sound in air? It can be shown that the speed of propagation of a longitudinal wave in an elastic medium is expressed by the formula

$$c=\sqrt{\frac{E}{\rho}},$$

where E is the bulk modulus of elasticity (the reciprocal of the compressibility of the medium) and ρ is the density of the medium. The meaning of E can be clarified by the following example.

Press the handle of a bicycle pump after closing the open end of the rubber hose. If the original volume of air under the piston was V_0 and the pressure was p_0, then, after increasing the pressure to a value p, the volume will decrease to V. The change in volume is equal to $V_0 - V$, and its relative change is $\frac{V_0-V}{V_0}$. The greater the force $F=p_0-p$ which compresses the air (or other gas), the greater is the relative change in volume; i.e., the relative change in gas volume is directly proportional to the applied force:

$$\frac{V_0-V}{V_0}=-kF.$$

In this formula* k is a constant designating the compressibility of the gas; the smaller the relative change in volume for a given force F, the lower is the compressibility of the gas. It follows from this formula that the compressibility is equal to the relative change in volume when the pressure changes by unity. The bulk modulus $E=\frac{1}{k}$, which is the reciprocal of the compressibility, is also called simply the three-dimensional elasticity.

Thus, elasticity is the force opposing compression of the air. The external pressure acting on the air pushes the air particles together, while the force of elasticity tends to expand the air. If these forces are equal, the air is in equilibrium. The external pressure in this way serves as a measure of the elasticity; and the elasticity of air and other gases is numerically equal to the absolute pressure which the gas exerts on a unit surface (1 cm^2). The formula for the velocity of sound can therefore be written

$$c=\sqrt{\frac{P}{\rho}}.$$

In this formula P is the pressure at sea level and 0°C. It is equal to 1033.6 gf/cm^2 and must be expressed in absolute units. Let us remember that in mechanics a unit force is that force which imparts to a mass of one gram an acceleration of 1 cm per sec per sec. This unit of force is called the dyne. Since, by Newton's law, force equals mass times acceleration, and the acceleration due to gravity is equal to 980.6 cm/sec^2, the force with which the earth attracts one gram is 980.6 abs. units. Thus, the atmospheric pressure P, expressed in absolute units, is $1033.6\times 980.6 =$ $= 1,013,500$ abs. units. The absolute unit of pressure is called the bar. One bar is the pressure exerted by one dyne of force acting on 1 cm^2.

The air density ρ at a temperature of 0°C and normal atmospheric pressure (density is the mass in grams contained in one cm^3) is equal to 0.001293. If these values for P and ρ are introduced into the last formula,

* Since the quantities $F=p_0-p$ and V_0-V have opposite sign (for compression V_0-V is negative and p_0-p is positive), the minus sign has been added, in order to make the coefficient k positive.

the velocity of sound is 280 m/sec. This value for c was first obtained theoretically by Newton. This differs greatly, however, from the actual speed of sound in air, which is 331.5 m/sec, as mentioned above.

The difference exists because, in developing the formula, it was not taken into consideration that as air is compressed the pressure and therefore the elasticity of the air will increase. In addition, air, like every gas, warms up with compression and cools off with rarefaction. The change in air temperature results in an additional change in its elasticity; at compression, due to the increase in temperature, the elasticity increases and at rarefaction it decreases slightly.

Of course, this additional change in the elasticity of air under compression can be obtained only if compression is performed in such a way that the heat generated has no time to dissipate. In the same way, if rarefaction is produced quickly, the resulting change in temperature will not have time to be compensated. Such processes, during which there is no exchange of heat with the surrounding medium, are called adiabatic processes. If an equalization of temperature takes place (that is, if the temperature remains constant) the process is said to be isothermal.

In the foregoing the change in elasticity due to compression and rarefaction was discussed, but it was not taken into consideration that these compressions and rarefactions are attended by temperature changes. Changes in temperature are found to cause additional changes in the elasticity of the air, a circumstance which was first commented upon by Laplace.

Laplace showed that the ratio between the elasticity for adiabatic compression and the elasticity for slow compression (during which the temperature of the compressed air has sufficient time to become equal to the temperature of the surrounding medium) is equal to the ratio between the quantities of heat necessary to heat a unit mass of air by 1°C at constant pressure and at constant volume respectively. This quantity is called the ratio of specific heats at constant pressure c_p and at constant volume c_v (for air, $\frac{c_p}{c_v} = 1.41$). If these additional changes in the elasticity of air are taken into account, the formula for the velocity of sound becomes

$$c=\sqrt{1.41\frac{p}{\rho}}.$$

It is easy to confirm by calculation that this formula gives just the velocity of sound which has been observed experimentally, that is, 331.5 m/sec (at 0°C).

Thus, the velocity of sound increases due to the temperature changes produced by the sound wave itself; and the propagation of sound is an adiabatic process. These temperature changes are very small; they have no influence on the average temperature of the air, because, although the temperature increases somewhat during compression, it decreases during rarefaction.

Dispersion. Effect of Temperature on Sound Velocity. There is a widely held opinion that, if the frequency of sound is lowered more and more, then at very low frequencies (of the order of a few cycles — infrasonic region) the difference in temperature between the compression and rarefaction resulting from passage of a sound wave has time to become equalized. In other words, at low sound frequencies, the phenomenon of dispersion

occurs, a decrease in the velocity of sound so that it approaches the value obtained by Newton. The French scientist Esclangon, who investigated the acoustics of guns and projectiles, as well as the propagation of infrasonic vibrations in air, attempted to observe experimentally this change in the velocity of infrasonic waves. He even published data showing an apparent decrease in sound velocity with decreasing frequency. Later measurements of sound velocity at low frequencies, however, showed the results of Esclangon to be erroneous; no change in velocity was observed, down to frequencies of one or two cycles.

It can be shown rather simply that, if there really is a transition to the Newtonian velocity of sound, it will not be at low, but at very high, frequencies.

In fact, the distance between rarefaction and compression points in a sound wave is equal to half its wave length ($\frac{\lambda}{2}$). If the frequency is low, the wave length is long; for a frequency of 5 cycles, $\lambda = 66$ m and $\frac{\lambda}{2} = 33$ m. Temperature equalization must take place over a distance $\frac{\lambda}{2}$ —that is, over several tens of meters at low frequencies. The time necessary for the equalization of temperature variations depends on the thermal conductivity of air; but this thermal conductivity is very small. Therefore, although the frequency of the sound is low and the vibration period of the air particles is long, owing to the large distance between compressions and rarefactions the temperature has no time to become equalized. On the contrary, at very high frequencies where wave length is very small, it can be expected that, in spite of the small time interval between compressions and rarefactions, there is time enough for temperature equalization. It can be shown that such equalization can take place at frequencies:

$$f = \frac{c^2}{\varkappa} c_v \Big|_{\rho = \text{const}},$$

where c is the velocity of sound, c_v the specific heat at constant volume, and $\varkappa$ the coefficient of thermal conductivity. For air this frequency is calculated to be about 10^{12}-10^{13} cycles. Such high hypersonic frequencies have not so far been produced artificially.

In the discussion of waves on the surface of water, it was noted that the propagation velocity of such waves depends on the wave length, that is, dispersion occurs. Sound waves of different wave length, and therefore different frequency, propagate in air with the same velocity. Thus, when sound propagates through air, dispersion is not observed.

If this were not true, it would not be possible to enjoy music; sounds of one frequency (one tone) would reach us first, and then the sounds of another frequency; and it would appear as if the orchestra were not producing them simultaneously.

The above formulas describing the velocity of sound appear to indicate that this velocity becomes greater with an increase in the pressure P or with a decrease in the air density ρ. Such a conclusion, however, is incorrect; as the pressure increases or decreases the air density does also, and so the ratio $\frac{P}{\rho}$ remains constant. The velocity of sound in air is the same at great heights (e. g., on mountains, where the air is rarefied and the pressure is only a fraction of that at sea level) as it is in valleys.

But this holds true only when the temperatures in valleys and on mountains are the same.

The velocity of sound is independent of the air pressure but dependent on the temperature*. The higher the temperature of the air the greater is the speed of sound propagation in it. If the temperature increases by 1°C, the velocity of sound increases by about 0.5 m/sec. If at 0°C the velocity of sound is 231.5 m/sec, then at normal room temperature (18°C) this velocity will be 342 m/sec. An easily remembered formula for the effect of temperature on the sound velocity in air is obtained by inserting the values of P and ρ for air:

$$c = 20\sqrt{T}\ \mathrm{m/sec},$$

where T is the absolute temperature (if the centigrade temperature is 0°, then $T = 273°$; a room temperature of 18°C corresponds to $T = 291°$).

The velocity of sound is different in different gases. The velocity for several gases at 0°C is given below:

Air	331.5 m/sec	Carbon dioxide	261 m/sec
Hydrogen	1265 m/sec	Oxygen	316 m/sec

§ 3. Propagation of sound

In the first chapter, during the discussion of water waves, we became familar with the basic laws of wave motion and with the nature of wave propagation. It has been mentioned that sound waves differ from water waves in physical nature and in structure. However, the basic laws of wave motion can be applied to sound waves. Reflection, refraction, diffraction, interference, and the other phenomena typical of wave motion on water also occur during the propagation of sound.

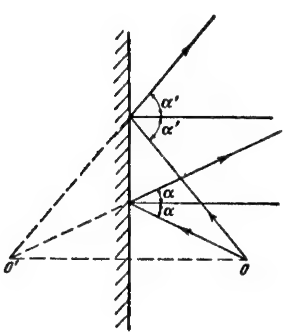


Figure 37. Reflection of sound waves from a flat wall

Reflection of Sound. When sound waves encounter an obstacle in their path, they are reflected from it. Reflection plays an important role in acoustics, e. g., with regard to sound propagation in closed rooms. Sound waves incident on a wall at angles α or α' (Figure 37) are reflected at these same angles; to an observer the sound seems to originate at a point O', which is the mirror image of the sound source (the virtual source). A geometric construction as in Figure 37 can be used only if the dimensions of the reflecting surface are larger than the wave length of the sound. If this is not true, then diffraction begins to play an important role, and the idea of a "sound

* According to the Clapeyron equation

$$PV = RT \text{ or } \frac{P}{\rho} = RT,$$

where V is the gas volume, T is its absolute temperature, and R is a number called the gas constant. Thus, the ratio $\frac{P}{\rho}$ in the formula for the velocity of sound is dependent on the temperature.

ray", as discussed at the end of the first chapter, loses its meaning.

Diffraction. The phenomenon of diffraction, which is a bending of waves near obstacles whose dimensions are smaller than or comparable to the wave length, causes the waves to travel along curved lines. A voice is heard "around a corner", but the speaker is unseen; noises in the street are heard through an open window in a room. The sound waves bend around obstacles (are diffracted by them), and so sound reaches us in the same way as waves on water bend around an obstacle (Figure 27) or pass through apertures in a partition (Figure 26). Wave lengths in the audible range are of the same order of dimension as (or even larger than) ordinary generators and receivers of sound and everyday objects. Therefore diffraction plays a very important role in the propagation of sound waves. If the length of a sound wave is small in comparison with the dimensions of obstacles, the bending is far less noticeable. It is then possible to speak of "sound rays" and to consider the sound propagation as rectilinear. In this case there is a sharply delineated region of sound shadow behind the obstacle (Figure 25), and it may be supposed that the sound propagates in accordance with purely geometrical laws. In acoustics such cases are met with particularly often when the sound wave length is small (that is, at high ultrasonic frequencies).

Superposition of Sound Waves. Interference. If two stones are dropped simultaneously into a pool, their waves are propagated independently of each other on the water surface and the superposition of waves occurs; the same may take place with sound waves. Air transmits simultaneously very different sounds, of various frequencies and amplitudes; and these sounds are propagated in different directions. Sounds of different frequencies (or wave lengths) pass through the air independently of one another.

The phenomenon of interference results from the superposition at a single point of two or more sound waves having the same frequency and exhibiting phase differences which do not change with time. Interference is a characteristic property of wave motion in general, and all the facts about interference which applied to water waves also apply to sound waves. Interference was one of the most important arguments in favor of the wave nature of light; it is also one of the arguments for the wave nature of sound. As a matter of fact, it is impossible to explain how two sound rays which come together can cancel one another, without introducing the idea of wave motion.

It was seen how the interference of waves on a water surface may be obtained by means of the vibrations of two small balls in a "wave bath". In the same way, if two sound sources of the same frequency and about the same intensity are provided, interference of sound waves may be observed.

Standing Waves. An important example of interference is the formation of standing waves, due to reflection. Exactly how standing waves are formed can be clearly demonstrated by a simple example. Let us attach one end of a cord (a few meters long) to the wall, and hold the other end in the hand (Figure 38). If a single, sudden up-and-down movement of the hand is made, a single wave will begin to travel along the cord. This wave will be reflected at the fixed end, will return to the hand, will be reflected from it also, and will travel again to the wall. If the cord is quickly and periodically moved up and down, two superposed trains of waves are obtained:

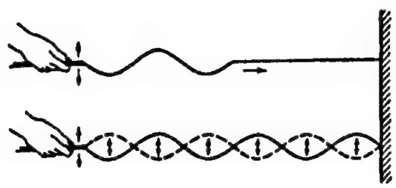


Figure 38. Formation of standing waves by means of vibration of a cord fixed at one end

waves going toward the wall and reflected waves returning from the wall. The superposition of these waves results in a complicated and changing mode of vibration, but at certain rates of shaking of the cord standing waves will be obtained. Certain parts of the cord will be at rest, while other parts undergo maximum vertical displacements with respect to the undisturbed position of the stretched cord. Those points of the cord which do not move are called **nodes**, and those at which the vibration amplitude is a maximum are called **loops**. Standing waves are formed when the length of the cord is some integral number of half wave lengths ($\frac{\lambda}{2}$, λ, $\frac{3}{2}\lambda$,...).

In the same way, interference between arriving and reflected sounds of a given frequency results in standing sound waves. For instance, if a

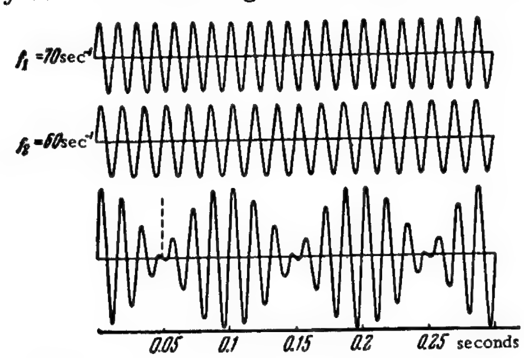


Figure 39. Beats, formed by means of vibration of two tuning forks with frequencies of 60 and 70 cycles

tuning fork with a frequency of 3000-4000 cycles is made to vibrate, then increases and decreases in the intensity of the sound are clearly heard as a wall is gradually approached. A decrease in sound corresponds to a node of the sound pressure, an increase to a loop. A more complicated picture of the formation of standing waves is obtained if sound is propagated in a closed room, in a pipe, or in a closed space generally.

Beats. The very interesting and frequently observed phenomenon of beats is caused by interference. If two sources produce sound waves with slightly different frequencies, then decreases and increases in the intensity of the resulting sound are heard. Suppose, for instance, that at a certain moment air compressions from two tuning forks, with vibration frequencies of 60 and 70 cycles respectively, reach the ear (Figure 39). The resultant sound will be louder than the separate sounds of each tuning fork. Since the frequencies of the tuning forks differ by 10 cycles, after 0.05 sec a rarefaction will reach us from one of the forks, while from the other a compression arrives; and the resultant sound will be weakened. After another 0.05 sec the compressions will again coincide. It is possible to discover regular changes in the loudness of the sound, or **beats**, with a

frequency of 10 cycles (period of 0.1 sec). The beat frequency is thus equal to the difference between the vibration frequencies of the two sound sources.

Sometimes changes in sound intensity, or beats, are heard in the sound of an airplane passing over at a great height. These beats are produced only if the airplane has two or more engines. The sound of the airplane is mostly produced by the action of the engines (exhaust) and by the turning propeller. The frequency of the exhaust sound is determined by the speed of revolution of the engine and by the number of its cylinders. For instance, if the engine operates at 2100 revolutions per minute and if it has nine cylinders, then, since exhaust is produced once during every two revolutions of the drive shaft, the number of exhausts per second is

$$\frac{2100}{2 \cdot 60} \cdot 9 = 157.5.$$

The revolution speeds of the separate engines may differ somewhat (up to 1%). As a result, sound waves arrive from two sources of nearly equal frequency and beats are heard. Suppose that the second engine operates at 2113 rev/min. Then, the frequency of exhausts for this engine will be 158.5 cycles per second and beats with a frequency 158.5-157.5, or one cycle per second (one beat per second), are heard.

Plane Wave. When the formation of sound pulses due to the breaking of a rubber balloon or the crack of an electric spark was considered, a spherical wave was involved. The greater the distance from the sound source the flatter the front of the spherical wave becomes. In a very great number of cases waves which are not exactly plane may be considered as such without involving large error. Therefore the basic relationships characteristic of plane sound waves will be considered in detail.

When a sound wave is passing through air, compressions and rarefactions are produced. Thus, the sound wave causes pressure changes independent of the average external air pressure. This additional pressure is called the acoustic excess pressure — or simply the acoustic, or sound, pressure. The acoustic pressure, which will be denoted by p, is measured in absolute pressure units — bars.

It is also known that during propagation of a sound wave the air particles are subject to vibrations about their equilibrium positions, which take place at a definite velocity v, called the acoustic particle velocity. Thus, the particle velocity is the speed of motion of the particles of the air due to a sound wave passing through it.

For a plane wave the ratio between acoustic pressure and the acoustic particle velocity is important:

$$\frac{p}{v} = \rho c,$$

where ρ is the density of the medium and c is the speed of sound in the medium*. For air, ρc is equal to 41 absolute units.

* This expression is analogous to Ohm's law for the passage of an electric current through a conductor. If a voltage V is applied to a resistance R, a current I will flow through it. Ohm's law states that $\frac{V}{I} = R$.

In the "acoustical Ohm's law" the acoustic pressure p corresponds to the voltage V, the particle velocity v to the current I, and the characteristic acoustic impedance ρc to the ohmic resistance R. Such an analogy is, of course, strictly formal, since the physical nature of the passage of current through a conductor is completely different from the passage of sound waves through the air. But in modern acoustics such analogies are very often used in order to simplify the solutions of many problems.

If the frequency of a sound wave is f, the acoustic particle velocity may defined as

$$v = 2\pi f x,$$

where x is the amplitude of displacement of the particle from its equilibrium position.

From the previous formula we can obtain the following expression for the amplitude of displacement of the particle:

$$x = \frac{p}{2\pi\rho c f}.$$

To a person standing near a locomotive when the engineer blows the whistle, the acoustic pressure of the sound heard is about $p = 3000$ bars.

A sound of this intensity is at the threshold of pain, that is, it produces an auditory sensation of pain. It is easy to calculate the acoustic particle velocity in this case:

$$v = \frac{p}{\rho c} = \frac{3000}{41} = 73 \text{ cm/sec}.$$

Since the velocity of sound at 18°C is 342 m/sec, it is seen from this example that even for the loudest sounds the propagation velocity c remains about 500 times greater than the acoustic particle velocity v. The above formula also shows the smallness of the displacements of particles from their equilibrium position in a propagating sound wave. For instance, suppose that the frequency f is 1000 cycles and that the acoustic pressure p is 3000 bars; the amplitude of displacement is then

$$x = \frac{3000}{6.28 \cdot 41 \cdot 1000} = 0.012 \text{ cm} = 0.12 \text{ mm}.$$

The faintest sounds which can be heard by a human ear have an acoustic pressure of about $2 \cdot 10^{-4}$ bar. For the above frequency (1000 cycles) we obtain

$$x = \frac{2 \cdot 10^{-4}}{6.28 \cdot 41 \cdot 1000} \approx 0.8 \cdot 10^{-9} \text{ cm}.$$

This shows how sensitive the human ear must be if it can hear sounds when the air particles are making such minute displacements.

§ 4. Intensity of sound. Radiation pressure

Intensity of Sound. A sound wave carries with it a certain energy in the direction of propagation. The sound becomes audible because of energy which originates at the source of the sound vibrations and which is transported by the sound waves. The changes in air pressure which reach the eardrum set it vibrating; the greater these changes, the louder is the sound.

The quantity of energy transferred by a sound wave in 1 sec through an area of 1 cm^2 perpendicular to the movement of the wave, is called the intensity or the force of the sound. For a plane sine wave the sound intensity is

$$I = \frac{p^2}{2\rho c},$$

where p is the amplitude of the varying acoustic excess pressure.

The quantity I, which represents the energy flux, has a definite direction coinciding with the direction of wave propagation. This direction is

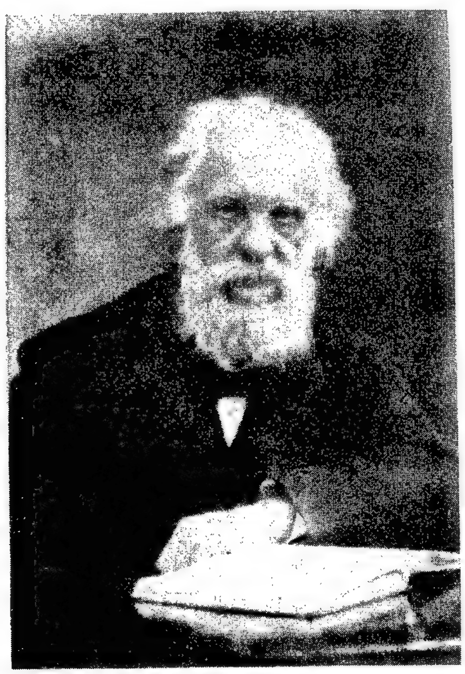
represented by the Umov-Poynting vector, so named in honor of the famous Russian physicist Umov and the famous English physicist Poynting. These two scientists, independently of each other, first established precise energy relationships describing wave propagation.

Note that the energy flux per second is just a measure of the power. Therefore, if the expression "force of the sound" is used here, the term "force" is not used in the same sense as in mechanics.

If the intensity of sound is to be represented in absolute units, it should be recalled from mechanics that: the unit of force is the dyne, which is the force which accelerates a mass of one gram by 1 cm/sec^2; the unit of work is the erg, done by 1 dyne in a displacement along 1 cm; and a unit of power corresponds to one erg of work accomplished during one second. An amount of power equal to 1 erg/sec is 10^{-7} watt.

If p, ρ, and c are expressed in absolute units, then the intensity I of the sound will be represented by the number of ergs which in one second pass across an area of 1 cm^2 (erg/sec · cm^2 or, in watts, 10^{-7} watt/cm^2). Using the previous formula it is easy to calculate the sound intensity of a plane wave. Let the acoustic excess pressure be 3000 bars (as in the preceding example), then

$$I = \frac{1}{2 \cdot 41}(3000)^2 = \frac{9 \cdot 10^6}{82} = 1.1 \cdot 10^5 \frac{\text{erg}}{\text{sec} \cdot \text{cm}^2} = 1.1 \cdot 10^{-2} \frac{\text{watt}}{\text{cm}^2}.$$



NIKOLAI ALEKSEEVICH UMOV
(1846-1915)

For the faintest sounds which can be heard

$$p \approx 2 \cdot 10^{-4} \text{ bar;}$$

and it is easy to calculate that

$$I \approx 10^{-16} \frac{\text{watt}}{\text{cm}^2}.$$

Measuring Sound Intensity. The Rayleigh Disk. Even for very loud sounds the energy transferred by the sound waves is extremely small. How can this energy be measured? In other words, how may the intensity of a sound be measured? Here, it is important to make absolute measurements, that is, measurements whose results can be expressed in definite units, so that the sound intensity is known to be, for instance, a certain number of watts per cm^2.

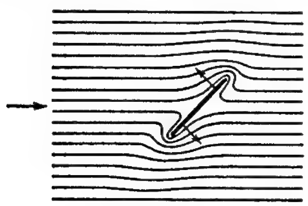


Figure 40. Flow around a disk

(The disk is at an angle of 45° to the flow)

One method for such measurement is by means of the Rayleigh disk. The Rayleigh disk is a small mica disk suspended on a thin quartz fiber. When sound waves fall on the disk, it turns, attempting to place itself perpendicular to the direction of wave propagation. This can be explained by considering the behavior of a disk in a steady stream of air. Figure 40 shows lines along which the air particles of a steady air stream move around a disk (streamlines); near the disk the lines become curved.

If flow is from left to right and the disk is situated as shown in the figure, then on the side of the flow the streamlines will be more curved at the lower edge of the disk than at the upper edge. The air particles at the lower edge must change their direction of motion sharply; this brakes them strongly and their speed decreases. On the other hand, the flow velocity decreases at the upper edge of the upstream side of the disk. On the other [downstream] side of the disk the contrary is true; streamlines will be more strongly curved at the upper edge than at the lower. The flow pressure on the disk at different points of its surface depends on the speed of the air particles at these points. The maximum pressure will be at those points where flow is completely stopped*. There are two such points of "stagnation" on the disk. At these points the forces are effectively concentrated and give rise to a couple tending to turn the disk with its face into the flow (see Figure 40).

For the same reason, if a piece of paper drops through the air, it flutters from side to side during its fall, attempting to turn in such a way that its surface is perpendicular to the direction of fall.

It will be noted in Figure 40 that if the direction of motion of the flow is reversed, then because of the symmetry of the streamline pattern the couple will not change; and the disk will tend to rotate in the same direction. Therefore, if the disk is in an air flow, the direction of which is changing periodically, it will turn as if in a constant air flow, placing itself perpendicular to the flow.

* According to the Bernoulli equation the sum of the hydrostatic pressure p and the part of the head $\frac{\rho v^2}{2}$ expended on displacement of the particles, where ρ is the density and v the velocity of flow, remains constant

$$p + \frac{\rho v^2}{2} = \text{const};$$

therefore, if v increases, p decreases.

It is known from the foregoing that during the propagation of sound the air particles make periodically repeated movements; the velocity of this motion (acoustic particle velocity) changes, too, and a change in pressure (the excess pressure) results. If the dimensions of the disk are small compared to the wave length of the sound (only in this case will measurements using a Rayleigh disk be correct), then the sound will represent for the disk an alternating air stream, and the considerations describing a disk in an alternating stream may be applied to a disk with sound "flowing around" it.

Usually, during sound-intensity measurements, the disk is positioned at an angle 45° to the direction of sound propagation, because at this angle the sensitivity of the disk is highest.

The angular distance through which the disk will turn is greater, the greater the acoustic particle velocity v (the speed of motion of the air particles in the sound wave). To measure the turning angle of the disk a small mirror is attached to it, which reflects light onto a scale marked with divisions (Figure 41). The elasticity of the quartz fiber is determined in advance; therefore v^2 can be found, and if ρc for the medium is known, the sound intensity may be computed.

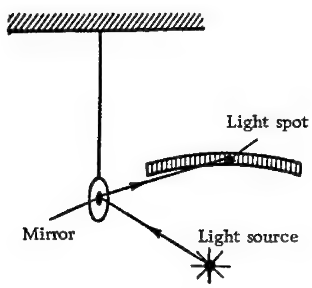


Figure 41. Measurement using Rayleigh disk

Precise measurements using this method were made by V. D. Zernov, a student of P. N. Lebedev. The Rayleigh disk is used even today for investigation of a sound field and for the calibration in absolute units of acoustical transducers (microphones).

Radiation Pressure. The Radiometer. Lebedev, who proved experimentally the existence of light pressure, was also interested in learning whether sound waves "press" upon obstacles which they meet during propagation. His student, A. B. Al'tberg, constructed an instrument called a radiometer, for measuring sound radiation. This instrument (Figure 42)

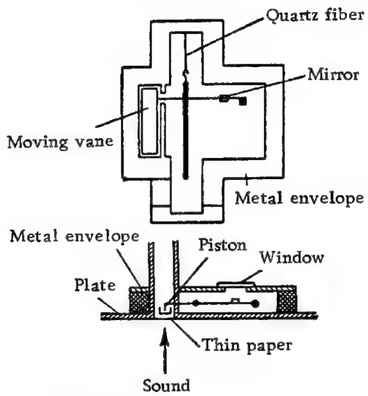


Figure 42. The radiometer of Al'tberg

consists of a very sensitive rotary balance. On a small arm, which is able to turn about its axis, there is suspended a light moving vane. A

small mirror is attached to the arm and the whole assembly is balanced by a small weight. Between the moving vane and the walls of the cavity within the body of the instrument there is a clearance of a fraction of a millimeter, allowing the vane to move freely. To avoid the influence of air currents (due to air temperature changes or other causes), to which the moving vane is very sensitive, the opening of the instrument is covered by thin cigarette paper or fine tissue. Paper or tissue has only negligible resistance to the passage of sound, but prevents air currents from reaching the disk.

Sound waves cause the mirror attached to the arm to turn; in this way it is proved that sound waves cause pressure on an obstacle.

This pressure (called the radiation pressure) should not be confused with the varying acoustic excess pressure which was discussed earlier. Let us try to clarify the physical meaning of radiation pressure.

It has been noted that, during the propagation of sound waves in a medium, alternate compressions and rarefactions take place by which the gas particles are displaced (in addition to their irregular thermal movements) at a definite velocity. In regions of compression the velocity of the particles is in the direction of propagation, and in regions of rarefaction it is in the opposite direction. It is therefore clear that resistance to the movement of the particles will be smaller when the particles move from a region of compression into a region of rarefaction than when they move from a region of rarefaction into one of compression. Since the acoustic excess pressure is the product of particle velocity and the characteristic acoustic impedance of the medium (see page 37), the pressure in the direction of propagation will be somewhat larger than in the opposite direction. Thus, a constant (radiation) pressure of the sound in the direction of wave propagation exists.

The English physicist Rayleigh has shown theoretically that at normal incidence the mean sound pressure on a wall which has dimensions far greater than the sound wave length and reflects sound completely is equal to $\frac{2I}{c}$, where I is the sound intensity and c the velocity of propagation (if the obstacle absorbs sound completely, then radiation pressure of the sound is $\frac{I}{c}$). In this way the radiometer can be used to measure the intensity I of the sound, provided the velocity of propagation is known. Such radiometers are especially used in ultrasonics, for measurement of the intensity of ultrasonic vibrations in liquids; this will be discussed later.

It may be observed that the above pressure value is strictly true for an isothermal process; in the adiabatic process which actually takes place the radiation pressure is equal to $(\gamma + 1)\frac{I}{c}$ (where $\gamma = \frac{c_p}{c_v}$). Rayleigh has developed these formulas for the case of sound in a closed pipe with rigid walls which contains an ideal gas (i.e., a situation where sound fills the tube completely). The above formulas show that the radiation pressure at the far wall of the tube is numerically equal to the energy transported per unit volume of the tube plus a certain additional amount caused by the heating of the gas due to the sound, provided the sound propagation is considered to be adiabatic.

The French scientist Langevin considered the case, more important in practice, of the radiation pressure upon an obstacle in free space (the

example of the radiometer). He concluded that the pressure upon a completely absorbing obstacle is exactly equal to the energy transported per unit volume by the incident beam of sound rays (analogous to light pressure). The apparent discrepancy between the conclusions of Rayleigh and Langevin was cleared up by the French physicist Brillouin, who found that the Rayleigh pressure consists of two components. The first part corresponds to the Langevin pressure — the pressure which the sound waves produce when encountering an obstacle, and which has a directional (vector) nature. The second part is the hydrostatic pressure arising in all directions; only this pressure acts upon the sides of the pipe, and it represents the less important part of the radiation pressure. In unconfined spaces a change in pressure is compensated by a change in volume, and we have only to consider the so-called Langevin pressure on a wall. This directional pressure has the same value in open and closed systems, which explains the accuracy of measurements made with the radiometer.

In general, light pressure has a very small value. Lebedev finally succeeded in measuring this pressure after the unsuccessful attempts of a great number of scientists during nearly 200 years. The total pressure which light from the sun exerts on our earth is about 100,000 tons (this is of course insignificant in comparison with the force of gravity). Nevertheless, it is well known that a great number of astronomical phenomena are caused by light pressure (e.g., the distinctive form of comet tails). Measurement of the radiation pressure of sound using a radiometer is considerably easier. Radiation pressure is often dealt with in ultrasonics, but this will be discussed later.

Decibels. The range of changes in sound intensity is enormous: the intensity of the weakest sounds (the threshold of hearing) is 10^{-16} watt/cm^2; and the intensity of sounds which cause a sensation of pain is around 10^{-2} watt/cm^2. Thus, the intensity of a loud sound is 10^{14} times that of a very weak sound. In order to avoid dealing with such enormous numbers — in acoustics, as well as in many other scientific fields — a logarithmic scale is used and the term decibel (db) is introduced*. If the intensity of one sound is I_1 and that of another sound is I_2, then it is considered that the first sound is louder than the second by K db, where K is calculated from the formula

$$K = 10 \lg \frac{I_1}{I_2}.$$

* The introduction of the logarithmic scale into acoustics was originally based on the Weber-Fechner law, according to which the auditory sensation S is proportional to the logarithm of the stimulus $\frac{I}{I_0}$:

$$S = A \lg \frac{I}{I_0},$$

where I is the sound intensity, I_0 is the sound intensity at $S = 0$ (threshold of hearing), and A is a constant. Further investigations have shown, however, that A depends on the sound frequency, so that the Weber-Fechner law does not have the generality which was earlier ascribed to it. Therefore it is hardly justifiable to explain, as is often done, that use of a logarithmic scale in acoustics is due to the logarithmic nature of hearing. Rather it is a mathematical convenience; remember, for instance, that in astronomy a logarithmic scale is used to define the brightness of stars; such a scale is also used to denote the range of electromagnetic waves; etc. For the effect of sound frequency upon the sensitivity of the ear, see below, Chapter III, § 1.

For instance, if the intensity of one sound is 1000 times the intensity of another sound, then the first sound is louder than the second by 30 db*.

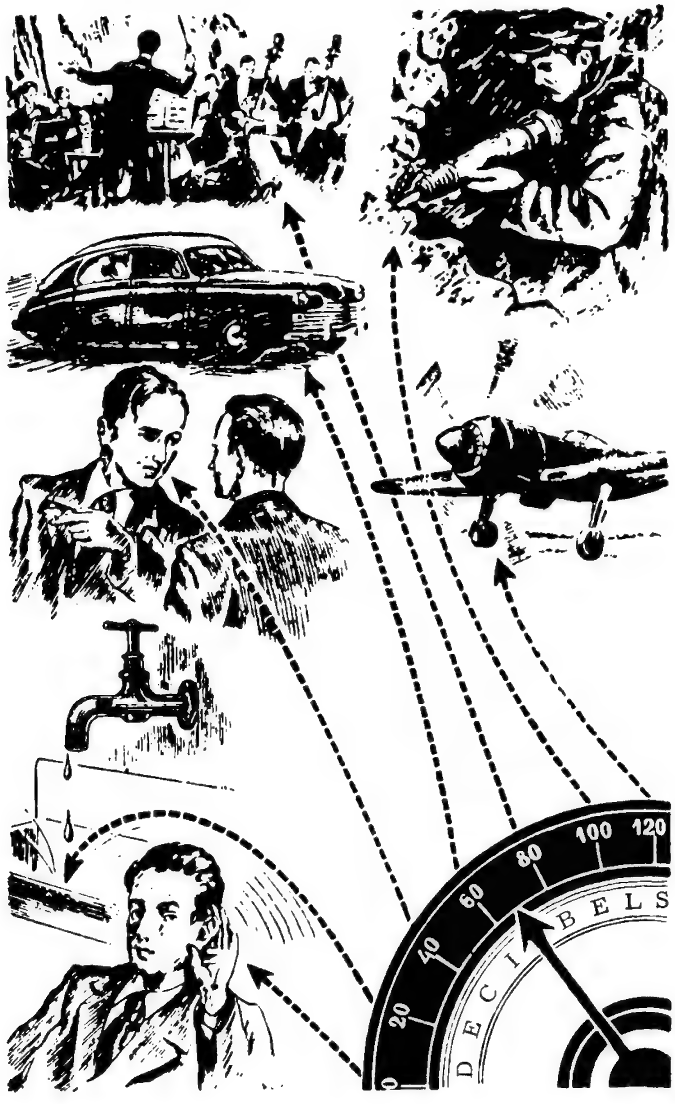


Figure 43. The loudness of noises in decibels with respect to the threshold of hearing (10^{-16} watt/cm^2)

* The intensity of sound is proportional to the square of the acoustic excess pressure. Therefore, the number of decibels K by which a sound with excess pressure p_1 differs from a second sound with excess pressure p_2 is defined by the formula

$$K = 10 \lg \frac{p_1^2}{p_2^2} = 20 \lg \frac{p_1}{p_2}.$$

For better understanding of the relation between the number of decibels and the sound-intensity ratio I_1/I_2 or excess-pressure ratio p_1/p_2 the following table is given:

N, db	0.5	1	2	3	5	10	20	40	60	80	100	140
I_1/I_2	1.12	1.26	1.59	2	3.16	10	10^2	10^4	10^6	10^8	10^{10}	10^{14}
p_1/p_2	1.06	1.12	1.26	1.41	1.78	3.16	10	10^2	10^3	10^4	10^5	10^7

Figure 43 shows the intensity of some common sounds on a decibel scale, calculated with respect to the threshold of hearing (10^{-16} watt/cm^2): 20 db—dripping water heard at a distance of $L \sim 1$ m; 40 db—low talking, $L \sim 1$ m; 60 db—light car on an asphalt road, $L \sim 5$ to 10 m; 80 db—fortissimo of a symphony orchestra; 100 db—pneumatic drill, $L \sim 1$ m; 120 db—airplane engine, $L \sim 5$ m. Here the extent of the range of changes in sound intensity with respect to the threshold of hearing is obvious.

§ 5. Weakening of sound with distance

The Weakening of Spherical Sound Waves. It is well known that as we go away from a sound source the intensity decreases gradually, until finally the sound is not heard at all. What is the reason for the weakening of sound with distance? There are a number of causes of this phenomenon and one of them is as follows. Normal sound waves travel away from a source as spherical or, in general, diverging waves. A spherical wave fills a volume which is steadily increasing with time; and so the movements of air particles caused by the sound are communicated to an ever-increasing mass of air. Therefore, as the distance increases, the movement of the air particles becomes weaker and weaker. This weakening may be expressed as a function of distance from the source.

Let a source Q be surrounded by a sphere of radius R; the surface area of the sphere is $S = 4\pi R^2$. If the intensity of the sound at the source is I_0 and it does not change with time, then all the sound energy radiated by the source will pass through the sphere, and

$$I_0 = 4\pi R^2 \cdot I_1,$$

where I_1 is the sound intensity per unit area (cm^2) of the sphere. Thus we obtain

$$I_1 = \frac{I_0}{4\pi R^2},$$

so that the intensity of a spherical wave decreases in inverse proportion to the square of the distance from the source. Therefore, to transfer sound over a considerable distance it is desirable to concentrate it toward a given direction; to be heard better, a speaker puts his hands to his mouth or uses a megaphone.

Absorption of Sound. Influence of Viscosity and Thermal Conductivity of the Medium. The weakening of sound intensity with increasing distance from the source is not only due to the distribution of energy throughout larger volumes ("geometrical" causes). Sound waves also gradually lose their energy because of absorption. When the motion of a sound wave is not restricted by its surroundings, then absorption is due mostly to the viscosity of air—or, in other words, to the internal friction experienced

by the air particles when they move as a result of passage of the wave; in this way part of the sound energy is converted into heat.

It has been experimentally established that sound absorption depends to a large extent on sound frequency. It can also be shown theoretically that the energy losses of a sound wave are inversely proportional to the square of the wave length and therefore directly proportional to the square of the sound frequency. The absorption of sound at 10,000 cycles is 100 times greater than the absorption of sound at 1000 cycles, and it is 10,000 times larger than that of sound at 100 cycles. This explains, for instance, the fact that near a gunshot a sharp report is heard, while at a distance from the gun the sound appears to be softer. Anticipating the following discussion somewhat, it may be pointed out that the report of a shot, like every short sound pulse, consists of a whole group of sound frequencies, from low infrasonic frequencies to frequencies of several thousands of cycles. It is just these high frequencies present in the sound of the shot which make it appear sharp. But since high-frequency sound is much more strongly absorbed in air than low-frequency sound, high-frequency sounds practically do not reach us when we are far from the gun.

The absorption of sound depends not only on the viscosity of air but also on its thermal conductivity. It should be recalled first of all what thermal conduction is.

If different parts of a body such as a metal rod have different temperatures, then heat passes from hotter parts of the body to cooler parts. Such heat transfer is called thermal conduction*.

The influence of thermal conduction on sound absorption may be observed by means of an experiment with a gas-filled vertical cylinder. Inside the cylinder a well-fitting piston moves without friction. If a small weight is placed on thepiston, the gas will be compressed. This compression takes place at a finite rate. Since the pressure change in the gas is not distributed instantly, the pressure directly under the piston will be higher than in the rest of the gas. Because gas heats up under compression, the temperature of the gas directly under the piston will be higher than that of the rest of the gas. As a result there will be a difference of temperature between the gas in the cylinder and the surrounding medium, and part of the heat will be conducted through the thermally conducting walls of the cylinder into the surroundings. Besides, if compression of the gas was rapid, part of the work was done in overcoming the internal friction (viscosity) of the gas. For infinitely slow compression these effects have no influence and work is done without loss. Therefore, compression of a gas at a finite rate requires more work than an infinitely slow compression. If the weight is removed from the piston, the gas will expand at a finite rate. The pressure of the gas on the piston and the gas temperature immediately below the piston will be lower than in the rest of the gas, and lower than that of an infinitely slow expansion. Therefore, in comparison with an infinitely slow expansion, the work done by the gas will be less.

It follows that the compression and expansion of a gas, if they take place at a finite rate, are irreversible processes accompanied by loss of energy.

* Thermal conduction should be distinguished from convection. In convection heat transfer is due to particles traveling from one region of the body to another, that is, it is due to transfer of mass (in liquids or gases). Thermal conduction, however, depends on heat transfer by means of molecular collision and is not accompanied by a sensible transfer of matter.

The work which must be done in the system (which includes the piston and the gas below it) to compress it to a certain volume is greater than the work done by the system during re-expansion to the original volume. Because of heat exchange between the walls of the cylinder and the surrounding medium during compression of the gas at a finite rate, the amount of heat lost to the surroundings is larger than the amount of heat returned to the system during expansion.

If the piston is made to vibrate within the cylinder, the indicated losses make necessary a certain expenditure of energy in order to sustain continuous vibration; otherwise the vibrations will die out.

During propagation of sound waves, neighboring layers of air (or liquid or solid) are compressed and expanded at a finite rate. The resulting difference in temperature between the compressed and the rarefied layers leads, because of thermal conduction, to heat exchange and equalization of temperature. Since, within an element of volume, more heat is transferred to the surrounding medium during compression than is returned during expansion, the medium is heated up. In other words, the sound waves lose energy, and this energy serves to increase the mean temperature of the air (medium). This is absorption of the energy of the sound waves.

It was noted above that sound propagation is adiabatic, that is, that the temperature difference between the compressed and rarefied layers has no time to become equalized during a half cycle of a sound wave. But this means that in a completely adiabatic process no sound absorption can take place due to heat transfer. This would actually be the case if it were not for thermal conduction. Thermal conduction destroys the adiabatic nature of sound propagation and leads to additional energy absorption by means of heat exchange.

But it must be pointed out that the deviation from an adiabatic nature of the sound is insignificant for practical purposes and does not introduce any essential change in the value of the propagation velocity, so that everything said above (§ 2 of this chapter) remains correct. A change in sound velocity because of deviation from adiabatic conditions becomes noticeable only at very high hypersonic frequencies.

The viscosity and thermal conductivity of air have about equal importance for sound absorption, the influence of viscosity being slightly greater. The effects of thermal conduction become more important when sound is propagated along a solid wall; in this case more important differences in temperature exist between neighboring air elements and between the air and the wall.

Coefficient of Sound Absorption. In order to evaluate quantitatively the sound absorption, an absorption coefficient α is introduced, a quantity which shows how the amplitude of a plane wave decreases with distance. The initial amplitude A_0 of the wave decreases over a distance x down to some value A_x. As shown by experiment, this decrease takes place in accordance with the exponential law*

$$A_x = A_0 e^{-\alpha x},$$

* The decrease in amplitude with distance may take place not only according to an exponential law but also according to a more complicated law (see Chapter VIII, which discusses the absorption of waves of finite amplitude). There also exists absorption which is considerably higher than that indicated by the above formulas.

where $e \sim 2.7$ is the base of natural logarithms.

For $x = \frac{1}{\alpha}$

$$e^{-\alpha x} = e^{-1} = \frac{1}{e}$$

and the decrease in amplitude is $\frac{A_x}{A_0} = \frac{1}{e}$.

Thus, the absorption coefficient is the reciprocal of some distance x_1 over which the amplitude of a propagated wave decreases e times:

$$\alpha = \frac{1}{x_1}.$$

The larger the absorption coefficient, the smaller is the distance over which the amplitude of the wave decreases to a given value.

A theory of sound absorption which takes into account only the influence of the shear viscosity of the medium gives the following expression for the coefficient of absorption α:

$$\alpha = 26.3 \frac{f^2}{c^3 \rho} \eta ,$$

where f is the sound frequency, c the sound velocity, ρ the density, and η the shear viscosity of the medium.

For air at 20°C, $\rho = 1.29 \cdot 10^{-3}$ g/cm^3, $c = 3.43 \cdot 10^4$ cm/sec, and $\eta = 1.71 \cdot 10^{-4}$ g/cm·sec. The above formula gives

$$\alpha = 0.87 f^2 \cdot 10^{-13} \text{ cm}^{-1}.$$

For instance, if $f = 1000$ cycles, then

$$\alpha = 0.87 \cdot 10^{-7} \text{ cm}^{-1}$$

and the distance x_1 over which the amplitude of the sound wave decreases e times (that is, to 37% of its initial value) is

$$x_1 = \frac{1}{\alpha} = 115 \text{ km} \quad (!)$$

If we take into consideration not only the viscosity but also the influence of thermal conduction, then

$$\alpha = 1.24 f^2 \cdot 10^{-13} \text{ cm}^{-1},$$

and instead of 115 km the distance is 80.6 km.

If the intensity of the sound rather than the amplitude is required, it may be remembered that sound intensity is proportional to the square of the amplitude. For instance, if the amplitude of a sound wave decreases by a factor of two, then the sound intensity will decrease by a factor of four. Therefore, the coefficient of absorption for sound intensity will be twice the coefficient of absorption for amplitude. Using the above example ($f = 1000$ cycles) the distance over which the intensity of sound in air is reduced to 37% of its initial value is 40.3 km.

Evidently such low sound absorption does not correspond to reality; sound is propagated through the atmosphere with a much higher absorption rate. The reasons for this will be considered later.

Chapter III

SOUND RECEIVERS AND RADIATORS. OSCILLOGRAPHS

§ 1. Sound receivers

As a result of very tiny vibrations of bodies in air, various sounds are generated — rustling, creaking, tapping noises, etc. These have different frequencies (and consequently different wave lengths), different intensities and propagation directions and, finally, different modes of vibration; air transmits simultaneously all these sounds. In addition to random sounds, such as noise, there exists a great diversity of ordered sounds: speech, music, singing, etc. For vibrations of different frequencies sounds of different pitch are heard. The higher the frequency of vibration, the higher is the pitch of the sound; the larger the amplitude of pressure variation in the sound wave, the higher is the intensity of the sound.

Sensitivity of the Ear. The human ear not only can hear sound over an enormous range of acoustic excess pressures —from several ten-thousandths of a bar to several thousand bars —it can also analyze sound in terms of frequency and intensity. We recognize friends and acquaintances by their voices; from a medley of sounds we single out the one which interests us*. The ear is not equally sensitive to all frequencies. Figure 44 shows the range of hearing of a normal human ear.

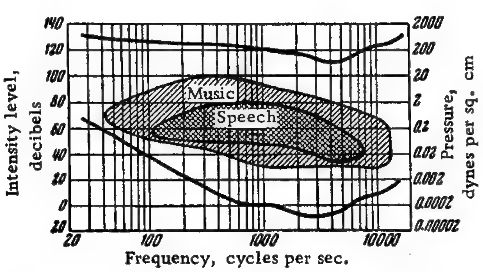


Figure 44. Diagram of sound perception by the human ear

The lower curve represents the "threshold of hearing"; it corresponds to the weakest sounds. The upper curve corresponds to loud sounds,

* The reader can find detailed information on hearing and speech in: Rzhevkin, S.N. Slukh i rech' v svete sovremennykh fizicheskikh issledovanii (Hearing and Speech in the Light of Contemporary Physical Investigations). —Ob"edinenie nauchno-tekhnicheskikh izdatel'stv. 1935.

perception of which causes a nearly painful sensation. Between these two curves lies the whole range of sounds which we can hear, plotted in terms of frequency as well as intensity. The shaded parts of the diagram represent the sounds of music and speech which are most often encountered.

Microphones. Contemporary development of acoustics and radio technology has made possible the creation of the "artificial ear", which enables the detection of elastic vibrations from very low frequencies to frequencies of tens and hundreds of millions of vibrations per second. In the range of audible frequencies (16 to 20,000 cycles) such an artificial ear is called a microphone.

A microphone is an instrument which transforms sound into electrical oscillations which can subsequently be transmitted over wires (as, for instance, a telephone line) or as radio waves.

The Carbon Microphone. There exist a great many types of microphones. The most widely used are carbon microphones; they are found, for instance, in telephone receivers. The simplest carbon microphone is shown in Figure 45. The diaphragm case of the microphone contains fine carbon powder, compressed by a thin metal or carbon diaphragm. The diaphragm is in contact with the powder and with the metal case. Through the back of the case a terminal passes which is insulated from the case by an ebonite bushing. If an electric battery is connected to the case and to this terminal, current will flow through the carbon powder. The carbon powder has the property of changing its electrical resistance as a function of the force with which the individual powder grains are pressed together. When the powder is compressed by the diaphragm, its resistance decreases and the current through the microphone increase; while, if the pressure on the diaphragm decreases, the resistance of the powder increases and the current through the microphone decreases. The sound wave acts upon the diaphragm as a pressure variation with a certain frequency. This change in pressure causes the current through the microphone to vary with the same frequency. When the pressure changes are small, the variations of current through the microphone are proportional to the sound intensity: the greater the pressure variations at the diaphragm, the greater are the changes in the current through the microphone. In this way the changes in electrical current correspond to the frequency and the amplitude of the sound impinging upon the microphone diaphragm.

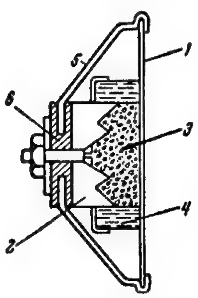


Figure 45. The simplest carbon microphone

1) diaphragm, 2) carbon backing, 3) powdered carbon, 4) felt ring, 5) diaphragm case, 6) ebonite bushing

The carbon microphone is the simplest and the most widely used type of microphone, but it has definite drawbacks. First of all, though it has a high sensitivity, its response threshold is high. This means that the microphone cannot pick up sounds of low intensity; when electrical current passes through the carbon powder, its resistance varies slightly, but rapidly and irregularly, and the microphone is "noisy". It turns out that, even with the best types of carbon microphones, this "noise" is equivalent to the action upon the microphone of an acoustic pressure variation of the

order of 0.01 to 0.02 bar. For sounds with a comparable pressure amplitude the variations in the current through the microphone are masked by the noise of the carbon powder.

Another defect of the carbon microphone is its great irregularity in sensitivity, with respect to the sound frequency. For different frequencies the microphone has different sensitivities; therefore, it distorts the sound, emphasizing certain frequencies and weakening the effect of others. The highest sound frequencies to which a carbon microphone is sensitive are in the 10-12 kc range.

Electrodynamic Microphone. Of higher quality but less sensitive is the electrodynamic, or moving-coil, microphone, shown schematically in Figure 46. The operation of such a microphone is based on the laws of electromagnetic induction: if a conductor is moved in a magnetic field, an electromotive force (e. m. f.) will be induced in it; if the conductor is closed through a resistance, a current will flow through it. This effect is made use of, by the way, in the working principle of all dynamos.

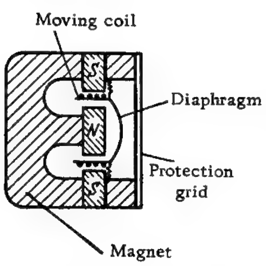


Figure 46. Electrodynamic (moving-coil) microphone

The electrodynamic microphone consists of a magnet with a ring-shaped gap, in which a strong magnetic field is concentrated. In the gap is placed the moving-coil, which consists of a number of windings of thin copper wire, and which is connected to a light diaphragm. When sound pressure causes the diaphragm, together with the coil, to vibrate, the windings of the coil intersect the magnetic field and an e. m. f. is induced in them. If the terminals of the coil are closed by a resistance, a current will flow through it, which will vary in accordance with the variations in the sound pressure acting upon the diaphragm of the microphone.

The sensitivity of the electrodynamic microphone is much lower than that of the carbon microphone, but on the other hand the quality of its sound reproduction is considerably higher.

The electrodynamic microphone differs from the carbon microphone by its reversibility, i. e., it can be used not only as a receiver but also as a radiator of sound. Dynamic loudspeakers, which are widely used in radio receivers, are devices of this type. In order to make an electrodynamic microphone operate as a sound emitter, an audio-frequency voltage must be applied to the terminals of its coil*.

Capacitor Microphone. The capacitor microphone is one of the high-quality microphones used in broadcasting and sound recording. The

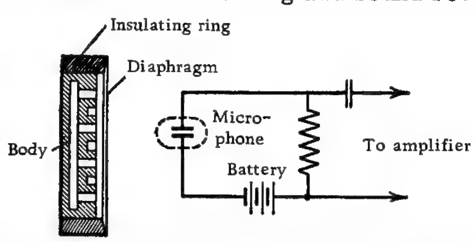


Figure 47. The capacitor microphone and its electrical connection

* For a more detailed description of an electrodynamic loudspeaker see § 3 of this chapter.

construction and electrical connection of a capacitor microphone are shown in Figure 47. The basic part of the microphone is a capacitor, one plate of which consists of a very thin, tightly stretched membrane of duralumin, and the other plate of which is a fixed piece of metal. Voltage from a battery is applied to the plates of the capacitor.

When a sound wave strikes the membrane (i.e., the movable plate of the microphone capacitor), it begins to vibrate at the sound frequency, under the action of the varying pressure. The distance between the movable and the fixed plates changes, which means that the capacitance of the capacitor formed by these two plates also changes. When the membrane vibrates, i.e., when the capacitance changes, an alternating current flows through it, and a corresponding voltage drop is produced across the resistance R and then amplified. The capacitor microphone, in addition to the electrodynamic microphone, is reversible; it can operate similarly to a receiver but also as an emitter of sound.

Under the action of sound waves an e.m.f. appears across the terminals of the microphone. The frequency of the changes in this e.m.f. is equal to the sound frequency and its amplitude is proportional to the amplitude of the sound. But this e.m.f., even for comparatively strong sounds, does not exceed a few thousandths of a volt. Weak sound signals, which are still comparatively easily perceived by the ear, produce only a few microvolts (millionths of a volt) at the output of the microphone.

But by means of electronic equipment the alternating voltages obtained from a microphone can be amplified from a few microvolts to dozens and, if necessary, to hundreds of volts; and the power may be increased from a few billionths of a watt to several watts or even kilowatts.

Such an amplification of the e.m.f. generated at the terminals of the microphone is realized using electronic amplifiers. The modern development of acoustics, as well as acoustical applications in quite diverse fields of science and technology, are due to the achievements of contemporary electronics. Without electronics it is not possible to make a single forward move in acoustics today.

It is not the task here to describe the function of the various electronic instruments which have wide application in acoustics — amplifiers, electronic oscillators, and electronic oscilloscopes. Nevertheless, we shall very briefly describe the possibilities which electronics provides for acoustics.

Connection of the Carbon Microphone. Figure 48 shows schematically the circuit associated with a carbon microphone, by means of which the alternating voltage produced by the sound is to be amplified. For this purpose a step-up transformer is usually included in the microphone circuit. If the primary winding of the transformer, which is connected to the microphone through a battery, has one tenth as many turns as the secondary winding, then an alternating voltage which is ten times as great as the voltage generated in the microphone circuit will appear at the grid of the amplifier tube. An amplifier tube will amplify this voltage several dozen times, so that across its plate-load resistor there is not just the few millivolts (thousandths of a volt), which may have existed at the microphone terminals, but rather several tenths of a volt.

This voltage can be further increased by adding a second amplifier tube, so that the alternating output voltage is increased to several volts.

Contemporary electronic tubes can amplify alternating voltages dozens of times, and this amplification can be achieved even at very high frequencies,

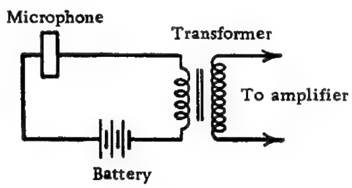


Figure 48. Connection circuit of a carbon microphone

to tens and hundreds of millions of oscillations per second. Electronic amplifiers consisting of several tubes can amplify a voltage several million times. Almost the smallest alternating current or voltage which can be produced can be amplified to a required value.

Here, we must say "almost", because there is a definite minimum voltage which can still be amplified. This limit is about a fraction of a microvolt and exists because of the electronic nature of an electric current. Since an electric current consists of moving electrons, a different number of them reaches the anode of a tube at different moments of time. As a result, the mean current through the tube varies (fluctuates). This phenomenon can be illustrated by the following example. If birdshot is poured out onto a table in a stream, the number of pieces of shot reaching its surface will vary continually. A typical noise is heard, caused by the different pieces of shot striking the table. By analogy to this, the current fluctuation in a tube is called the "shot effect". The shot effect, or internal noise of a tube, which is similar in nature to the noise in a carbon microphone, masks the signal which is to be amplified*.

In addition, if an electrodynamic or a capacitor microphone is used as a sound receiver, the thermal motion of the air molecules will cause the moving system of the microphone to be bombarded by molecules and consequently to perform small random deviations about its equilibrium position (Brownian movement). This limits definitely the sensitivity of such microphones; it may be noted that the Brownian movement determines the sensitivity limit of a mirror galvanometer, the mirror of which also makes random movements under the action of molecular impact. Essential in an electrodynamic microphone may also be the thermal fluctuations of electrons, which cause the appearance of random electromotive forces at the coil terminals; these also influence the sensitivity limit. However, these phenomena become noticeable only at a very high microphone sensitivity and have so far no practical importance (with the exception of certain special precision measurements); the threshold of response of these microphones is determined in practice by inevitable acoustical disturbances in the form of all sorts of noises and sounds, as well as by the disturbances due to electrical and mechanical fluctuations, which were mentioned above.

* In addition to the "shot effect", the amplification limit of an electronic tube is determined by a number of other fluctuation effects.

§ 2. Loop oscillographs and electronic oscilloscopes

With the aid of a given microphone, combined with an amplifier, weak sounds may be amplified and transmitted over long distances through wires or on radio waves; in addition, the characteristic properties of sound itself can be studied: frequency, amplitude of sound pressure, and the distribution of sound in space (sound field). Very important in such investigations is the oscillographic recording of the electrical oscillations which correspond to sound or mechanical vibrations.

While considering the oscillation of a pendulum, we pointed out the possibility of recording the mechanical vibrations on moving paper. The recording of mechanical, as well as electrical, oscillations and also the direct observation of the oscillation — the mode of oscillation, the frequency and amplitude, and the damping — are today realized in many ways. The basic instruments for these purposes are the loop oscillograph and the electronic oscilloscope.

Loop Oscillograph. In the loop oscillograph the variations in current are recorded by a device which makes use of a loop. This device, a vibrator, is shown schematically in Figure 49.

Between the pole pieces of a magnet a loop 1, made of thin phosphor bronze wire, is drawn over a pulley 3; a small mirror 2 about 1 mm² in area is attached to the loop. The tension in the loop is regulated by the spring 4, which allows some variation of the natural frequency of the moving system. The current to be recorded is supplied to terminals K_1 and K_2, to which the ends of the loop are connected.

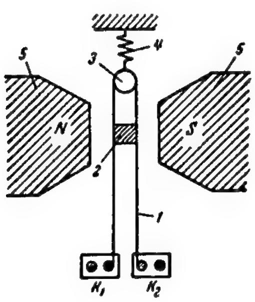


Figure 49. Schematic diagram of the loop vibrator

1) loop, 2) mirror, 3) pulley, 4) spring, 5) magnet

When current flows in the loop, the loop turns under the action of the magnetic field, and the mirror turns with it. For small turning angles of the mirror, this angle is proportional to the current. The moving system is made extremely light, in order to make the natural frequency of oscillation of the loop as high as possible. Then the moving system will follow as exactly as possible variations in the external force (in this case — variations in current). If the natural frequency of vibration of the moving system is close to the frequency of the external force, the vibrations of the mirror establish themselves slowly, and the obtained record is distorted.

This system is placed in a container, filled with special oil in order to increase the damping of the oscillations of the moving system. Heavy damping ensures an undistorted record of the variations in current.

A pencil of light rays from a special lamp in the illuminator falls on the mirror of the vibrator, is reflected from it, and passes through a system of lenses onto photographic paper or film. The roll of photographic paper or film is pulled through, with a definite and strictly constant speed, by a special device driven by a motor. If a constant current passes through the loop, the mirror does not move and the light spot from the mirror will trace a straight line on the photographic paper. If the current varies, the mirror vibrates; and instead of the straight line a curve will be recorded

which repeats all the oscillations of the current. The speed of movement of the photographic paper may be changed from a few millimeters per second to several meters per second, so that with a loop oscillograph slow as well as rather rapid processes can be recorded.

Instead of a single vibrator contemporary loop oscillographs contain several vibrators, which allow simultaneous recording of several processes. For instance, in loop oscillographs used in seismic prospecting for useful minerals (see Chapter X) the number of vibrators may be 24 or more.

Figure 50 shows a recording of the noise made by pieces of shot dropping onto a metallic surface. The noise was picked up by a microphone, which was connected to an amplifier; the alternating voltage from the amplifier was applied to the loop. Above the noise curve another loop recorded a 250-cycle sinusoidal current, obtained from an audio oscillator. The recording shows that the noise represents a very complex sound, the intensity and mode of vibration of which vary with time in an extremely random way.

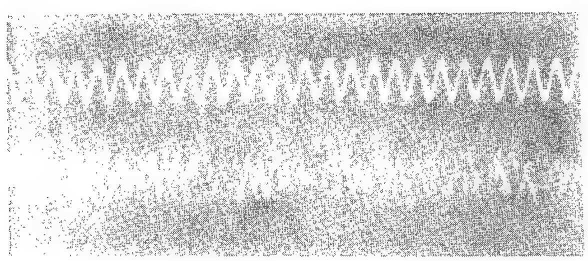


Figure 50. Record of shot noise.

Above the noise curve another loop recorded a 250-cycle sinusoidal current

Electronic [cathode-ray] Oscilloscope. The investigation and recording of the wave forms of sound oscillations can also be realized quite clearly using the electronic oscilloscope — an instrument which receives very wide application in radio, television, radiolocation, and practically every field of physical experiment.

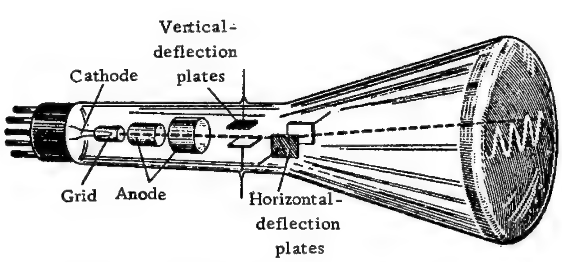


Figure 51. Schematic drawing of a cathode-ray tube

The basic element of the electronic oscilloscope is the cathode-ray tube, shown schematically in Figure 51. It is an evacuated glass envelope, inside of which is an electron gun, consisting of a cathode, a grid (control element), and two anodes—the first and the second. The electrons emitted

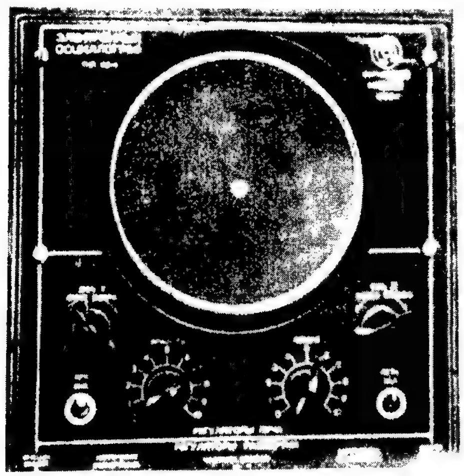


Figure 52. Light spot on the screen of an electronic oscilloscope

In this photograph, as in Figures 54 and 56-58, only the upper half of the front panel of an electronic oscilloscope is shown

by the cathode (which is heated by current) flow toward the second anode of the tube, to which a positive d.c. voltage of approximately 1000 volts or more* is applied. Corresponding d.c. voltages are also applied to the grid and the first anode; and these electrodes of the electron gun serve to focus the electron beam into a thin electron ray, similarly to the focusing of light rays by a lens and an objective. The thin electron beam passes through the hollow cylindrical anodes and through the gaps between two pairs of deflection plates, finally striking the screen of the tube. This screen is coated with a special fluorescent substance, and when it is struck by the fast-moving electron beam, a bright luminous spot appears (Figure 52). Two

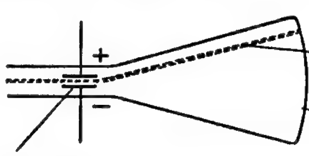


Figure 53. Deflection of an electron beam by plates

* There are some special-purpose cathode-ray tubes which operate even at comparatively low voltages — at around 100 volts.

pairs of deflection plates, one pair oriented perpendicular to the other, deflect the electron beam when voltage is applied to them. The beam is deflected up or down (plates on the left in Figure 51) and to the right or left (plates on the right in Figure 51), depending on the polarity of the voltage applied to the plates.

Figure 53 shows the case when the upper plate has a positive, and the lower plate a negative, voltage.

Since electrons are negatively charged particles, they are attracted to the upper plate and repelled by the lower plate; the beam is deflected upward, and so is the spot on the screen. The amount of deflection is proportional to the voltage applied to the plates. If an alternating voltage is applied to the vertical-deflection plates, the electron beam will move upward during the half-cycle when the upper plate is positive, and downward during the next half-cycle, when this plate is negative.

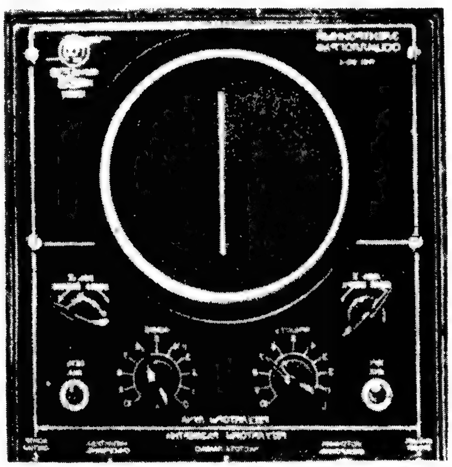


Figure 54. Vertical luminous line

For a frequency less than 14 or 15 cycles, it is possible to see the light spot move up and down on the screen of the tube. For higher frequencies the eye cannot follow the motion of the point and a bright line is observed (Figure 54), the length of which is proportional to the voltage on the plates*. In order to observe a waveform on the screen of the cathode-ray tube, instead of just a vertical line characterizing the amplitude of oscillation, an alternating voltage of saw-tooth form must be applied to the horizontal plates of the tube (Figure 55). This type of alternating voltage can be obtained with the aid of electronic circuits; when a saw-tooth voltage is applied to

* Certain types of fluorescent screens possess the property of persistence; when the luminous spot moves over the screen, it leaves behind it a trace, which remains visible for a certain time. A screen coated with a green luminous substance has a persistence of about 8 msec; one coated with blue has a persistence of several microseconds. In a number of cases it is desirable to have long persistence (for instance, for observation of short-time processes); certain fluorescent substances give a persistence of 10 seconds or more.

the horizontal plates, the luminous spot will move across the screen with a constant velocity.

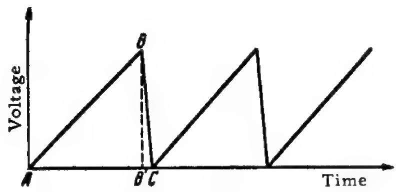


Figure 55. Saw-tooth voltage (sweep)

For instance, let the saw-tooth voltage be zero (point A in Figure 55), and let the luminous spot (the trace of the electron beam) be at the left side of the screen. With increasing voltage the spot will move with constant velocity to the right and, at the maximum voltage (point B), will reach a certain point on the right side of the screen. At this moment the voltage amplitude will rapidly fall to zero (point C) and the luminous spot will return to its original position. The time interval B'C is called the return time; this time is small in comparison with AB', the time of forward movement, and can be made equal to a fraction of one percent of it. The saw-tooth voltage which is applied to the horizontal plates of the tube is called the time sweep, or simply the sweep. When no voltage is applied to the vertical plates, a horizontal, luminous straight line is obtained on the screen of the tube (Figure 56).

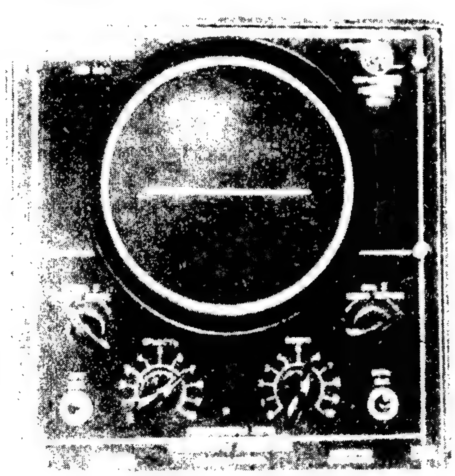


Figure 56. Horizontal luminous line

If, simultaneously with the application of a sweep voltage to the horizontal plates, some voltage which is to be examined is applied to the vertical plates (for instance, a voltage which varies sinusoidally), then, similarly to the recording of a sine wave by a light spot from the vibrator of a loop

oscillograph when photographic paper is passed before it, the electron beam will trace out a sine curve on the screen. If the sweep time coincides with the period of the sinusoidal oscillation applied to the vertical-deflection plates, then on the screen of the tube one period of the sine curve will be displayed. If the sweep time is longer than the period of the oscillation being studied, then several complete periods of the sine curve will be observed, the number of which is determined by the ratio of the sweep time to the period of oscillation.

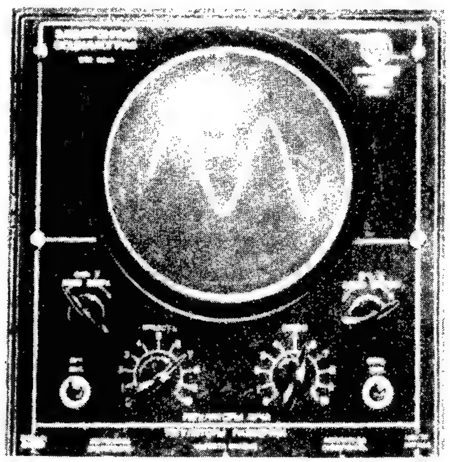


Figure 57. Two periods of a sine curve

A sinusoidal voltage is applied to the vertical plates of a cathode-ray tube. A sweep voltage is connected, the period of which is twice the period of the sinusoidal oscillation

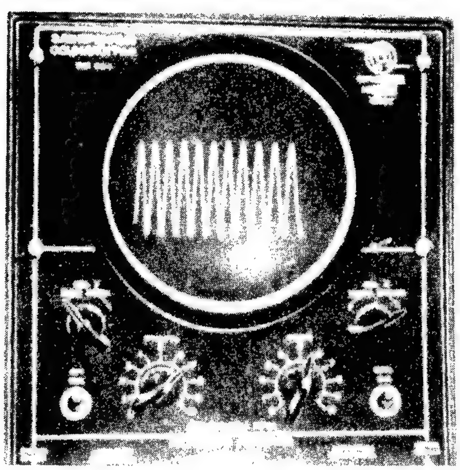


Figure 58. Eleven periods of a sine curve

The sweep frequency can be varied continuously, so that any number of complete periods of the sine curve may be obtained on the screen. Figure 57 shows a photograph of the screen of an electronic oscilloscope when the sweep time is twice the period of the sinusoidal oscillation, and two periods of the sine curve are displayed; in Figure 58, 11 periods of the sine curve are shown.

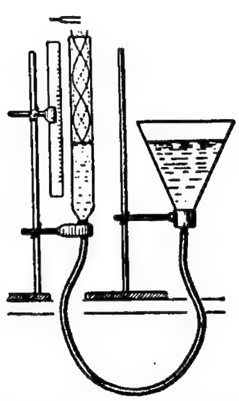


Figure 59. Resonance of an air column in a pipe under the action of a vibrating tuning fork

The height of the column can be changed by raising or lowering the water funnel

In order to obtain a motionless picture on the screen of the tube, the ratio between the sweep frequency and the frequency of the sine wave being studied must be perfectly constant. If this is not true, the pattern on the screen will move; this "creeping" of the figure is greater, the larger the variation of sweep frequency with respect to signal frequency. By means of synchronization the sweep time of an electronic oscilloscope is made precisely equal to some integral multiple of the period of the oscillation being studied; thus, when the signal frequency changes, the sweep frequency changes in equal proportion, and no relative change occurs in the signal and sweep frequencies.

By means of the electronic oscilloscope complicated forms of current and voltage oscillation can be observed and measured. In addition to direct observation, the picture is often photographed.

§ 3. Sound sources

Sound waves in air are due to vibrating bodies, and every vibrating body may be a source (emitter) of sound. In whistles, sirens, organ pipes, musical wind instruments, and in the human vocal apparatus itself, the vibrating sound source is the air. In stringed musical instruments the string, connected to the body of the instrument, is the source; and in contemporary loudspeakers it is some sort of vibrating surface.

Later, sound production by the piezoelectric and magnetostrictive effects will be discussed, as well as sound of thermal and aerodynamic origin.

Pipes and Sirens. The Voice. Let us perform the following experiment, using a glass pipe which is open at its upper end. The narrow lower end of the pipe is covered by rubber tubing which is connected to a water-filled vessel (Figure 59). At the open end of the pipe a vibrating tuning fork is placed. If the water level is changed by raising or lowering the vessel of water, the sound will be heard to intensify and reach considerable loudness, and then to become weak again. The intensification of the sound is a result of resonance. Under the action of an external sinusoidal force (the vibrating prongs of the tuning fork) forced oscillations of the air column in the pipe arise; when the natural frequency of these oscillations coincides with the natural frequency of the tuning fork, the amplitude of the air-particle

displacement in the pipe increases. The sound will continue for a certain time even after the tuning fork has been taken away from the opening of the pipe, but with the air column in the pipe performing natural oscillations.

As a result of reflection of the sound from the water surface, standing waves arise in the pipe; the mechanism of their formation is similar to the formation of standing waves while jerking a cord fixed at one end. It is easy to find by experiment that resonance occurs when the length L of the air column is equal to an odd multiple of $\lambda/4$, where λ is the wave length of the sound in air

$$L=\frac{n\lambda}{4} \qquad (n=1,\ 3,\ 5,\ldots).$$

Since the sound is reflected almost completely from the water surface (the reason for this will be discussed later) practically no energy will penetrate from the air into the water; it is not difficult to understand that in this case there must always be a displacement (and velocity) node and a pressure loop at the air-water interface. However, the above condition for the formation of standing waves in a pipe also means that at the open end of the pipe there is a pressure node, and therefore the energy flux through the open end of the pipe is also equal to zero. This means that there is no energy exchange between the oscillations of the air column in the pipe and the surrounding air.

However, sound is heard from the pipe. It is clear therefore that the above conclusion is incorrect. An erroneous result was obtained because the foregoing formula for L was taken to be precise, while it is only approximate.

The quantitative theory taking into account the oscillation of the air column directly over the open end of a pipe gives, for natural frequencies of the pipe, instead of the approximate formula

$$L=\frac{n\lambda}{4} \qquad (n=1,\ 2,\ 3,\ldots)$$

the more precise formula

$$L+0.63R=\frac{n\lambda}{4} \qquad (n=1,\ 3,\ 5,\ \ldots),$$

where R is the radius of the pipe. If the opening of the pipe is placed flush into an infinite flat plate (baffle), which is practically the case when the pipe has a flange comparable in size to the wave length of the sound, the final correction is not $0.63\,R$ but $0.8\,R$. Taking into account the vibration of the air column above the open end of the pipe therefore leads to an apparent increase of $0.63\,R$ at the open end of the pipe. Between the opening of the pipe and the next displacement node there must be a section of about $\frac{\lambda}{4}-0.63\,R$, as shown in Figure 59.

The end correction must be doubled for a pipe which is open at both ends. In such a case, for a round pipe, the following formula is true

$$L+1.37R=\frac{n\lambda}{2} \qquad (n=1,\ 2,\ 3,\ldots).$$

Let us now take a glass test tube (or a metal or wooden pipe) closed at one end, and let us blow across the open end, a sound of a definite frequency will be heard, i.e., a simple musical tone. The pitch of this tone will depend on the length of the pipe; the longer the pipe, the lower is the

frequency of the sound. The explanation for the appearance of the sound is as follows: as a result of blowing at the open end of the pipe a certain rarefaction of the air in the pipe occurs. However, as soon as the rarefaction has taken place, the air jet will be sucked into the pipe somewhat and will create in it an excess pressure due to air compression. This excess pressure will proceed to equalize, again creating a rarefaction in the pipe. The compressions and rarefactions will happen one after the other, and the air column in the pipe is compressed at one moment and rarefied at another.

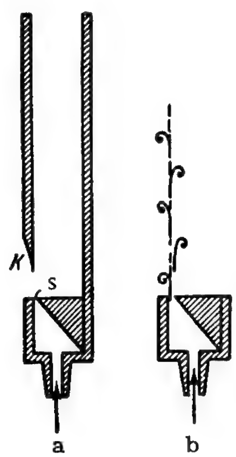


Figure 60. a) Cross section of an organ pipe; b) periodical formation of vortexes during the passage of an air jet through a slit

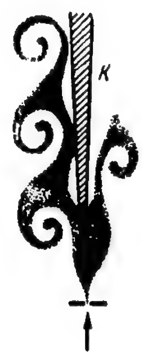


Figure 61. Vortexes in an organ pipe; air containing smoke is blown through the pipe

Because of reflection from the closed end, standing waves will arise in this air column and will continue as long as air is blown across the open end of the pipe. The vibrations of the air column in the pipe are transmitted to the surrounding air and thus sound is radiated. The wave length of the sound radiated by such a whistle, as we know from the foregoing, is equal to four times the length of the pipe. But this is only a very crude explanation. In the previous experiment with the tuning fork we dealt with forced and free oscillations of an air column. Here, the much more complicated phenomenon of self-oscillation is involved. Self-oscillation of a gas column also takes place in organ pipes.

In order to obtain sound from a pipe which is closed at one end, it is necessary to apply some skill—to blow with a certain force and with a certain position of the lips. To avoid these inconveniences, and also to obtain stronger oscillations from blowing air into the tube, the air stream in the organ pipe (or whistle) is directed toward a sharp edge. Figure 60a shows the cross section of an organ pipe with one end closed; through the lower pipe the air from the organ box enters a chamber which is closed from above and has a thin slit S.

Let us examine in more detail the process of sound formation in an organ pipe, and let us clarify why this process should be considered a self-oscillation.

A photograph of a stream of smoke shows that at the slit S a complicated periodic formation of vortexes takes place, as shown schematically in Figure 60b. These periodic vortexes, apparently emanating from the slit, pass one after the other either to the left or to the right of the sharp edge K (Figure 61). Periodic pushes act on the air column in the pipe, and as a result vibrations arise in it. These vibrations, in turn, have a reciprocal action on the process of vortex formation at the slit. The vibrations of the air column in the pipe attain a frequency which is near to one of its natural frequencies, the particular one depending on the speed at which air is blown through the slit.

The frequency and the nature of the oscillations of the air column in an organ pipe are determined by the properties of the oscillating system itself; the regulation of the energy input for the maintenance of the oscillations of an air column in an organ pipe represent a typical case of self-oscillation, which has been discussed in the first chapter.

Beside closed pipes, open organ pipes are also used; the wave length of the sound from such a pipe is half as great (i.e., the frequency is twice as high) as that from a closed pipe of the same length.

Especially strong sounds are obtained by means of sirens, which are used for signaling on ocean-going vessels, for factory whistles, or for air-raid warning. The construction of a siren and the principle of its operation can be briefly described as follows. A siren consists of a pipe, the lower end of which is closed by a fixed disk with a row of holes in it. Against this disk is placed a rotating disk, which is turned by means of compressed air, a motor, or a spring device. The rotating disk also has a number of

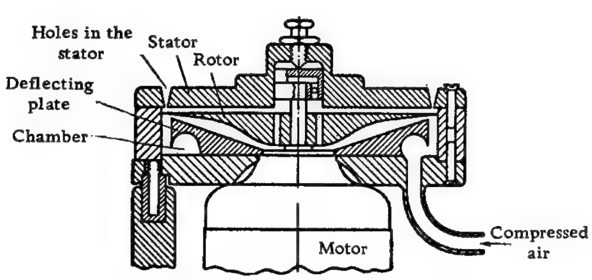


Figure 62. Construction of a powerful siren (frequency range from 3 to 30 kc)

holes. When the disk rotates, these holes alternately coincide and do not coincide with the holes in the fixed disk and, accordingly, the path to the pipe opens and closes. An air stream at a pressure of several atmospheres is blown into the pipe through the fixed disk and is interrupted by the rotating holes. These interruptions result in a series of pulses or shocks in the air, which create a strong sound the frequency of which is determined by the number of shocks per second, i.e., by the number of holes in the disks and the velocity of rotation of the movable disk The characteristic wailing sound of a siren is explained by the fact that, after the siren has been started, the rotation speed of the turning disk increases progressively and then, having reached its highest value, decreases when the motor is

shut off; accordingly, the frequency of the emitted sound increases progressively and then decreases.

Thus, the sound emitted by a siren is the result of breaking up a stream of air.

Figure 62 shows the construction of a powerful siren intended for various acoustical investigations (precipitation of smoke in factory chimneys, condensation of fog, investigation of sound propagation, etc.). Compressed air enters a chamber from which, after passing a deflecting plate, it passes through gaps between the teeth of a spinning rotor (the rotor has the shape of a disk 15 cm in diameter; it is made from a special aluminum alloy and has 100 identical teeth). The rotor is driven by a 1.2 kw motor, which can make from 133 to 340 revolutions per second. The stator also has 100 holes, with diameters somewhat smaller than the rotor teeth. The siren (Figure 63) is provided with an exponential horn (see below) and produces sound in a range between 3 and 30 kc. The acoustical power of such a siren reaches 150 watts. The sound intensity produced by this siren is so great that in its vicinity special precautions must be taken in order to protect the auditory apparatus.

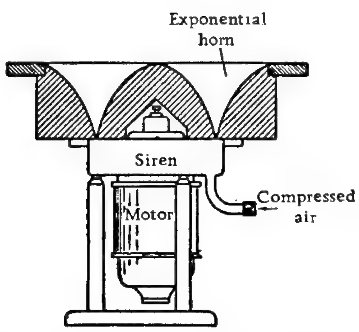


Figure 63. External view of a powerful siren

The formation of sound in the so-called reed organ pipes (Figure 64) and reed-type musical instruments also occurs as a result of breaking up the air jet; the reed vibrates under the action of the air stream and converts it into a series of separate pushes.

The human vocal apparatus represents a highly developed type of reed instrument, in which the vocal chords play the role of reeds. These special elastic chords close the upper end of the windpipe, which is analogous to the air pipe. The air expelled from the lungs passes through the slit formed by the chords and causes them to vibrate; the air stream is interrupted at the natural frequency of oscillation of the chords; and as a result sound is emitted. By changing the tension of the vocal chords, their natural frequency of oscillation, and consequently the frequency of the sound produced, is varied. The frequency and the nature of the oscillations of the vocal chords are defined by the properties of the vibrating system itself—the vocal apparatus; also, the regulation of the energy input in order to maintain the oscillation is effected by the vocal chords, i.e., by a mechanism which is a part of the vibrating system. Therefore, the vibrations of air

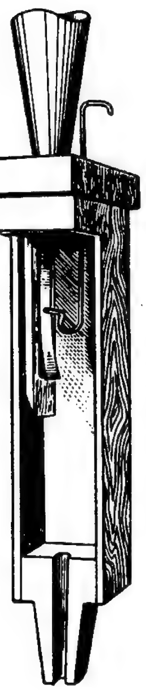


Figure 64. Cross section through a reed organ pipe

in the cavity of the mouth, just as the vibrations of an air column in an organ pipe, represent a typical case of self-oscillation.

Sound Radiation. Influence of Size of Vibrating Surface. Vibrating bodies have different capacities for radiating sound waves. Some bodies are good sound emitters, while others, even when vibrating with large amplitudes, are poor emitters. The ability to radiate sound depends on the dimensions of the surface of the body. The larger this surface, the better is the sound emission.

This can be easily demonstrated by means of a vibrating tuning fork. When the tuning fork vibrates, each of its prongs produces compression of the air on one side and rarefaction on the other side. The difference in pressure for the two sides of the prong equalizes quickly, because the dimensions of a tuning-fork prong are small and the equalization of pressure in air takes place at the velocity of sound. Thus, during vibration the prong of the tuning fork essentially transfers air from one side of it to the other, and nearly all the energy of vibration is expended on this transfer.

Sound is produced by the vibrating fork only because this transfer of the air is not complete. Figure 65a shows the movement of the prongs of a vibrating tuning fork, and Figure 65b shows the circulation of air currents around these prongs. If a plate AB — shown by a dashed line in Figure 65b — is placed near one prong of the tuning fork (so that this plate does not touch the prong), it will hinder the transfer of air and the sound from the tuning fork will increase.

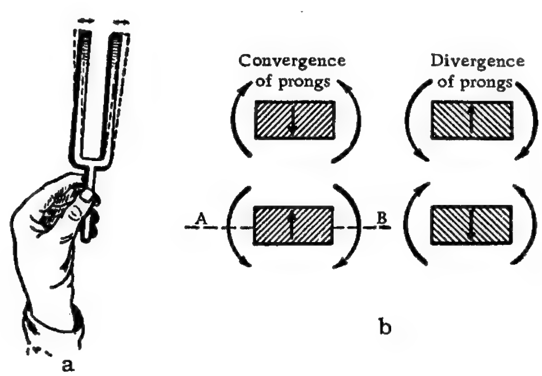


Figure 65. a) Movement of the prongs of a vibrating tuning fork; b) direction of air-particle movement around the prongs of a vibrating fork

The plate AB, in the position indicated by the dashed line, reduces circulation of air about the prong, which results in an increase of sound radiation by the tuning fork

Normally, other methods are used in order to increase the radiation of sound from a tuning fork. The most usual method consists in placing the fork on a wooden box, which is open on one or both sides. The vibrations of the tuning-fork prongs are transmitted through its stem to this box, where they cause vibrations of the air column in it. Such a box is called a resonator; it radiates sound in the same way as the pipes which were discussed above. If the box is closed on one side and its length is equal to a quarter wave length of the sound generated by the tuning fork, the vibrations of the air column will be the strongest (as in a pipe closed at one end). Under such conditions resonance occurs: the frequency of the external force (vibration of the tuning fork) coincides with the natural frequency of vibration of the air in the resonator box. The surface of he resonator box also radiates and thus contributes to the amplification of he tuning-fork note.

The dependence of sound radiation on the size of the vibrating surface is also essential for strings. A string by itself radiates an insignificant amount of sound energy, since its thickness is small in comparison with the wave length of the sound radiated by it. Sound production by stringed instruments —piano, violin, harp, and others —occurs only because the strings, at the point where they are attached, transmit their vibrations to the body of the instrument, which, together with the air in it, is the actual source of sound.

In order to reduce the harmful effect of pressure equalization, the vibrating surface should have larger dimensions than the wave length of the sound radiated. If the period of vibration of the sound-radiating surface is equal to T, then during this time the pressure pulse which is responsible for equalizing the pressure on the two sides of the surface will be able to travel a distance equal to the length of the radiated wave $\lambda = cT$. If the size of the radiator is larger than the wave length λ, this pulse will not be able to travel around the surface, and pressure equalization will occur to a much smaller degree.

While the above discussion gives some indication of the role of the size of the vibrating surface during sound radiation, it is too rough, as well as oversimplified. Let us consider this question in more detail.

Generally speaking, every vibration of a body in air (and also in a liquid) is accompanied by two phenomena:

1) sound radiation occurs and sound waves propagate from the vibrating body into the surrounding space;

2) the body carries along with it particles of the medium (in the case of the tuning fork, air is transferred from one side of the prong to the other) so that in its immediate vicinity periodic changes in the local motion of particles of the medium take place.

This last circumstance has the result that, together with the vibrating body, a certain mass of the medium also vibrates—known as the additional or adherent mass. In order to maintain the local air flow the power which causes the body to vibrate must have a wattless or, as it is called, reactive (inertial) component, which is determined by the adherent mass; as a consequence, all the power applied to the body is not used for radiation, but only a part of it (active component).

The reactive component becomes apparent in the phase difference between the sound pressure P, produced by the vibrating body, and the acoustic

particle velocity v; when the pressure increases, the velocity v does not immediately reach the value which corresponds to the given P.

It has been noted that, the lower the sound frequency, the greater is the wave length of the sound and the larger must be the surface of the vibrating body, in order to produce a sufficiently powerful sound. At low frequencies, however, it is practically impossible to utilize surfaces with dimensions larger than the wave length, since the surface would become too large. For instance, at a frequency of 50 cycles the wave length in air is about 7 m. Therefore, in order to increase the intensity of low-frequency sound. methods other than an increase of the surface dimensions are employed, as will be discussed later.

Telephone. One of the most widely used sound sources is the telephone. A microphone transforms mechanical vibrations of the air particles (sound) into electrical oscillations; in contrast, the telephone transforms electrical oscillation into mechanical vibration (sound). Normally, in telephone receivers and radio headphones, an electromagnetic mechanism is used; but there is also a telephone with a moving coil, corresponding to the electrodynamic microphone, but which is used as a sound source.

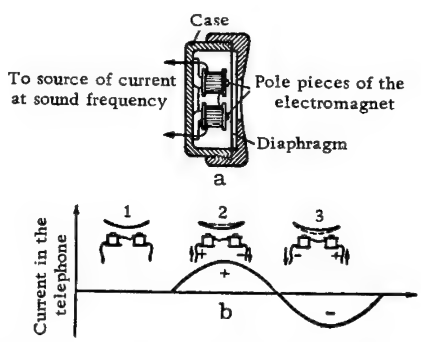


Figure 66. a) Schematic diagram of the telephone; b) sequence of operation of the telephone; dashed line indicates the original position of the diaphragm

An electromagnetic telephone is shown schematically in Figure 66a. Within the case of the telephone is an electromagnet, consisting of an iron horse-shoe magnet with pole pieces around which coils are wound (in addition to iron, alloys with good magnetic properties may be used). A thin round iron diaphragm is placed a small distance from the pole pieces.

When no current flows through the coil, the magnetized core attracts the diaphragm; but since the diaphragm is fastened at its edges and is elastic, it does not touch the pole pieces, but only bends slightly (Figure 66b left). When alternating current passes through the coils the force of attraction changes. During the positive half-cycle of the alternating current the magnetic field generated in the pole pieces adds to the effect of the permanent magnet, and the diaphragm bends more strongly (Figure 66b center). During the negative half-cycle of the alternating current the generated magnetic field will have a direction opposite to the field of the permanent magnet; the attraction decreases and the diaphragm moves away

(Figure 66b right). As a result, the diaphragm vibrates about the original position at the frequency of the alternating current passing through the coils of the telephone. Since this current is of sound frequency, a sound wave is generated in air the frequency and waveform of which correspond to the oscillations of the current in the coils.

It is very important for the magnetic field of the alternating current through the telephone coil to remain at all times smaller than the magnetic field of the core itself under conditions of no current. If this were not the case, or if the core were not magnetized at all, the telephone diaphragm would vibrate much more weakly, and at a frequency double that of the alternating current. In fact, if the core is not magnetized, the diaphragm will be attracted to it during positive as well as negative half-cycles of the current. This is because the iron is attracted equally by the north as well as by the south pole of a magnet. Consequently, if the core of the telephone electromagnet is not magnetized in advance, or if the changes in the magnetic field of the core, caused by the passage of current through the coil, are larger than the magnetic field of the core itself, then, for every period of current oscillation in the coil, the diaphragm will perform two oscillations and frequency doubling takes place. Thus, the permanent magnet increases the sensitivity of the telephone and eliminates distortion of the frequency of the sound.

For greater sensitivity of the telephone and a more uniform effect upon the diaphragm several electromagnets are placed in the telephone; however, in such a case the shape of the core may become rather complicated. The telephone can operate not only as a sound source but as a receiver of sound vibrations as well, i.e., it can be used as a microphone.

The telephone represents a very sensitive transducer; its diaphragm begins to vibrate in the presence of very weak currents through the coil of the electromagnet. But the telephone cannot produce sound which can be heard at any distance — in order to hear the sound the telephone must be held to the ear. There are, however, a great number of different sound radiators of considerable loudness, first of all the dynamic loudspeaker.

Electrodynamic Loudspeaker. The electrodynamic loudspeaker works on the same principle as the electrodynamic, or moving-coil, microphone;

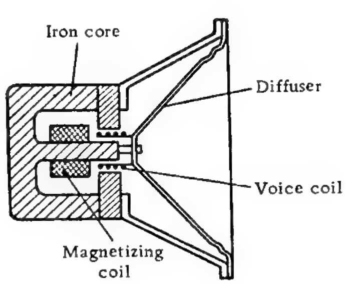


Figure 67. The parts of an electrodynamic loudspeaker

however, in order to obtain sufficient acoustic power, the dimensions of the loudspeaker are much larger.

Figure 67 shows the cross section of a cone, or diffuser, loudspeaker. A paper diffuser, in the form of a truncated cone, is glued to a short cylindrical frame (made also of paper). On the frame is wound the voice coil, through which the signal current from the amplifier passes. In addition to sufficient power, a high-quality loudspeaker must produce uniform sound radiation over a wide range of frequencies — e.g., from 30 to 10,000 cycles. The reproduced sound (for instance, music) may have a variety of quickly changing frequencies and amplitudes; and the moving system of the loudspeaker, as a result of its mass, elasticity, and friction, possesses several natural frequencies of oscillation. Near these frequencies resonance may appear; if the frequency of the external force coincides with the natural frequency of the loudspeaker, the sound will be distorted. In order to avoid this situation the natural frequency of the moving system is made as low as possible (about 100 cycles); in addition the damping of the system is increased, so that the establishment of oscillation is barely detectable by the ear.

The diameter of the diffuser is normally 20-30 cm, i.e., it is much smaller than the wave length of the low-frequency sound which is to be produced. In order to improve the reproduction of low-frequency sounds by the loudspeaker, it is often placed into a recess in a board, which acts as a baffle, reducing the harmful influence of pressure equalization on both sides of the diffuser; the chassis of a radio receiver has the same function.

Electrodynamic loudspeakers vary considerably in size and, accordingly, in the acoustic power radiated. Ordinary household loudspeakers in radio sets have a power of about several watts. The power of a loudspeaker is understood to be the electrical power supplied to it from the amplifer. But the acoustic power radiated comprises, for the diffuser loudspeaker, only 2-3% of the electrical power supplied; the other 97-98% of the power is used to heat the voice coil and is lost in other ways.

The efficiency of a powerful electrodynamic loudspeaker provided with a horn is considerably higher; about 50-60% of the electrical power supplied can be radiated by the loudspeaker as sound energy.

Horns. Effective sound radiation can take place only if the dimensions of the vibrating surface are larger than the wave length of the radiated sound. For low sound frequencies this problem is solved by the use of a horn; a horn makes it possible to obtain effective radiation over a comparatively wide frequency range.

A horn is a pipe with a gradually increasing cross section. Thus, for a conical horn, the cross-sectional area S increases with the distance x in accordance with the law

$$S = Ax^2,$$

where A is some constant. Since the sound intensity I is proportional to the square of the sound pressure p,

$$I \sim p^2$$

and since, on the other hand, the sound power per unit area [intensity] is inversely proportional to the area,

$$I \sim \frac{1}{S},$$

we have for a conical horn

$$p \sim \sqrt{I} \sim \frac{1}{\sqrt{S}} = \frac{1}{x\sqrt{A}}.$$

It follows that in a conical horn the pressure amplitude decreases in inverse proportion to the distance, i.e., in the same way as in a spherical wave. Therefore, a conical horn only increases the axial concentration

of energy (see below); namely, it increases the directivity of the radiation, but does not improve the ratio between the active and reactive components of the radiation resistance.

The most successful form of horn is the exponential horn, in which the cross section increases in accordance with the exponential law

$$S = S_0 e^{\beta x},$$

where S_0 is the cross-sectional area at the beginning of the horn (its most narrow part, called the throat) and β is a certain constant, called the index of widening [flare factor] of the horn. In an exponential horn the acoustic pressure decreases considerably more slowly than in a conical horn; the large end opening of the horn ensures a high active component of the radiation resistance. By means of an exponential horn a small radiator can be given advantageous operating conditions, and quite high acoustic power may be obtained from it at low frequencies.

The Thermophone and Ionophone. Periodic changes in the temperature of the medium (for instance, by means of a thin metal wire with alternating current flowing through it) also lead to sound radiation. The thermophone works on this principle, and is used for a number of absolute acoustical measurements. The thermophone consists of a thin (10-20 microns) foil (normally made of platinum) in a suitable holder, through which an audio-frequency current is passed. In order to avoid frequency doubling (see the description of the telephone), a d.c. current of a value larger than the a.c. current is also made to flow through the foil.

The thermal principle is also applied in the ionophone.

Radio technicians working with powerful radio transmitters noticed that when a more or less stable discharge appeared on antenna installations (the so-called corona discharge), the sound transmitted by the radio station was heard in the vicinity of the discharge. The investigation of this phenomenon showed that the discharge was a sound source because the variation in volume, associated with the discharge, took place in phase with the modulation frequency. On this principle is based the sound source known as the "ionophone". The ionophone is a sound-producing instrument which utilizes changes in the state of a ionized gas (air) in order to radiate sound and ultrasound waves. Figure 68 shows schematically such an instrument. In order to ionize the air, voltage from a high-frequency generator is used; the frequency of the generator is about 20 megacycles and its voltage is 8-10 kv. The output of this generator is modulated by a low-frequency voltage from an amplifier. The high-frequency voltage, modulated by the audio-frequency signal, is applied to the platinum electrode 1, which is soldered into a tube of fused quartz 2; the second electrode 3 (at ground potential) is a metal ring around the quartz tube. Between these electrodes, which constitute a condenser, a capacitive current passes (the so-called displacement current). Sometimes the quartz tube is surrounded by an evacuated housing 4. Finally, the quartz tube is connected to an ordinary horn 5. The discharge appears on the platinum electrode; the high temperature of the discharge gap and the high frequency create conditions favorable to a quite high ionization of the air in the tube. The discharge itself does not produce sound when a nonmodulated high-frequency voltage is applied, and spark discharge, accompanied by "crackle", is absent.

When modulated high-frequency voltage is applied, however, the volume taken up by the discharge changes in proportion to the audio-frequency voltage. The state of a unit volume of ionized air in the discharge space depends basically on its temperature, according to the equation $pV = RT$. A change in the temperature T causes a change in the product pV. The rate of temperature change is determined by the quantity of heat generated per unit time in the discharge space, and by the rate of heat loss; the heat-loss mechanism is not as yet completely clear.

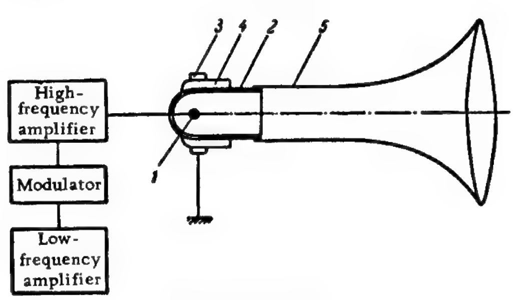


Figure 68. Ionophone

1) platinum electrode, 2) quartz tube, 3) annular electrode, 4) metal housing, 5) horn

A great advantage of the ionophone is its wide frequency range — from very low frequencies (determined by the properties of the horn) up to about 100 kilocycles. In the audible-frequency range it has a very uniform frequency response and therefore high-quality reproduction. Among its drawbacks are the relatively complicated construction and the very short service life (about 500 hours) of the electrode, due to disintegration.

Public Address Systems. Powerful loudspeakers, especially of the horn type, are very important for installations providing sound to large areas and transmitting speech or music to many listeners. Special installations, with powerful amplifiers or groups of loudspeakers in order to spread sound over a whole area occupied by listeners, are used in the squares and streets of towns during demonstrations and mass gatherings, in open-air theaters, stadiums, and airfields. Powerful mobile loudspeaker units were used during the Great Patriotic War as a means of agitation and propaganda among the enemy armies. Powerful loudspeakers are also used in industry, to control the work in large factory areas, in control centers of railway stations, in ports, during the construction of buildings, etc. One of the tasks of providing sound systems for large squares is that of ensuring equal listening conditions at all places to which sound is supplied, i.e., the maintenance, as far as possible, of a uniform loudness level.

Directivity of Sound Radiation. It has been noted that a vibrating rod which is larger than the length of capillary waves on a water surface propagates plane waves. The radiation of these waves takes place in a definite direction, perpendicular to the rod; thus the rod is a source of waves possessing the property of directivity. Attentive observation shows that, at a certain distance from the rod, the plane waves gradually turn into spreading circular waves, because of diffraction. It is found that the distance at which such spreading becomes noticeable is greater, for larger dimensions of the rod in relation to the length of the capillary waves generated by it. If the dimensions of the rod are considerably smaller than the wave length, or if, instead of the rod, a ball vibrates, then circular waves will appear which will spread in all directions and the source will not have the property of radiation directivity.

In the same way, if the dimensions of a diffuser are larger than the wave length of the sound which it radiates, the radiation will have a definite directivity. The larger the vibrating surface in relation to the wave length of the sound, the more directed is the sound radiated.

The directivity of the sound radiation can be shown in different ways. In the open air, where there are no obstacles to the propagation of sound, the sound field of a loudspeaker or any other sound source can be investigated using a microphone connected to an amplifier*; this is possible when there is no wind, or at least when the wind is weak.

Nearly all sound sources radiate with an intensity which is directionally nonuniform; and nearly all receivers react differently to sound waves arriving from different directions. For such investigations it is therefore necessary to know in advance the directional properties of the microphone (or to have a microphone which does not exhibit directivity).

If a microphone M, which is suspended several meters above a sound source (Figure 69), is moved, then when the microphone is exactly over the radiator J (position A, which corresponds to the axis of the radiator),

* Sound-field investigations are also carried out in special [anechoic] sound-measurement chambers, the walls of which absorb sound well and do not give reflection.

the instrument connected to the amplifier output gives a maximum indication. In positions B, C, and D the indications of the instrument will decrease correspondingly.

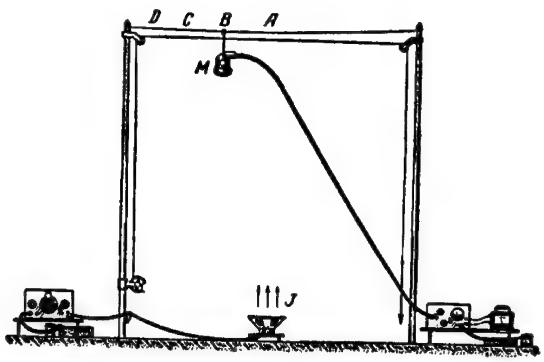


Figure 69. Directivity measurement in the open air

Left–sound generator; right–amplifier and measuring instrument.

If the displacement of the microphone with respect to the sound source is not along a straight line, but along the periphery of a circle of radius JA, and if, when recording the results of measurement on a graph, the indications of the instrument (which are readings proportional to the sound pressure at the point of microphone placement) are marked off in every direction from the center of the graph, information is obtained about the distribution in space of the sound energy from the sound source (using a graph in polar coordinates). Such a graph of the energy distribution is space around a radiator is called the r a d i a t i o n - d i r e c t i v i t y p a t t e r n*. Thus, the directivity pattern indicates the amount of sound energy concentrated in a given direction.

Figure 70 shows the directivity patterns of a vibrating disk which is set into a baffle, for different ratios of disk diameter to wave length of the sound which it radiates. Sound radiation by such a disk can be considered as radiation into the complete half-space. When the diameter *D* of the disk is smaller than $\lambda/4$, the intensity of radiation is uniform in all directions (curve 0 in Figure 70a). As the ratio of disk diameter to sound wave length increases, directivity appears and the radiation cone becomes more and more narrow (curves 1, 2, 3 in Figure 70a). Here, in addition to the basic beam of rays, so-called side lobes appear; after the basic sound rays an isobar of zero pressure appears and after it the next beam (Figure 70b and c). The angle between the first two zero-pressure lines (which enclose the basic beam) is called the divergence angle of the sound rays. It is seen from Figure 70 that as the sound frequency increases (the wave length

* It is, of course, simpler to obtain the directivity pattern by the method indicated in Figure 69, whereby the microphone is moved along a straight line and not along a circle. It is often used when rough information about the directivity is required. But in this case the mistakes caused by the increased distance traversed by the sound, in comparison with the distance JA, should be taken into account, particularly when the microphone is very far from point A.

decreases) the divergence angle of the sound rays decreases, and the radiation becomes more directed; on the contrary, when the frequency decreases (the wave length increases) this angle will increase, and the directivity becomes less and less*.

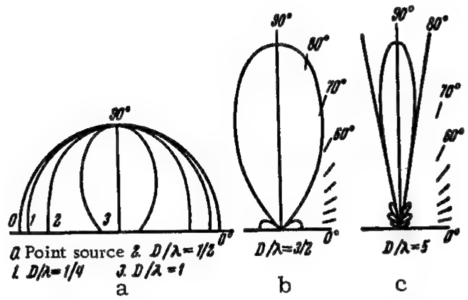


Figure 70. Directivity patterns of a disk with diameter D, vibrating in a large baffle—for different ratios D/λ

With an increase in the ratio of the dimensions of the vibrating surface to the wave length of the radiated sound, the radiation directivity becomes sharper.

In horn loudspeakers the directivity pattern is defined by the dimensions of the output surface (the mouth) of the horn, which can be considered to be closed by a vibrating disk equal in size to the mouth.

The mouth of a horn can have a large surface, so that the directivity pattern of horn loudspeakers can be made quite sharp. In the often-used exponential horns the directivity pattern has the approximate form of an elongated ellipsoid of revolution.

In the same way as a radiator, a microphone can exhibit directivity of reception, and everything which pertained to the directivity of radiators is valid for sound receivers as well. A radiator and a receiver of equal dimensions have the same directivity patterns.

The type of directivity pattern (Figure 70) for sound radiation by a vibrating disk can be explained using Huygens principle. If the radius R of the disk is considerably smaller than the wave length λ, the distances between the point at which the sound intensity is measured (point of observation) and any other points on the vibrating surface of the disk are practically the same; these distances differ by some values Δ, which are small in comparison with λ. If the surface of the disk is divided into small elements, then, in accordance with Huygens principle, each of these elements can be considered as the center of an elementary spherical wave. For $R \ll \lambda$, independently of whether the point of observation is on the axis or not, the elementary spherical waves arrive at this point in phase and therefore intensify each other; here, spherical waves are involved, the source of which does not exhibit directivity.

* There is a relation $\sin 2\alpha = 1.2\frac{\lambda}{D}$ between the divergence angle 2α, the diameter D of the disk, which vibrates as a piston, and the wave length λ of the sound. This formula allows determination of the divergence angle of the sound from a radiator, provided its dimensions and the wave length λ of the sound are known.

When the wave length λ decreases (or R increases), the differences Δ become comparable to λ. Because the distance L from the source to the point of observation may be great (i.e., $L \gg R$), then in the axial direction the differences Δ will remain small as before, and on the axis the elementary waves will be mutually intensified. But in directions which are not axial, due to the quite high values of Δ and to interference, a partial compensation of the sound pressure takes place. At small λ (large R) compensation may become complete, and directions of zero pressure will appear.

Thus, the type of directivity pattern is explained by interference effects. But it should always be remembered that all radiation is accompanied by diffraction effects and, in general, the directivity of the radiation is determined by the combined action of interference and diffraction.

Thus, for instance, we have indicated that, in the case of radiation of plane capillary waves by a vibrating rod, after a certain distance the plane waves change into diverging ones, because of diffraction. For vibrations of a piston (or any other surface) experiment and theory indicate that for $R \gg \lambda$ the waves are plane, or nearly plane, at a small distance from the piston. But at a certain distance along the axis the radiation from the piston begins to diverge because of diffraction. From this moment on, the sound intensity decreases and finally becomes (at large distances) inversely proportional to the square of the distance from the source. The quantitative theory shows that radiation has a tendency to become dispersed considerably, beginning at a distance $L_{cr} = \frac{R^2}{\lambda}$. The point on the axis near L_{cr} indicates the beginning of the change of the beam of sound rays from parallel to divergent.

Pulsating Sphere (Zero-Order Radiator). All the great variety of different sound sources can be described in terms of very simple acoustic radiators (or their combinations), the simplest of which is the pulsating sphere. Sound radiation by a pulsating sphere lends itself quite easily to quantitative calculation. Such a sound source is called a radiator of zero order. A pulsating sphere is shown schematically in Figure 71; although the radius of the sphere changes, its center remains fixed. Visually, a pulsating sphere may be represented by a rubber balloon, into which air is pumped during the first half-cycle and out of which air is pumped during the second half-cycle. It should be noted therefore that for pulsation of the sphere some external source of air is necessary, which imparts to the walls of the sphere a radial motion of definite velocity. Theory shows that for such a simple source the sound radiation is determined by the "active radiation resistance"

$$R_{act} = \rho c S \frac{\left(\frac{2\pi a}{\lambda}\right)^2}{1+\left(\frac{2\pi a}{\lambda}\right)^2},$$

where ρc is the characteristic acoustic impedance of the medium, a is the radius of the sphere, $S = 4\pi a^2$ is the surface area of the sphere, and λ is the wave length of the radiated sound. The mean value W_{act} of the acoustic power radiated during one period is thus*

$$W_{act} = \frac{1}{2} R_{act} v_m^2,$$

where v_m is the maximum value of the oscillation velocity of the sphere surface. The reactive (or wattless)

* This formula recalls the expression for the average electrical power which is generated during one period in a conductor with ohmic resistance R, through which a current I flows:

$$W_{el} = \frac{1}{2} R I^2.$$

radiation resistance has the form

$$R_{react} = \rho c S \frac{\frac{2\pi a}{\lambda}}{1+\left(\frac{2\pi a}{\lambda}\right)^2}.$$

Operating conditions will be most advantageous when R_{act} is larger than R_{react}. The above formulas for

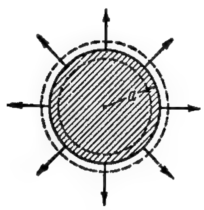


Figure 71. Pulsating sphere (first-order radiator)

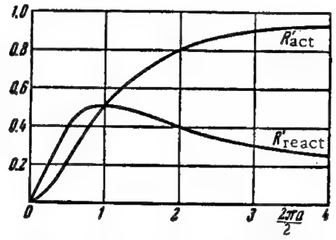


Figure 72. Active and reactive components of radiation resistance of a pulsating sphere as functions of $\frac{2\pi a}{\lambda}$

R_{act} and R_{react} show that this requirement is fulfilled when $\frac{2\pi a}{\lambda} > 1$, i.e., when the radius of the sphere is greater than the wave length of the radiated sound. Figure 72 is a graph of $R'_{act} = \frac{R_{act}}{\rho c S}$ and $R'_{react} = \frac{R_{react}}{\rho c S}$ as functions of $\frac{2\pi a}{\lambda}$. It is clear that for small values of $\frac{2\pi a}{\lambda}$ $(a < \lambda)$ R'_{react} is the determining factor, and that for large values of $\frac{2\pi a}{\lambda}$ $(a > \lambda)$ R'_{act} becomes considerably larger than R'_{react}.

It is seen from the above expression for the active radiation resistance, which determines the acoustic power radiated, that R_{act} is proportional to the characteristic acoustic impedance ρc of the medium and to the surface area S. It follows that the acoustic power radiated is greater for a larger acoustic impedance. This partly explains the greater ease of sound radiation into water as compared to radiation into air; for air (as noted earlier) $\rho c = 41$ cgs units, and for water $\rho c = 150{,}000$ cgs units.

Acoustic Dipole (First-Order Radiator). The next elementary radiator in the order of complexity is the so-called acoustic dipole, or radiator of the first order. This type of sound source is shown schematically in Figure 73. It represents a sphere which performs motion (oscillations), for instance, in the direction of the

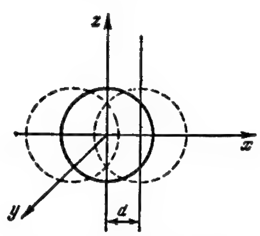


Figure 73. Acoustic dipole

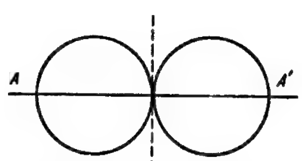


Figure 74. Directivity pattern of an acoustic dipole

x axis, so that the center of the sphere is displaced by a certain amplitude d; the radius of the sphere does not change as a result of this. This representation of a dipole is equivalent to another, which consists in representing the dipole as two spheres vibrating with equal intensity, at a distance $2d$ from each other in the x direction, and vibrating in antiphase —when one sphere causes a compression, the other causes a rarefaction. For this reason an acoustic dipole is also called a "double source".

A freely vibrating surface represents a double source, or acoustic dipole.

It is easy to see why a surface, radiating toward both sides, is called a double source: while one compresses the air, the other side produces rarefaction, and the faces of the surface can be compared to sources having opposite signs of pressure.

A typical double source is the tuning fork whose prongs vibrate in antiphase and at equal amplitudes.

Figure 74 shows the directivity pattern of an acoustic dipole with its axis directed along the line AA'; it has the shape of a figure eight, where the sound field in the direction perpendicular to AA' is zero. This type of directivity pattern is the result of interference.

In a number of cases not just one but two or a greater number of radiators are involved, all driven by the same oscillator. The sound field of a group of radiators becomes very complicated because of interference effects, with several maxima and minima of sound intensity in different directions. Groups of radiators are used when it is required to obtain a sharp (or some other specific) directivity pattern. Here, the ratio of distances between radiators to the wave length λ of the sound radiated plays an important role.

Quadrupole (Second-Order Radiator). An even more complex sound source is the quadrupole, or second-order radiator, which represents a combination of four pulsating spheres or two dipoles (Figure 75). In Figure 75 a single dipole is indicated by an arrow with the letter F, representing force. In the case of the

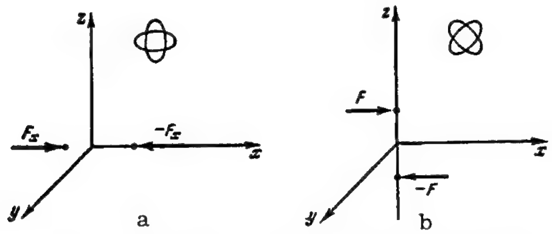


Figure 75. Longitudinal (a) and transverse (b) quadrupoles

longitudinal quadrupole, the directions of the dipole forces are opposite to one another, and the forces are directed along the line connecting them. Thus, it can be imagined that the quadrupole represents (as shown in the upper part of Figure 75) a sphere which, without changing its volume and keeping its center fixed, is transformed, during the first half-cycle, into an ellipse, with its major axis perpendicular to the x axis, and during the second half-cycle into an ellipse with its major axis parallel to the x axis (longitudinal quadrupole) (Figure 75a). The direction of the quadrupole forces may be perpendicular to the line connecting the dipoles; in such a case we have a transverse quadrupole (Figure 75b).

Chapter IV

SOME EXPERIMENTS WITH SOUND. SOUND ANALYSIS

§ 1. Some experiments with sound

With an audio oscillator, loudspeaker, microphone, and electronic oscilloscope a number of important and instructive experiments may be carried out.

Let us connect the output terminals of the audio oscillator to the terminals of the loudspeaker. Its diffuser will vibrate and sound will be generated. Let us now connect the microphone (which is on a stand with wheels) to the input terminals of the oscilloscope, using a shielded cable (Figure 76). When the sweep of the oscilloscope is turned on, a sine curve

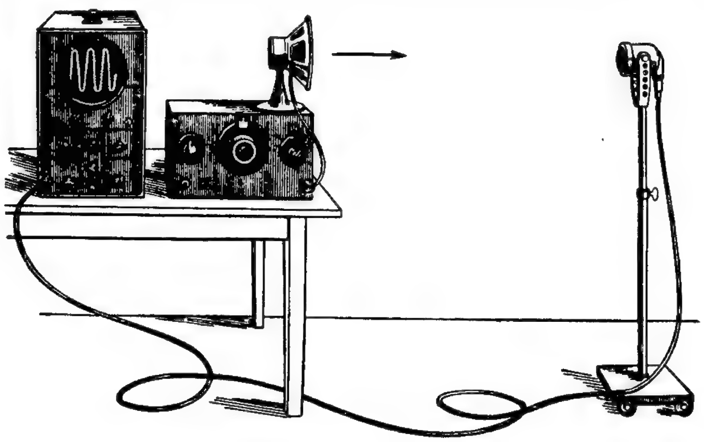


Figure 76. Experiments with sound, using an audio oscillator, loudspeaker, microphone, and electronic oscilloscope

is observed on the screen of the tube*. If the loudspeaker is disconnected from the oscillator, then instead of the sine curve a horizontal luminous line (sweep) is observed. This experiment proves that, during the passage of sound, an air pressure (the acoustic, or sound, pressure) varying sinusoidally arises at the point of microphone placement. If we disconnect the microphone from the oscilloscope and connect the oscillator to the same oscilloscope terminals, then on the scope a sine curve with the same frequency

* If the amplifiers in the oscilloscope (for the vertical- and horizontal-deflection plates) have only a very small gain and if the sensitivity of the microphone is low (e.g., capacitor microphone), then it is necessary to use a microphone amplifier after the microphone. Using a type EO-4 oscilloscope and a carbon microphone, no preliminary amplification is necessary.

as the previous sine curve appears. It follows, therefore, that the frequency of the sound-pressure changes is equal to the frequency of the audio-oscillator signal.

As the voltage supplied by the oscillator to the loudspeaker is increased, the sound becomes louder; accordingly, the amplitude of the sine curve observed on the oscilloscope increases. A previous statement which was made without proof may now be verified: the greater the amplitude of sound pressure, the louder the sound. It is equally simple to verify another previous statement: the higher the frequency of oscillation, the higher the pitch of the sound. For this purpose let us increase the oscillator frequency; the pitch of the sound becomes higher and at the same time the number of sine-wave periods observed on the oscilloscope increases.

As the frequency of the audio oscillator is increased, the sound becomes higher in pitch, weaker, and finally, at a frequency of about 20,000 cycles, sound is no longer heard at all. Meanwhile, a sine curve will be visible as before on the oscilloscope (as the frequency of the audio oscillator is increased, the sweep frequency must also be increased, in order to have the same number of sine-wave periods on the screen as previously). If the loudspeaker is disconnected from the oscillator, then again just a horizontal line appears on the oscilloscope. This experiment proves that the ear is not able to perceive sound of a frequency higher than approximately 20,000 cycles, as mentioned earlier.

Now if the microphone is moved away from the loudspeaker, the amplitude of the sine curve on the oscilloscope will be seen to decrease progressively. This indicates that the amplitude of sound pressure at the point of microphone placement decreases.

§ 2. Measurement of sound velocity using an acoustic interferometer

Acoustic Interferometer Using Traveling Waves. With this apparatus it is quite easy to make exact measurements of sound velocity. This can be done in the following way. The sweep of the oscilloscope is turned off. Then, with the microphone connected, a vertical luminous line will be visible on the oscilloscope, with a length which is twice the amplitude of the sine wave which is observed with the sweep on. Now the microphone is disconnected and to the horizontal-deflection plates of the oscilloscope is connected, instead of the sweep, the signal from the audio oscillator; this signal is varied until the resulting horizontal line is equal in length to the vertical line which was seen before. The microphone is reconnected to the input terminals of the oscilloscope. A figure will be observed which is similar to those in the photographs of Figure 77. These are Lissajous figures. Lissajous figures are obtained by the combination of two mutually perpendicular sinusoidal oscillations whose frequencies may be expressed as a ratio of whole numbers: 1:1; 1:2; 1:3; 2:3; etc. If both oscillations are sinusoidal and have equal frequencies (frequency ratio 1:1) and amplitudes, as in the present case, the Lissajous figures obtained will resemble those in Figure 77 and Figure 78 (upper row). Depending on the phase difference between these oscillations, the shape of the figures will change. If the phase difference is zero, the Lissajous figure will be a

straight line, inclined 45° to the horizontal and vertical axes. For a phase difference of 45° an ellipse is obtained; for a phase difference of 90°, a circle; and at 135°, again an ellipse. For a phase difference of 180° a straight line again appears, but its direction is perpendicular to the straight line which corresponds to a phase difference of 0°. Above 180° the forms

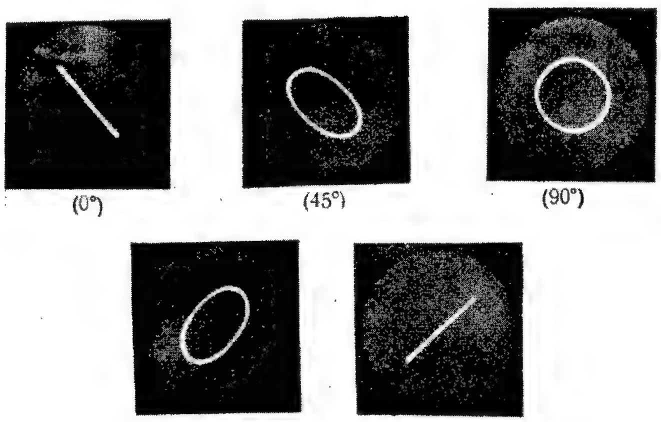


Figure 77. Lissajous figures for 1 : 1 frequency ratio for equal amplitudes of sinusoidal oscillation

Photographed from the screen of a cathode-ray tube. Phase difference indicated in parantheses.

of the figures repeat themselves, and at 360° a straight line again appears. If the amplitudes of the oscillations are not equal, the general form of the Lissajous figures remains the same; however, the ellipses become more flattened, the circle is transformed into an ellipse, and the slope of the straight lines obtained at phase differences of 0 and 180° is different. For intermediate phase differences intermediate forms of Lissajous figures are obtained. If the semiaxes of the ellipses are measured, then simple formulas indicate the phase difference which corresponds to the figure on the oscilloscope. Thus, the figure obtained makes it possible to measure the phase difference between electrical oscillations. In this application the oscilloscope represents an instrument which makes possible measurement of the phase difference between two sinusoidal oscillations. Such instruments are called phasemeters.

When two mutually perpendicular sinusoidal oscillations with different frequencies are combined, the Lissajous figures are more complicated. Stationary figures arise on the screen only when the ratio between the frequencies of the oscillations may be expressed as a ratio of whole numbers.

Let us return to the experiment and suppose that, having connected the instruments as stated above (Figure 79), a Lissajous figure appears which has the form of an ellipse. If the stand with the microphone is gradually moved away from the loudspeaker, the Lissajous figure on the oscilloscope

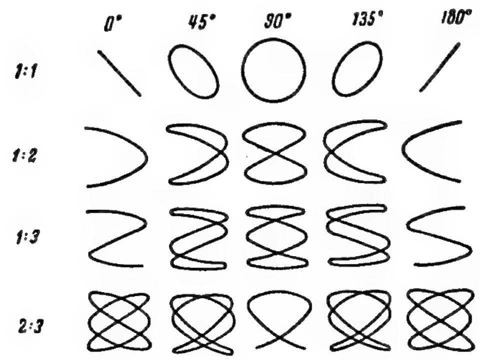


Figure 78. Lissajous figures for frequency ratios: 1:1; 1:2; 1:3; 2:3, with phase differences in 45° steps

is seen to rotate. It will take the position shown in Figure 78, top right (180°), and then will pass through all positions from right to left (to 0° or 360°); thereby giving figures corresponding to 45°, 90°, and so on.

Since it has already been verified that changes of sound pressure correspond to changes in electrical voltage, the existing phase difference between the sound pressures at the points of placement of loudspeaker and microphone corresponds to the phase difference of the voltages at the oscillator output and the microphone-amplifier output. Therefore, we can draw the following conclusions from the above experiment.

a) The phase difference between the sound pressures at the points of placement of loudspeaker and microphone does not change with time. If this were not the case, the figure on the screen of the cathode-ray tube would not be stationary.

b) The phase difference varies from point to point as the microphone is moved.

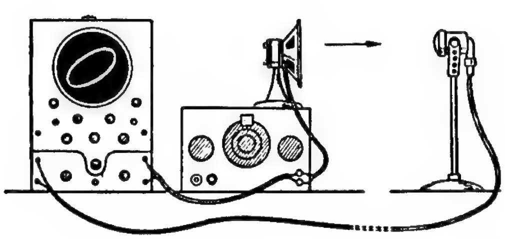


Figure 79. Connection of instruments for observation of phase-difference changes between loudspeaker and microphone

An explanation of these phenomena may be found using the following reasoning. Suppose that the wave length of the radiated sound is known, and the microphone is placed at a distance from the loudspeaker which is exactly equal to this wave length Let us attempt to see whether, with time, the phase of the sound pressure at the loudspeaker changes relative to the phase of the sound pressure at the microphone. Figure 80 shows successive

configurations of the sound pressure at these points*. We see from these graphs that during the entire period T the phases of pressure at both points coincide. When there is a maximum or a minimum of sound pressure at the loudspeaker, there is also a maximum or minimum at the microphone: the phase difference of sound pressures at these points is zero. If the distance is decreased to $\frac{\lambda}{2}$, the variations in pressure will be in antiphase. In the same way, if, instead of one wave length, the distance between loudspeaker and microphone is increased to any integral number of wave lengths, the pressure variations will be in phase; and if it is an odd number of half-waves, they will be in antiphase. Finally, when the distance is not an integral number of half-waves, the phase difference of the variations in sound pressure will have intermediate values.

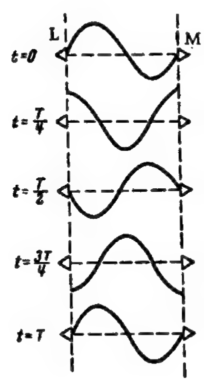


Figure 80. The phase of the sound pressure at the microphone relative to the phase of the sound pressure at the loudspeaker, during one period (T) of oscillation

The loudspeaker and the microphone are located one wave length from each other

Thus, the result of the above experiment becomes understandable; when the microphone is moved away from the loudspeaker, the difference in phase of the sound pressures at their points of placement must vary; accordingly, the difference in phase between electrical oscillations from the outputs of the audio oscillator and the microphone must vary, too.

Between any two given points of a region within which sound waves propagate, the phase difference of the pressure variations (and of other acoustic quantities: displacement of air particles, acoustic particle velocity, etc.) remains stationary with time.

It is known that one complete cycle of change of the Lissajous figures corresponds to a displacement of the microphone from the loudspeaker by one wave length of the sound. Therefore, by measuring the distance over which this figure will complete one full cycle of variation, the wave length λ is known. The sound frequency f may be read from the dial of the audio oscillator, and so it is easy to find the velocity of sound:

$$c=\lambda f.$$

In this way the experiment allows us to measure the velocity of sound. In order to perform this measurement more precisely the microphone must be moved over as large a distance as possible, in order to obtain a larger number of complete rotations of the Lissajous figure, because then the error in calculating the phase difference will become smaller. If the cathode-ray tube possesses good focusing and produces a thin luminous line on the screen, then near phase differences of 0° or 180° (straight line) phase variations of about 1° can be noticed (during this change the straight line becomes a highly elongated ellipse); therefore, the count of the

* In reality, for a separation of a single wave length the situation will be more complicated; but this is not essential here.

number of cycles of change of the Lissajous figures should begin and end at the straight-line position.

Now let us place the microphone at a certain distance D from the loudspeaker, and let us measure this distance. Suppose we find $D = 342$ cm. If it is recalled that the sound velocity at an air temperature of 18°C is 342 m/sec, and if the frequency is read from the audio oscillator as $f_1 = 5000$ cycles, then the wave length of the sound from the loudspeaker will be $\lambda_1 = \frac{34\,200}{5000} = 6.84$ cm, and the distance D will correspond to $N_1 = \frac{D}{\lambda_1} = 50$ wave lengths. Let us slowly increase the oscillator frequency. It will be seen that the Lissajous figures on the oscilloscope will change in the same way as they did when the microphone was moved. This is to be expected, because at higher frequencies the wave length decreases and therefore more wave lengths correspond to the distance D.

Let us adjust the frequency of the oscillator to $f_2 = 6000$ cycles. At this frequency $\lambda_2 = \frac{c}{f_2} = \frac{34\,200}{6000} = 5.7$ cm, and the number of wave lengths corresponding to the distance D will be $N_2 = \frac{D}{\lambda_2} = 60$. During the frequency change from 5000 to 6000 cycles the Lissajous figures will be seen to perform $N_2 - N_1 = 10$ complete cycles of change.

It is now easy to conclude that if the distance D and the frequency difference $f_2 - f_1$ are measured, and the corresponding number $(N_2 - N_1)$ of complete cycles of change of the Lissajous figures is counted, then the velocity of sound may be determined:

$$\text{at frequency } f_1,\ D = N_1\lambda_1 = N_1\frac{c}{f_1} \text{ and } f_1 = \frac{N_1 c}{D};$$

$$\text{at frequency } f_2,\ D = N_2\lambda_2 = N_2\frac{c}{f_2} \text{ and } f_2 = \frac{N_2 c}{D}.$$

Subtracting the first equation from the second,

$$f_2 - f_1 = \frac{(N_2 - N_1)\,c}{D},$$

and so

$$c = \frac{D\,(f_2 - f_1)}{N_2 - N_1}.$$

In contradiction to the first method of measuring the sound velocity c (where the sound frequency remained constant and the sound receiver was moved), in the second method (where the sound frequency changes and the distance between sound source and sound receiver remains stationary) the precise value of the velocity may be found only if this velocity is constant at all frequencies from f_1 to f_2. In other words, the second method of measurement is useful only when dispersion does not occur.

All the experiments for measuring the sound velocity which have been briefly described here can be easily made in a room, provided the instruments indicated above are available.

Changes in phase difference with changes in oscillator frequency can not only be observed on an oscilloscope, but also recorded with a loop oscillograph. The arrangement for such recording, using a special phasemeter and a loop oscillograph, is shown in Figure 81.

The voltages from the audio oscillator and from the microphone amplifier are applied to the phasemeter – an instrument which indicates changes in the phase difference between two sinusoidal oscillations. Without going

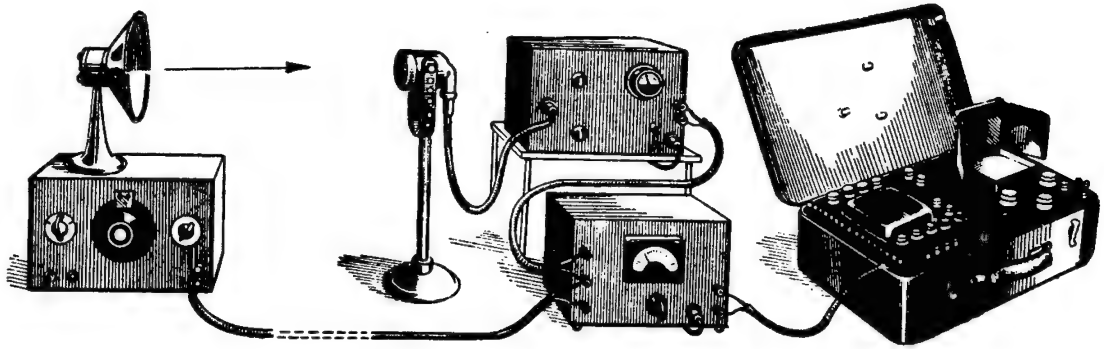


Figure 81. Measurement of sound velocity using an acoustic interferometer and recording equipment

Left—audio oscillator and loudspeaker; right—microphone, amplifier, phasemeter, and loop oscillograph.

Figure 82. Recording of phase-difference changes with a recording phasemeter and loop oscillograph.

Distance between loudspeaker and microphone, 14.7 m. Number N equal to 127; frequency change from 2000 to 5000 cycles.

into the details of operation of this instrument, we will only note that when the phase difference (between the two signals applied to its inputs) changes, the current through the loop galvanometer changes too. In other words, the current through this galvanometer is proportional to the phase difference $\varphi_2 - \varphi_1$. Thus, when $\varphi_2 - \varphi_1 = 0$ the current is equal to zero; for a phase difference $\varphi_2 - \varphi_1$ near to 360°, the current through the loop galvanometer is at a maximum. For instance, if the galvanometer current is 10 ma at $\varphi_2 - \varphi_1 = 360°$, then at 180° the current is 5 ma. As $\varphi_2 - \varphi_1$ passes through 0°, the current jumps from 10 ma to 0.

Figure 82 shows a recording of phase-difference changes obtained for a distance of 14.7 m between loudspeaker and microphone, and with a frequency change from 2000 to 5000 cycles ($f_2 - f_1 = 3000$ cycles). The regions of horizontal straight lines on the recording (on the left and on the right) correspond to switching off of the sound source; the sharp jumps in phase from top to bottom represent passage of the phase difference through 0°; and the distance from the upper point (where the jump begins) to the lower corresponds to a phase difference of 360°.

After the number N of inclined lines is determined from this recording (their number on the recording is 127), i.e., the number of complete 360° changes in phase difference, it is easy to find the velocity of sound using the foregoing formulas.

This method of sound-velocity measurement is the most exact: it is called the method of the acoustic "traveling-wave interferometer" (in contrast to the acoustic standing-wave interferometer, which will be discussed in the next chapter). In the present case, however, we are not dealing with pure interference as defined above, i.e., with the superposition of two (or in general several) waves at some point in space, but with the combination of two oscillations of the same frequency.

It is seen from the preceding formula that if the sound velocity c is known, then it is easy to measure the distance between sound source and receiver using the method of the acoustic interferometer, where

$$D = \frac{c(N_2 - N_1)}{f_2 - f_1}.$$

Radio Interferometer. Interferometric methods are used in radio physics to measure the velocity of radio-wave propagation or (if this velocity is known) for distance measurement. The instrument which makes possible such measurements is the radio interferometer. The original idea for such an instrument was due to L.I. Mandel'shtam and N.D. Papaleksi, who also constructed the instrument. Since we are now acquainted with the acoustic interferometer, it is easy to understand the principle of operation of the radio interferometer.

The working principle of the radio interferometer is illustrated in Figure 83. Consider the antenna of a radio transmitter to transmit radio waves with a frequency f_1. This radio station is called the key station. A radio receiver, at a distance D and tuned to the frequency f_1, receives the signal sent by the key station. By means of a special circuit the frequency of the received and amplified oscillations is converted into another frequency $f_2 = \frac{3}{2} f_1$, which is in turn radiated by a radio transmitter, adjacent to the receiver. The whole combination of receiver, frequency converter, and transmitter is called the reflecting station. Radio waves of frequency f_2 arrive at the receiver of the key station, which is tuned to this frequency. The amplified electrical oscillations of frequency f_2 at the output of this receiver are applied to the horizontal-deflection plates of a cathode-ray tube; to the vertical plates is applied the voltage at frequency f_1 transmitted by the key station. A stationary Lissajous figure is obtained on the screen of the cathode-ray tube, corresponding to a frequency ratio $f_2 : f_1 = 3:2$. If the distance D or the frequency f_1 (and consequently the frequency $f_2 = \frac{3}{2} f_1$) changes, the Lissajous figure rotates, performing the cycle of changes shown in Figure 78. For an increased distance between the two stations the number of wave lengths corres-

ponding to f_1 (in the direction from the key station to the reflecting station) and the number of wave lengths corresponding to f_2 (in the direction from the reflecting station to the key station) will change. Accordingly, as in the case of the acoustic interferometer, the Lissajous figure on the screen of the cathode-ray tube will also vary. By measuring the distance over which the reflecting (or key) station was moved, and by counting the number of complete cycles of change of the Lissajous figures, the wave length and consequently the velocity of propagation of the radio waves can be determined* (this coincides with the velocity of light).

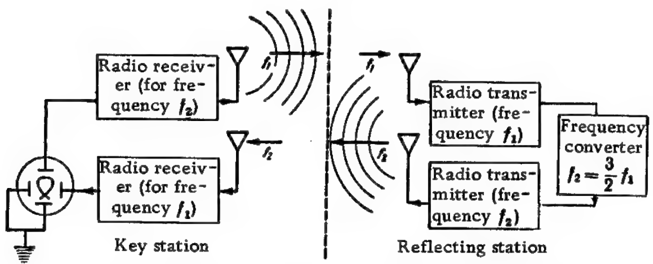


Figure 83. Diagram of the radio interferometer of Mandel'shtam and Papaleksi

If the distance D is known exactly, the propagation velocity of the radio waves can be measured using the method of the acoustic interferometer. On the other hand, if this velocity is exactly known—it is in fact equal to 300,000 km/sec—then the radio interferometer can be used as a precision instrument for measurement of large distances (radio range finder).

The similarity between the acoustic interferometer and the radio interferometer is thus seen to be very great; but there is also a difference. In the case of sound the waves propagate mainly in one direction; the sound, transformed into electrical oscillations, is conveyed through wires from the microphone to the electronic oscilloscope.

Radio waves, on the other hand, are propagated from the transmitter of the key station to the receiver of the reflecting station, and then back again from the transmitter of the reflecting station to the receiver of the key station. In this case the radio signal of the reflecting station, with a frequency f_2, is analogous to the wire connecting the microphone to the electronic oscilloscope. Frequency conversion in the radio interferometer is necessary for the following reasons. It is clear that it is impossible to perform the above operation of the radio interferometer with one frequency only. If this were attempted, the receiver of the key station, being tuned to frequency f_1 and being in the same place as the transmitter, would receive only the transmitter of the key station. The ratio $f_2 = \frac{3}{2}f_1$ was chosen for minimum influence of the transmitter upon the receiver in its vicinity**.

§ 3. Spectral representation of complex oscillations

We now have sufficient information at hand for the discussion of more difficult questions. So far we were mostly interested in very simple sinusoidal sound waves; this type of wave is observed, for instance, in a pure audible tone of definite frequency. But the sound waves propagating through the air normally have a more complex form, especially if the air particles are acted upon simultaneously by several waves which may, in addition, propagate in different directions. Such sounds (for instance, noise) have no stable form at all (cf. Figure 50).

* The formulas which relate the number of cycles of change of the Lissajous figures, the change of distance, and the propagation velocity of the radio waves will be different from the above formulas for the acoustic interferometer, because in the case of the radio interferometer two frequencies f_1 and f_2 are involved.

** In this case the receiver can be easily detuned for the harmonics (see next section) of the transmitter signal.

The combination of waves has already been considered, in the discussion of the phenomenon of interference. There the component waves all had the same frequency, or frequencies which differed very little from each other (forming beats), the difference being only one of amplitude and phase. For the addition of waves of different frequencies, amplitudes, and phases, even if the individual waves are sinusoidal, the resultant wave may have a very complicated form. In Figure 84 two sinusoidal oscillations A and B are shown. The addition of these oscillations gives a curve which is no longer sinusoidal, but of more complicated form. If the amplitudes of the added oscillations have some ratio other than the one shown, the resultant oscillation will be different from the curve in the given example.

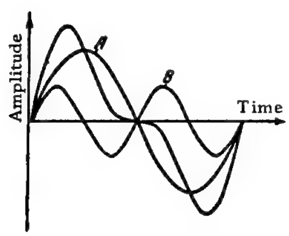


Figure 84. Addition of two sinusoidal oscillations A and B

The amplitude of oscillation A is twice the amplitude of oscillation B; the frequencies of oscillation differ by a factor of two

The resultant curve shown in Figure 84 repeats itself after a time T, its period. Thus, as a result of the addition of two sinusoidal oscillations, a periodic oscillation is obtained, but with a form more complicated than the form of the component oscillations. If not two but a larger number of sinusoidal oscillations with a number of frequencies are added, i.e., oscillations the frequencies of which have a ratio 1:2:3:4..., with different amplitudes and different phases, we shall obtain in each case a new form of resultant oscillation; but the result of the addition will always give a periodic oscillation.

But if, as a result of the addition of sinusoidal oscillations with various frequencies and different amplitudes and phases, a periodic oscillation of a complex form is obtained, is not the reverse also true? Is it not possible to represent every periodic oscillation, regardless of its form, as a sum of harmonic oscillations with various frequencies and different amplitudes and phases?

This question may be answered in the affirmative. If a number of sinusoidal oscillations of frequencies with a proportion 1:2:3:4, ... are added together, then, by a suitable choice of amplitudes and phases of oscillation, a periodic oscillation of any form can be obtained—triangular, square, sawtooth, and so on. In other words, a complex periodic oscillation can always be represented as a sum of simple sinusoidal or harmonic oscillations with different amplitudes and phases; the frequencies of the harmonics must be integral multiples of the fundamental frequency. For different forms of complex oscillations the amplitudes and phases of the fundamental frequency as well as of the harmonics differ from each other. This is Fourier's theorem, which plays an extremely important role in the theory of oscillations and waves. Figure 85 gives an example of how a square wave can be obtained by the addition of sinusoidal oscillations; in Figure 85a, as a result of adding three sine curves with a frequency ratio 1:3:5 (dashed curves), a curve is obtained which is similar to a square wave but still differs from it (thick curve); by adding eight oscillations (Figure 85b) a square wave is already rather well approximated.

If the frequencies of the component sinusoidal oscillations are plotted on the horizontal axis, and their amplitudes along the vertical axis, then the oscillation in Figure 85b may be represented as shown in Figure 86. Here, the length of each line corresponds to the amplitude of an individual harmonic oscillation. This representation of a complex oscillation is called spectral analysis. The spectrum in Figure 86 includes 8 frequencies.

The spectral analysis of complex oscillations has extremely great importance in the theory of oscillations.

Such a representation of the oscillations is visually very simple, but it does not show the phases of the harmonics. Therefore it is impossible to construct the form of the resultant oscillation from the spectrum alone. In a great number of cases, however, this is not required. For example, the ear does not react to phase relations of single components when it perceives a complex sound oscillation; here only the spectrum of the amplitude is essential.

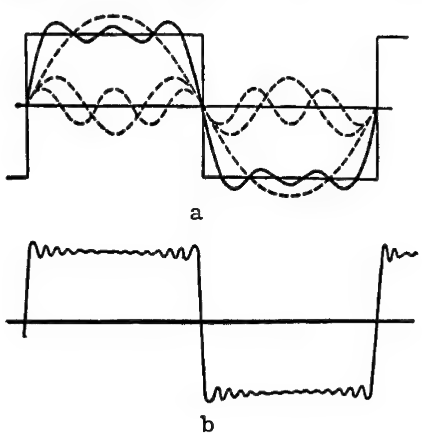


Figure 85. Addition of sinusoidal oscillations

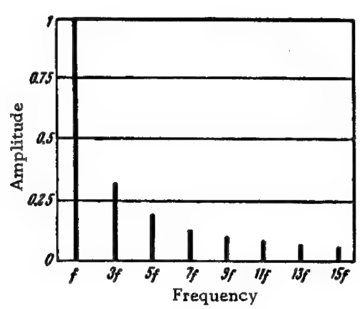


Figure 86. Spectrum of eight oscillations, corresponding to Figure 85b

Thus, any complex periodic process can be represented as a sum of harmonic oscillations whose frequencies are integral multiples and which have suitable amplitudes and phases. Nonperiodic processes, like the report of a gun, can also be so represented, but the sum and the spectrum for a nonperiodic process differ substantially from the spectral representation of a periodic process. This difference may be clarified by the following example.

Let a body capable of oscillation (for instance, a light weight and a rigid spring) be pushed periodically. Suppose that the natural frequency of oscillation of the body is 400 cycles and the frequency of the pushes is 50 cycles; thus every push is imparted to the body after it has complete 8 oscillations. The character of these oscillations is shown in the left part of Figure 87a; after every push the body performs damped oscillations. The mechanical oscillations of the body can be transformed into electrical ones, and with the aid of some type of harmonic analyzer the spectrum of the oscillations of the body under the action of the pushes can be obtained. The analysis

will give us the spectrum shown in Figure 87a on the right. This spectrum consists not of one vertical line (as for a simple sinusoidal oscillation) but of a number of lines placed every 50 cycles. The lowest (fundamental) frequency will be the frequency of the pushes—50 cycles; the other frequencies are integral multiples of the fundamental frequency. Since the process shown in Figure 87a is periodical, the spectrum obtained as a result of measurement represents the line spectrum of a periodic process of complex form.

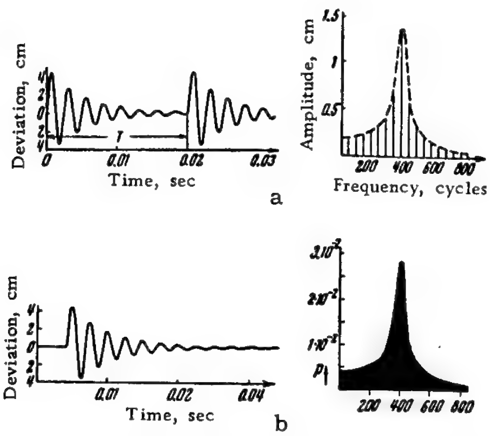


Figure 87. a) One push after every eight oscillations; frequency of pushes, 50 cycles; b) one single push and the corresponding continuous spectrum

For some other frequency of pushes the spectral lines will be placed differently, but the spectrum envelope will be the same as that for a push frequency of 50 cycles. As the frequency of pushes is decreased continuously, the analysis of the oscillation shows that the number of spectral lines increases more and more. Thus, at a frequency of pushes of 25 cycles, the lowest frequency will be 25 cycles; the harmonics will be distributed in the same way as for a push frequency of 50 cycles, but there will be twice as many of them. In this case the envelope of the spectral lines will be the same (for a corresponding choice of scale for the ordinate) as for a push frequency of 50 cycles (dashed line in Figure 87a).

Finally, let a single push act upon the oscillating body, after which the oscillations are damped comparatively quickly (Figure 87b). The analysis of this typical nonperiodic process will show that here no separate spectral lines occur. The spectrum of a single damped oscillation of a body is called a continuous spectrum; in Figure 87 it is represented by a shaded area. The envelope of this continuous spectrum is similar to the resonance curve; the maximum of the envelope occurs at a frequency equal to the frequency of the damped oscillation of the body; and on both sides of this frequency the amplitude falls (the drop is sharper the smaller the damping).

The difference between the spectra of a periodic oscillation (line spectrum) and a nonperiodic oscillation (continuous spectrum) can be easily checked in simple experiments with a piano. Let us press down the pedal

of the piano and thus release its strings. If some musical tone is produced in the room, then, after the tone has stopped, the piano will produce a sound with the frequency of this tone; the string of the piano which has a natural frequency near to the frequency of the tone will "respond" to it. But if a sound of complex form is produced, for instance, if a chord is played on another musical instrument, then not one but several piano strings will "respond"; the frequency components of the compound sound will act on the corresponding strings of the piano (line spectrum). A sharp, abrupt sound will cause all the piano strings to resound, because all frequencies in the audible range are present in such a sound (continuous spectrum).

We have seen that any complex periodic oscillation can be represented by a sum of sinusoidal oscillations with integral multiples of their frequencies. It turns out that a nonperiodic oscillation (single damped oscillation, pulse, "section" of a sinusoid, and other such forms) can be represented as a sum of harmonic oscillations, but the number of oscillations which comprise this sum is unlimitedly large (infinite), and the frequencies of these oscillations are distributed continuously over the whole spectrum*.

A single square pulse of duration T (this pulse corresponds as a first approximation to a gun shot or to an explosion) gives a continuous spectrum as shown in Figure 88. The assertion that the sound of a shot contains a collection of all possible frequencies, from the very lowest to the very highest (mentioned during the discussion of sound absorption) now becomes clarified.

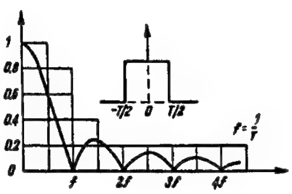


Figure 88. Continuous spectrum of a square pulse of duration T

An isolated "section" of a sinusoidal wave train containing, say, six periods of oscillation, also has a continuous spectrum, as shown in Figure 89.

Now the question may arise of why a part of a sinusoidal train does not represent a periodic process. We must, therefore, refine somewhat our concept of a periodic process.

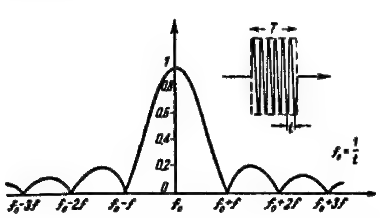


Figure 89. Continuous spectrum of a "section of a sinusoid" containing six oscillations with an overall duration $T=\frac{1}{f}$

The period of the oscillations within the "section of the sinusoid" is equal to $t=\frac{1}{f_0}$

Strictly speaking, a periodic process, including a sinusoidal oscillation, is a process which never began and which will never stop; the sinusoid must be unlimited [infinite] in time. Indeed, if this were not so, the state of the system at the beginning of the process or at the end would not be repeated, and the process would not be periodic. But such oscillations do not exist in nature, and the infinite sine curve is only an abstract concept. Therefore, it makes sense to speak only of the degree to which an actual process approximates a periodic sinusoidal process. If the number of oscillations during the time interval under consideration is large, it will

* Nonperiodic oscillations are expressed not by a Fourier sum but by a Fourier integral.

not introduce great error to replace a sinusoid which is limited in time by an infinite sinusoid; however, a "section" of a sinusoidal train, containing a small number of periods of oscillation cannot be replaced by an infinite sinusoid.

§ 4. Analysis of sound. Noise

Analysis of Sound. The resolution of a complex sound into a number of simple tones by means of Fourier's theorem is called harmonic analysis, or simply sound analysis. The ear is capable of distinguishing a number of simple tones in a complex oscillation; it can therefore perform a harmonic analysis of sound by perceiving separately the very simple oscillations of which it is composed, i. e., the ear can separate a complex sound into its harmonics.

Sound analysis has great importance in acoustics. For instance, in order to muffle the sound of automobile-engine exhaust, it is necessary to know which frequencies and amplitudes of oscillation make up this sound; from these data suitable construction for an absorber can be calculated. To muffle the noise of an airplane engine the sound spectrum of the engine must also be known.

Figure 90 shows the spectrum of noise generated by the Po-2 airplane.

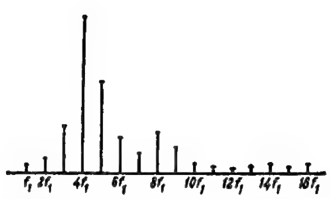


Figure 90. Noise spectrum of the Po-2 airplane in flight

Height 200 m; N = 1680 rev/min; orientation unknown

This aircraft has one engine and five cylinders and the propeller has two blades. If one cylinder operated, then the frequency of the sound would be $f_1 = \frac{N}{120}$ (N is the number of revolutions of the engine; one exhaust for every two turns of the shaft). Since there are five cylinders, the frequency of the sound generated by the exhausts (for equal force of the exhausts) is

$$f_e = \frac{N}{120} \cdot 5 = 5f_1.$$

The frequency of the sound generated by the two-blade propeller is

$$f_P = \frac{N}{60} \cdot 2 = \frac{N}{120} \cdot 4 = 4f_1.$$

Since not all the exhausts are of equal force, frequencies $f_1, 2f_1, 3f_1, \ldots$ are present in the noise spectrum. Most pronounced are frequencies $4f_1$ and $5f_1$, which correspond to the basic noise of the propeller and engine.

We will indicate another important application of sound analysis. For intelligent construction of musical instruments it is necessary to perform an analysis of the sounds of these instruments. Figure 91 shows the form of the sound oscillations of a piano (frequency 128 cycles) and of a clarinet (frequency 275 cycles), as well as their sound spectra. It is seen from the spectrogram that the piano sound contains harmonics up to the eighteenth inclusive, and the clarinet sound has up to the twelfth harmonic (second and fourth are absent).

We have already said that such sounds as noise have no stable oscillation form at all and are typical nonperiodic processes. Noise therefore

has a continuous oscillation spectrum, in which all frequencies are present. Therefore the expression "white noise" is often used, in analogy to the expression "white light".

"White light" (for instance, the light of the sun) is a combination of all possible frequencies of light oscillations and is accordingly also characterized by a continuous spectrum.

But the nature of one noise can be different from that of another noise

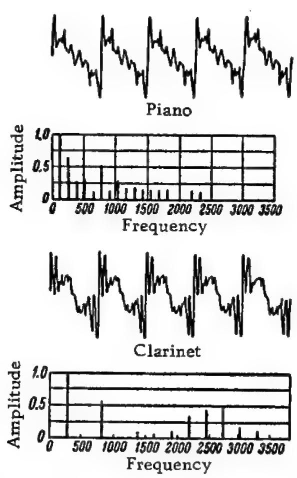


Figure 91. Form of oscillations and sound spectra of a piano (frequency 128 cycles) and of a clarinet (frequency 275 cycles)

For instance, street noise is different from the noise of an airplane engine. Every noise has its typical properties, and in the continuous spectrum of a given noise certain sound frequencies are emphasized or reduced.

A high level of noise in a factory containing many machines reduces working efficiency considerably; and noise also strongly disturbs mental work. In order to combat noise it is necessary first of all to know their sound spectra, in order to deaden the basic frequencies which are present in the noise. Here sound analysis also helps, but the physiological properties of hearing must also be taken into account. The ear possesses properties with respect to sensitivity to different frequencies and sound intensities (recall the diagram of Figure 44), which distinguish it from a microphone. Therefore, when a noise analysis is made, these properties are either taken into account, or specially constructed microphone amplifiers are used, with sensitivities to different frequencies corresponding to the sensitivity of the average human ear. Amplifiers of this kind, together with a microphone, are often used for measuring noise levels in decibels and are called noisemeters.

The Helmholtz Resonator. One of the first methods of sound analysis was suggested by the famous German physicist Helmholtz. To analyze sound Helmholtz used a set of hollow metal spheres, which had natural frequencies from the very highest to the very lowest audible frequencies (Figure 92). Such resonators are called Helmholtz resonators.

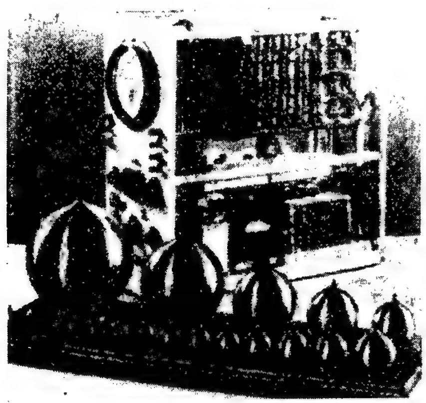


Figure 92. Group of Helmholtz resonators; to the rear, a modern analyzer with a cathode-ray tube

Figure 93 shows the cross section of a resonator; the input orifice is on the right side of the picture, and the opposite side has a small extension to be placed in the ear*.

Every air cavity which has an opening—a bottle, flask, or jug—represents an acoustic resonator. When the frequency of the sound arriving at the cavity coincides with the natural oscillation frequency of the air in it, sound oscillations are produced in the cavity; when the external disturbance has ceased, the cavity continues to sound for some time. We have here the phenomenon of resonance—a system oscillates under the action of a periodic external force, when its frequency coincides with the natural frequency of the system.

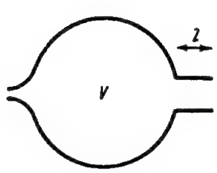


Figure 93. Cross section of Helmholtz resonator

When a complex sound—a periodic oscillation of complex form—reaches the Helmholtz resonators, the separate sinusoidal components of this sound act on the corresponding resonators; some simple tone which is contained in the complex tone causes sounding of whichever resonator has a natural frequency nearest the frequency of this tone. By listening in turn to the sounding of the resonators, and by evalutating the sound by ear, an analysis of the components contained in the complex tone can be made.

Modern Methods of Sound Analysis. Today, the direct analysis of sound is normally replaced by analysis of the electrical oscillations. The sound is received by a microphone, transformed into electrical oscillations, and amplified. Of course, the microphone and the amplifier must not introduce any distortion; the voltage at the amplifier output must reproduce exactly the sound oscillations received by the microphone. For the analysis of electrical oscillations two principal methods are used: either these oscillations are recorded on an oscillograph and the harmonic analysis of

* The natural oscillation frequency of a Helmholtz resonator can be calculated by the formula

$$f = \frac{c}{2\pi}\sqrt{\frac{S}{lV}},$$

where c is the velocity of sound, S is the area of the input orifice l is the length of the throat, and V is the volume of the resonator.

the obtained curve carried out graphically, using Fourier analysis, or else special analyzers are used, which are based on various electronic circuits. There are a great number of different types of these analyzers in existence, which make it possible to analyze sound quickly and to obtain the form of the oscillation spectrum. Analyzers with cathode-ray tubes make it possible to observe the spectrum of the sound on an electronic oscilloscope.

Figure 94 shows a simplified block diagram of one of these analyzers. The electrical oscillations to be analyzed (for instance, from the output of a microphone) are amplified and applied to a row of filters, which have their inputs connected in parallel*. Each of these filters passes a definite frequency band, say 1/3 of

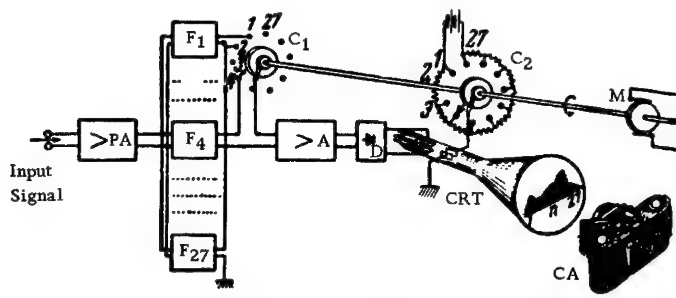


Figure 94. Simplified block diagram of a sound analyzer

PA—preamplifier; F_1 .... F_{27}—filters; A—amplifier; D—detector; C_1, C_2—commutators; B—battery, M—motor; CA—camera; CRT-cathode-ray tube

an octave (36-48, 48-60, 60-72, 72-96 cycles, etc.), and they cover the whole range of investigated frequencies (the frequency band of the described analyzer is from 36 to 18,000 cycles; the number of filters is 27). The number of such filters can reach several dozen. Let a sine voltage of 65 cycles be applied to the input of the analyzer; then only at the output of the third filter, which passes the band 60-72 cycles, will the voltage be different from zero; at all the other filters no voltage appears. In the case of a signal of complex form the filters, being spectral instruments, will separate corresponding frequency components of the input signal—they will resolve this signal into a spectrum. After filtering, the detection (rectification) of the signal components takes place, i.e., the d.c. component of the voltage is obtained. All filters have a voltage gain of unity; when equal voltages of different frequencies are applied to the inputs of the filters (corresponding to the pass bands of the filters) the same voltage amplitudes are obtained on the output of each filter.

The output of each filter is connected to the contacts of a commutator C_1, which are arranged in a circle. A wiper moves over these contacts; the wiper is caused to rotate by the motor M and connects the corresponding contact (in the diagram contact 4) to the amplifier A, which is connected through the detector D with the vertical-deflection plates of the cathode-ray tube. Thus, the outputs of the filters are connected one after the other to these plates. On the same shaft there rotates another wiper-contact which connects a circular system of contacts, symmetric to the first (second commutator C_2). If in the first commutator the wiper closes contact 4, then the same contact is closed by the wiper in the second commutator. Between the contacts of the second commutator are connected resistance wires, through which a current from a source B of d.c. voltage flows. The wiper of the second commutator is connected to the horizontal-deflection plates of the cathode-ray tube. During the making of the contacts these plates receive a d.c. voltage which increases by steps, and the luminous point on the screen moves from left to right. At the passage from the 27th contact to the first the luminous point jumps from the right side of the screen to its original position (return sweep). In this way a stepped sawtooth sweep is obtained.

During one second the wipers complete 10-15 turns, i.e., the pattern on the screen repeats itself 10-15 times; therefore the eye sees a stationary picture—a "photograph" of the spectrum of the input signal in the usual system of coordinates. This pattern can be photographed.

* The filters can be constructed in different ways; they can be ordinary resonance types, consisting of capacitance and inductance, or filters containing RC-circuits (resistors and capacitors), and so on.

The described method of automatic analysis is feasible only with periodically repeating processes. If the process is not periodic, it can in some cases be transformed into a periodic one; this is often done in the following way.

The process is recorded by some method of sound recording (a phonograph record or tape recorder). Copies of the recording are then made and by some means or other all the records are connected in series. The obtained disk or tape is reproduced; and the microphone of the sound analyzer will thus receive a periodically repeating process from the loudspeaker.

One other type of analyzer will be described briefly—the heterodyne analyzer. The block diagram of this analyzer is shown in Figure 95.

The amplified signal from the microphone is applied to a mixer, to which a voltage from a local oscillator (the heterodyne) is also applied. The frequency of the signal and that of the local oscillator are mixed (added together) in the mixer, which may be a vacuum tube with two grids. To one of the grids is applied the signal from the amplifier; to the other, the signal from the local oscillator. In such a mixer the mixing of the signals takes place in the tube, i.e., electronically. If the frequency of the signal is f_c and the frequency of the local oscillator is f_r, then, as a result of mixing, combination frequencies appear: $2f_c$, $2f_r$, $f_c + f_r$, $f_c - f_r$, and so on. The narrow-band filter after the mixer passes only the sum frequency $f_c + f_r$, and no others.

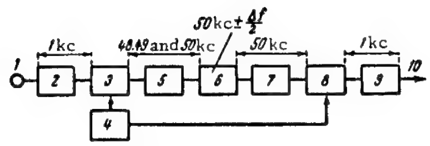


Figure 95. Block diagram of a heterodyne analyzer

1) microphone, 2) amplifier, 3) mixer, 4) local oscillator, 5) high-frequency amplifier, 6) narrow-band filter, 7) high-frequency amplifier, 8) mixer and filter, 9) audio-frequency frequency amplifier, 10) output to indicator

If the signal frequency is 1 kc and the heterodyne frequency is 49 kc, then, after mixing, the filter, which is tuned to the frequency $f_c + f_r$, selects the frequency 49 + 1 = 50 kc. The voltage at this sum frequency is then amplified and applied to a second mixer. In it the signal $f_c + f_r$ is mixed with the frequency f_r of the local oscillator, and the low-frequency filter singles out the difference frequency $f_c + f_r - f_r = f_c$, which is the signal frequency (in this example, 1 kc). After suitable amplification the signal voltage with a frequency f_c is applied to some indicator—a meter, an electronic oscilloscope, or, for recording, a loop oscillograph.

We have seen how the heterodyne analyzer works if the signal frequency is 1 kc. For some other signal frequency (for instance, 2 kc) the tuning of the local oscillator is varied until the sum frequency of the signal and of the local oscillator again becomes equal to the frequency to which the filter is tuned, i.e., 50 kc $\pm \frac{\Delta f}{2}$, where Δf is the pass band of the filter*. In this way the analysis of a complex oscillation is performed in the heterodyne oscillator, using the method of consecutive "interrogation"—variation of the heterodyne frequency. To obtain the range of frequencies for which such an analysis can be performed in the range of audible frequencies (for instance, 30 cycles to 20 kc), the heterodyne frequency must vary over a range from 30-50 kc. The heterodyne analyzer may be transformed into an automatic analyzer; for this purpose the heterodyne frequency is changed over its whole range several times (10-15) per second ("warble tone"); this may easily be achieved by driving the rotor of the tuning capacitor of the local oscillator by means of a small constant-speed motor.

The two types of analyzers discussed above are based on the switching in of filters in sequence, i.e., on the sequential "interrogation" of the indications of each filter. Therefore this method of analysis is called the sequential method.

* By the pass band Δf is meant the difference between the highest and the lowest frequencies passed by the filter. Here it is assumed that at the highest and the lowest frequencies the amplitude of the signal passed constitutes a definite fraction (for instance 0.7) of the maximum amplitude at the filter output.

When performing the analysis of sound processes, and also of electrical and mechanical oscillations, the following practical questions are always encountered: what is the precision of the analyzer, and what is the time required for analysis using a given analyzer? As we know, the spectrum of the input signal may be either a line spectrum or a continuous spectrum. Even in a line spectrum the spectral lines (frequencies) may be very near to each other. Therefore, an analyzer having filters with relatively wide pass bands cannot separate spectral lines with frequencies close to each other. The capacity to separate, or resolve, two spectral lines very close to one another is called the resolving power of the analyzer; this capacity determines the working precision of the analyzer. Therefore, in order to increase the resolving power, it is necessary to decrease the pass band of the resonators (filters) or, since the resonance curve of the resonator becomes sharper as the pass band is narrowed, to increase the selectivity of the resonator.

But we know from the first chapter that the sharper the resonance curve of a system, the longer the time for establishment of the process in the system, and thus the longer the time needed for analysis, since it is necessary for analysis that the analyzed signals not be distorted; and transient processes cause distortion. It is desirable to achieve a minimum analysis time and maximum resolving power. From what was said above these demands appear mutually contradictory, since the greater the resolving power, the longer the time needed for analysis. We have therefore to choose a compromise solution: if the analysis must be made in a minimum time Δt, which is particularly important for rapid processes, the pass band Δf of the filters must be increased, i.e., the resolving power of the analyzer must be decreased.

Theory shows that there is an important relation between Δt and Δf:

$$\Delta f \, \Delta t \geqslant A,$$

where A is a certain constant of the order of unity (the exact value of A depends on the choice of Δf and Δt).

Visualization of Speech. If we have a complete set of resonators (filters) which are tuned to different frequencies, the analysis may be made by simultaneous action of the complex oscillation upon all the resonators together. This method of analysis is called the simultaneous method. It

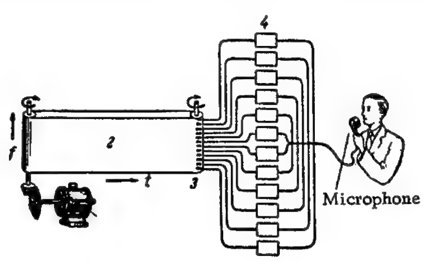


Figure 96. Block diagram of the set-up for visual observation of speech spectra as functions of time (acoustic spectrograph)

1) motor, 2) fluorescent screen in the form of an infinite strip, 3) lamps, 4) filters

can be used for the investigation of rapid (or transient) processes. We shall consider the application of the method of simultaneous analysis for the visualization of speech. In addition to the clarification of speech individualities and the investigation of the phonetic properties of language and of music, the visualization of speech makes it possible to judge speech defects and to observe to a certain degree living speech by visual, rather than audible, means.

Figure 96 shows the block diagram of one possible method for speech visualization using simultaneous analysis. The sound signal (speech) from the microphone is suitably amplified and applied simultaneously to 12 filters; the filters each have pass bands of 300 cycles and cover the range from the very lowest audible frequencies up to 3500 cycles (in the diagram the lowest filter passes the lowest frequency). The output of each filter is

Figure 97. Spectrogram of the vowels a, i, o, u [Russian sounds]

connected to a lamp; the lamps are placed one above the other at equal distances. The light from the lamp falls on the fluorescent screen, coated with a high-persistence substance; this screen is pulled along at a constant velocity by an electric motor. The greater the voltage amplitude at the filter output (the larger the effect in the signal of the frequencies corresponding to the pass band of the filter), the brighter is the glow of the lamp.

Figure 98. Sound spectrograms of heartbeats

Left, normal heart; center and right, disturbances in the normal operation of the heart

Thus, on the screen the sound spectrogram of the investigated variable process appears in terms of the usual rectangular coordinates; along the horizontal axis is the time, and along the vertical axis is the frequency; the degree of illumination of the screen corresponds to the intensity of the sound.

Figures 97 and 98 show examples of spectrograms.

Chapter V

ULTRASONIC WAVES IN AIR

So far we have dealt only with sound waves in air the frequencies of which were between 16 cycles and 20 kc. However, the range of elastic vibrations is not limited to this part of the spectrum. As already mentioned, elastic vibrations with frequencies above 20,000 cycles are called ultrasonic.

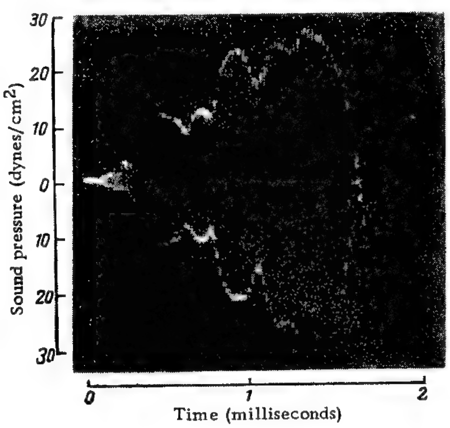


Figure 99. Ultrasonic scream of a bat at a distance 10 cm from its mouth, photographed from the screen of an electronic oscilloscope. The ultrasonic frequency of the pulse is 48 kc

Some animals possess the ability to hear sounds of higher frequencies than those heard by man. Thus, birds react very painfully to ultrasound with a frequency of the order of 25 kc; special powerful ultrasonic generators have been used to frighten away seagulls that polluted drinking water reservoirs. The seagulls, as well as other birds, avoided entering the region of ultrasonic radiation.

Small insects in flight produce sounds of very high frequencies.

Bats, who have very weak eyesight or even none at all, find their way about in flight and catch their prey by means of ultrasonic pulses, following each other at repetition frequencies between a few cycles and several dozen cycles per second.

These pulses consist of ultrasonic vibrations at frequencies between 20 and 50-60 kc (the frequencies differ for different species of bats). When

their auditory apparatus receives the ultrasonic pulses reflected from obstacles, the bats can determine the direction and the distance of the reflecting body.

In Figure 99 an oscillogram is shown of an ultrasonic pulse from a bat, received by an ultrasonic receiver and photographed on the screen of an electronic oscilloscope.

§ 1. Generation of ultrasound. The piezoelectric effect

During the last 20 years ultrasound has played an ever-increasing part not only in scientific research, but also in the solution of a great many technical and practical problems—in submarine signaling and communication, ultrasonic flaw-detection for metals and alloys, medicine, etc.

The nature of ultrasonic waves is in no way different from that of sound waves. However, due to their high frequencies (and consequently their small wave lengths) they possess a whole series of special properties.

Such sound sources as a telephone or an electrodynamic loudspeaker are not capable of radiating frequencies exceeding 15,000-20,000 cycles. This is obvious, since at high ultrasonic frequencies very great forces would be required to cause a telephone or loudspeaker diaphragm to vibrate with sufficient amplitude. Actually, in the case of a body vibrating sinusoidally, the velocity is proportional to the vibration frequency, while the acceleration is proportional to the square of the frequency. From the principles of mechanics we know that force equals mass times acceleration. Therefore, if the frequency is to be increased, for example, 10 times, then the magnitude of the force producing it must be increased 100 times in order for a telephone or loudspeaker diaphragm to vibrate with the same amplitude in both cases. At high ultrasonic frequencies very great forces would be required in order to impart sufficient vibration amplitude to a diaphragm.

At present, the generation of high-frequency mechanical vibrations is based mainly on utilization of the so-called piezoelectric and magnetostriction effects.

Piezoelectric Effect. A whole series of crystals—including quartz, tourmaline, and Rochelle salt—possess a remarkable property. If a plate is cut out of one of these crystals in a certain way, then on compressing or stretching the plate an electric charge appears on its faces—positive on one side, negative on the other. Such a crystal is said to possess a piezoelectric effect. The root "piezo" means pressure; and so piezoelectricity refers to the formation of electrical charges as a result of mechanical pressure. The formation of electrical charges on the surface of a crystalline plate subjected to pressure is called the *direct piezoelectric effect*. It should be noted that, in addition to the direct piezoelectric effect, there is also an *inverse piezoelectric effect*, whereby the dimensions of a plate are changed under the influence of an electric field. If the plate is covered on both sides with metal (e.g., aluminum foil), and the pieces of metal (electrodes) are connected to an electric battery, then the thickness of the plate is changed somewhat (e.g., contraction occurs); with a change in the polarity of the voltage, the contraction of the plate is replaced by expansion*.

* The piezoelectric phenomenon was discovered in 1880 in France by the Curie brothers, as was the inverse piezoelectric effect (in quartz), but at a somewhat later date.

Figure 100. Natural quartz crystals

Quartz Plates as Generators and Receivers of Ultrasound. Figure 100 shows some natural crystals of quartz, and in Figure 101 there is a drawing of a hexagonal prismatic quartz crystal. The principal crystallographic axes X, Y, Z are known respectively as the electrical, the mechanical, and the optical axes [for uniaxial crystals]. A plate can be cut out of the quartz at any angle with respect to these axes. The ones most frequently used in ultrasonic applications are the so-called "X-cut" (otherwise known as "Curie-section") plates, which are cut out perpendicularly with respect to the electrical axis (X-axis) of the crystal.

If such a plate is stretched in the direction of the X-axis (i.e., across the plate), then a positive charge forms on the surface toward the positive side of the X-axis, while a negative charge forms on the opposite side. If compression is substituted for the tension along the X-axis, the signs of the charges are reversed (Figure 102a). If the compression or the tension is applied parallel to the Y-axis, then electrical charges again

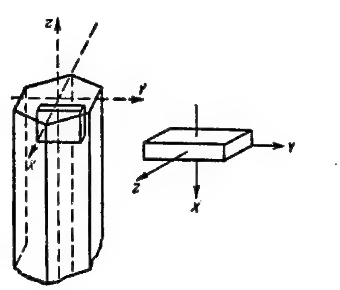


Figure 101. The principal part of a quartz crystal—a hexagonal prism—showing the position of the principal (crystallographic) axes, and an X-cut plate

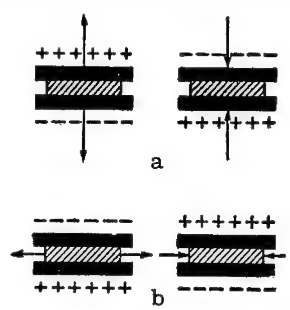


Figure 102. a) Tension and compression of an X-cut plate, applied parallel to the X-axis; b) tension and compression of an X-cut plate, applied parallel to the Y-axis

form on the same surfaces, but their respective signs are reversed compared to the ones formed by stress parallel to the X-axis (Figure 102b). Compression or tension parallel to the Z-axis does not produce charges. It is worthy of note that the magnitude of the charge formed, which we shall designate by e, proves to be strictly proportional to the mechanical force applied:

$$e = d \cdot F,$$

where F is the force in dynes and d is a constant called the piezoelectric modulus. For the X-cut

$$d = 6.4 \cdot 10^{-8}.$$

For the inverse piezoelectric effect, the change in the thickness Δl of the plate, induced by the action of the electric field, is proportional to the electrical potential applied:

$$\Delta l = d \cdot V,$$

where V is the applied potential in absolute electrostatic units. For example, let us calculate the change in thickness of the plate if a potential of 1000 v is applied to its electrodes. In this case

$$\Delta l = d \cdot V = 6.4 \cdot 10^{-8} \cdot \frac{1000}{300} = 21.3 \cdot 10^{-8}\,\text{cm} = 21.3\ \text{Å}$$

(where 1000 was divided by 300 in order to convert to absolute electrostatic units). Thus, the plate will become thicker or thinner (depending on the polarity of the applied voltage) by 21.3 Å, i. e., by two parts in 10,000,000 of a centimeter.

In addition to the X-cut (Curie section), ultrasonics makes frequent use of the Y-cut. A Y-cut plate is so cut out that its lateral faces are perpendicular to the mechanical axis (Y-axis) of the quartz crystal, and the electrodes of such a plate are placed over these faces.

If an electrical potential is applied to the electrodes, then the Y-cut plate is subjected to a shearing strain, in contrast to the X-cut plate, which under similar conditions experiences a force of tension or compression across the plate. If, on the other hand, the plate is subjected to a force bringing about a shearing strain, then electrical charges are formed on the electrodes. Y-cut plates are used as generators and receivers of transverse waves (shear waves). The piezoelectric modulus for the Y-cut is $1.7 \cdot 10^{-8}$, i.e., approximately 4 times less than for the X-cut.

Usually, the electrodes covering the plate of the crystal are made of foil or else of highly polished metal plate. A common practice is to metallize the plate, giving it a thin coating of silver or chromium. If an alternating voltage is applied to the electrodes of an X-cut plate, then the plate will be alternately stretched and compressed in rhythm with the oscillations of the voltage. The faces of the plate will oscillate with respect to each other, and if one of them is in contact with some medium (e. g., air) in which elastic waves can propagate, then waves will be generated. But the piezoelectric modulus for quartz is extremely small; 1000 v produces a change of only $2 \cdot 10^{-7}$ cm in the thickness of the plate. Hence, in order for the amplitude of the vibrations to be considerable, it is usually necessary to use a frequency of the applied alternating potential which is equal to the natural frequency of vibration of the plate itself, i. e., it is necessary to make use of the resonance phenomenon.

Here we must proceed somewhat ahead of our subject and mention that elastic waves of compression and rarefaction can be generated and propagated not only in a gaseous medium, but also in solid bodies; the plate of crystal represents such a body. While the plate is executing lateral vibrations, compressions and rarefactions take place in it, spreading with a definite velocity—the velocity of propagation of elastic waves in quartz. These waves on reaching the surface of the plate are reflected and begin to move in the reverse direction.

As a result of the superposition of waves going in opposite directions, standing waves will be formed at some frequencies, in a way similar to the formation of standing waves for the vibrations of a rope or in organ pipes.

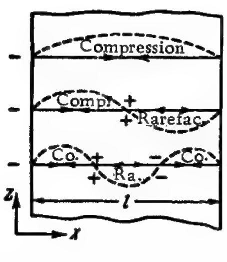


Figure 103. Distribution of pressure and charge in an X-cut plate; the plate is $\frac{\lambda_1}{2}$, λ_2, and $\frac{3\lambda_3}{2}$ (λ is the wave length)

From Figure 103 it will be seen that standing waves are formed in cases where some odd multiple of half wave lengths is equal to the thickness of the plate. If the thickness l of the plate is equal to one wave length, i.e., two half wave lengths, then vibration of the plate is impossible, since the electrical charges on both faces of the plate would be of the same sign. The plate will vibrate at its fundamental frequency when its thickness equals $\frac{\lambda}{2}$; the vibrations will correspond to the third harmonic when the thickness is $\frac{3\lambda}{2}$; to the fifth harmonic when the thickness is $\frac{5\lambda}{2}$; and so on.

Since the frequency is determined from the formula, $f=\frac{c}{\lambda}$, and since, according to the above $\lambda=\frac{2l}{n}$ (where $n = 1, 3, 5, \ldots$, and l is the thickness of the plate), therefore the natural frequencies of the plate are given by the formula

$$f=\frac{nc}{2l},$$

where c is the velocity of elastic waves in the plate.

The velocity of elastic waves* parallel to the X-axis in quartz equals 5400 m/sec, and so the natural frequency of the plate (the fundamental, or first harmonic) is given by the formula

$$f=\frac{2700}{l}\text{ kc/mm}.$$

The experimental value for f differs somewhat from the one calculated theoretically

$$f_{\text{exp.}}=\frac{2880}{l}\text{ kc/mm}.$$

The discrepancy between the experimental and theoretical values of f is apparently due to the fact that in the theoretical formula given above the transverse vibrations of the plate are not taken into account (the elongations along the Y- and Z-axes, which occur simultaneously with the compressions along the X-axis).

Thus, an X-cut plate 1 mm thick has a natural frequency of 2880 kc. It may be noted that a Y-cut plate of the same thickness has a natural

* We refer here to the longitudinal elastic waves formed in a plate executing lateral vibrations, as distinguished from transverse waves (see Chapter IX).

frequency of 2000 kc. This occurs because the velocity of elastic waves in quartz along the Y-axis is somewhat different from the velocity along the X-axis.

Modern methods of cutting quartz crystals makes it possible to cut plates with thicknesses of a few hundredths of a millimeter, i.e., thinner than tissue paper. A natural frequency of $50 \cdot 10^6$ cycles (50 megacycles) is obtained with a thickness $l \approx 0.05$ mm (X-cut). However, such plates are extremely fragile; they break easily and are seldom used for the generation of ultrasound. Therefore, in order to obtain very high ultrasonic frequencies, we must employ plates which are not too thin, and utilize the higher harmonics. For this purpose, to a strip with a fundamental frequency of, say, 1 megacycle, an alternating potential with a frequency of, say, 25 mc is applied from a generator; the plate will then vibrate at its 25th harmonic.

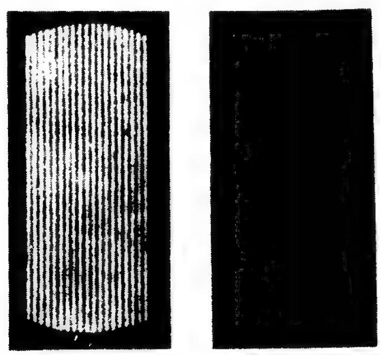


Figure 104. Compressions and rarefactions in a quartz plate executing lateral vibrations at its 19th and 39th harmonics

The photograph was obtained by the dark-field method

Figure 104 represents a photograph (obtained by the dark-field method) of vibrations of a quartz plate, corresponding to the 19th and 39th harmonics. Compression and rarefaction bands in the plate are evident.

The power generated by the ultrasonic vibrations is proportional to the surface area of the plate. In order to obtain a larger surface area, mosaics are used consisting of a large number of plates of the same cut and thickness.

The alternating potential supplied to the electrodes of the quartz plate is produced by an electronic generator, the frequency of which is adjusted to be equal to some frequency which is characteristic of the plate—either its fundamental or a higher harmonic. The output voltage of the generator is from a few hundred to several thousand volts. Since quartz is an excellent insulator and possesses great dielectric strength, large potentials may be applied to it in order to obtain considerable amplitudes of vibration. Some investigators have increased this potential to as much as 60,000 v/cm*.

* In such cases the quartz plate was placed in transformer oil to avoid electrical breakdown at the edges.

Damping of vibrations in a quartz plate placed in air is extremely small, hence its resonance curve is extraordinarily sharp. This is of great significance both in ultrasonics and in radio applications. In radio, quartz plates are widely used for various purposes (generator frequency stabilization, quartz frequency standards, quartz clocks, filters, etc.).

The vibration amplitude of an X-cut quartz plate at resonance is determined from the formula

$$x_k = \frac{\overline{m}}{(2k-1)\pi} d \cdot V,$$

where k is the order of the harmonic, and

$$\overline{m} = \frac{\rho_0 c_0}{\rho c}$$

(ρ_0 and c_0 are respectively the quartz density and the velocity of longitudinal waves in quartz; ρ and c are the same for the medium in which the ultrasound is propagated; and V is the amplitude of the alternating electrical potential).

Under static conditions the change in thickness is

$$\Delta l = d \cdot V,$$

hence, the increase in the amplitude of displacement under resonance conditions, as compared with static conditions for the first harmonic ($k = 1$), is determined by the ratio

$$\frac{x_1}{\Delta l} = \frac{\overline{m}}{\pi} = \frac{\rho_0 c_0}{\pi \rho c}.$$

For air, $\rho c = 41$, while for quartz, $\rho_0 c_0 = 1.43 \cdot 10^6$, so that

$$\frac{x_1}{\Delta l} \sim 10^4 .$$

Thus, for vibration of an X-cut quartz plate the amplitude of vibration at resonance will be approximately 10^4 times greater than under static conditions (the applied potential being the same), i.e., the resonance curve will be very sharp. In case the quartz plate vibrates in water, $\rho c = 1.5 \cdot 10^5$, and

$$\frac{x_1}{\Delta l} \sim 3.$$

Hence for a plate vibrating in water the resonance curve will be flatter; the vibration amplitude at resonance is only 3 times greater than that without resonance (results for a plate vibrating in transformer oil are about the same).

It should not be assumed, however, that an indefinitely high potential may be applied to a quartz plate, even apart from the danger of electrical breakdown. The mechanical deformations occurring in quartz due to the piezoelectric effect at high alternating voltages may, for resonance, reach values exceeding the mechanical breaking point of quartz, and the plate will be fractured.

In the case of a plate vibrating at resonance in oil or water, greater electrical potentials may be applied than in air. It has been found, for instance, that in transformer oil a flat plate may produce ultrasonic power exceeding 300 watt/cm^2 (see Chapter VIII).

We have discussed an X-cut plate of quartz as a source of ultrasonic vibrations. If such a plate is acted upon by an alternating pressure, then

due to the direct piezoelectric effect an alternating potential will be induced between the lateral faces of the plate. Such an alternating pressure, for example, will be exerted upon a plate placed in a medium in which sound waves are propagating. Therefore, a plate can also serve as a receiver of ultrasonic vibrations, or a high-frequency microphone. Owing to the smallness of the piezoelectric modulus of quartz, the induced electrical potential is extremely small, even for high ultrasonic pressures. In order to make the signal greater, a radio-frequency amplifier is used, tuned to the corresponding frequency.

Ultrasonic Generators and Receivers Employing Rochelle Salt Crystals. Besides quartz, Rochelle salt crystals have come into widespread use. For various applications in acoustics, artificial crystals are specially grown under laboratory conditions; their weight may reach several kilograms.

Quartz and Rochelle salt differ sharply in their properties. Quartz is very strong, while Rochelle salt is fragile; quartz is neither soluble in water nor in acids, but Rochelle salt dissolves easily in water. Quartz is heat-resistant, with a high melting point of 1470°C, and it loses its piezoelectric properties at 570°C; Rochelle salt melts at 58°C and loses its piezoelectric properties at 54°C. The piezoelectric effect of quartz is affected very little by temperature, while that of Rochelle salt is highly temperature-dependent. However, Rochelle salt possesses a piezoelectric effect many times greater than that of quartz. This explains the wide

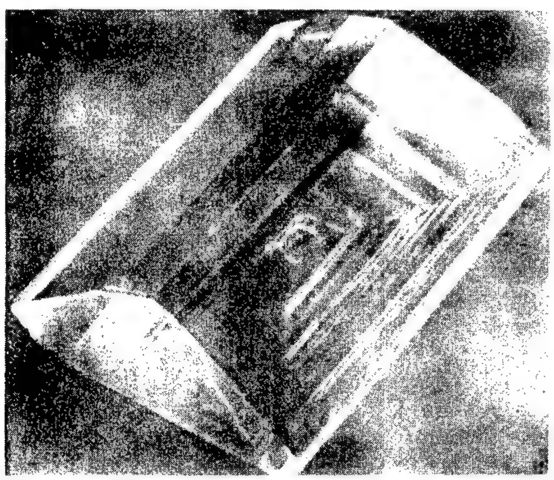


Figure 105. Part of a Rochelle salt crystal

application of Rochelle salt (all its disadvantages notwithstanding) in modern acoustics. Figure 105 shows a photograph of part of a Rochelle salt crystal. The piezoelectric properties of such a crystal are more complicated than those of quartz. Figure 106 shows part of a Rochelle salt crystal on which are marked the principal axes X, Y, and Z. A plate cut out of a Rochelle salt crystal perpendicularly with respect to the X-axis has different piezoelectrical properties than an X-cut quartz plate.

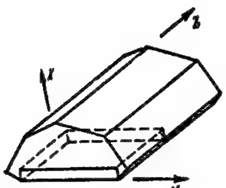


Figure 106. The arrangement of the crystallographic axes in a Rochelle salt crystal

The plate has been cut perpendicular to the X-axis

If an electrical potential is applied to the faces of the Rochelle salt plate which are perpendicular to the X-axis (Figure 107), the plate undergoes a shearing strain instead of the change in thickness in the X-direction which is seen in the case of a quartz plate cut similarly; on the other hand, if a Rochelle salt plate is subjected to a shearing strain, electrical charges appear on the faces perpendicular to the X-axis.

In order to use a plate of a Rochelle salt crystal as a piston, i.e., to subject it to tension and compression, it turns out to be necessary to cut out the plate perpendicularly with respect to the X-axis, but at an angle of 45° to the Y- and Z-axes (Figure 108). Such a cut is called a "45° X-cut". For such a plate a potential applied to the faces perpendicular to the X-axis will cause compression, as shown in Figure 109; and conversely, compressing the plate along the X-axis will cause charges to appear on the faces perpendicular to the X-axis. Thus the radiating faces of the plate are mainly its end faces.

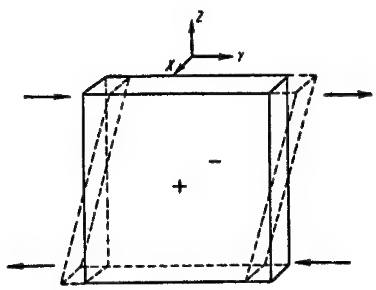


Figure 107. On the application of a force couple to a Rochelle salt plate cut out perpendicularly with respect to the X-axis, electrical charges appear on the faces perpendicular to the X-axis. If electrical charges are applied to these faces, then the plate will change its shape (as shown by dotted line), as the result of a shearing stress

In order to obtain a greater surface area for an ultrasonic generator or receiver, several plates of this cut are stacked together (Figure 110). Very often bimorph elements of Rochelle salt are used, especially for the generation and reception of audio-frequency oscillations. These are composed of two 45° X-cut Rochelle salt plates which are cemented together.

If such a 45° X-cut Rochelle salt bar with two electrically conductive faces is bent (Figure 111a), then electrical charges of equal polarity will form on the two faces. This will occur because, during bending, the upper part of the bar (above the dotted line) is compressed, while the lower part is stretched; and in accordance with Figure 109, for a given direction of bending electrical charges of equal polarity will form on both sides of the bar (for instance, both positive).

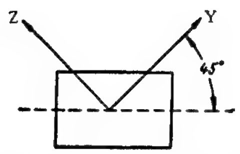


Figure 108. Rochelle salt plate cut out perpendicularly with respect to the X-axis and at a 45° angle to the Y- and Z-axes ("45° X-cut")

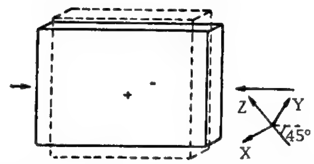


Figure 109. Electrical charges appear on those faces of a 45° X-cut plate of Rochelle salt which are perpendicular to the X-axis, as a result of compression applied in the direction shown by the arrows. Conversely, electrical charges produced on these faces will cause the plate to change its shape due to forces of tension of compression (dotted lines)

Let the bar be cut in half along the dotted line and let a third interface be introduced along this line, after which the two halves are cemented together (Figure 111b). The outer faces are now connected together and form one pole (similar to the pole of an electrical battery). The other pole is formed by the inner interface. Thus, we obtain the so-called bimorph cell, first proposed by N.N. Andreev. A bending of the bimorph produces a potential difference between its terminals or poles. On the other hand, if an electric potential is applied to its [inner and outer] surfaces, a bimorph element will be subjected to bending. Such cells find widespread use in electroacoustics—as microphones, pick-ups, vibration meters, etc.

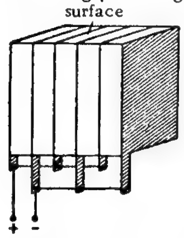


Figure 110. A stack of 45° X-cut plates of Rochelle salt

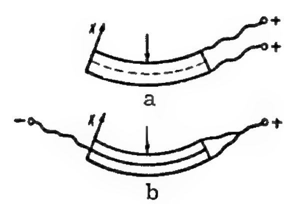


Figure 111. A bimorph element composed of two 45° X-cut plates of Rochelle salt

Let us discuss one more Rochelle salt crystal plate, which may vibrate as a piston-type radiator. Along the main axes, X, Y, Z of a Rochelle salt crystal, let us lay off equal segments, and let us draw a plane through the end points of these segments (Figure 112). A plate cut along this plane, or in any other plane parallel to it, will execute lateral vibrations, in the same way as a quartz plate.

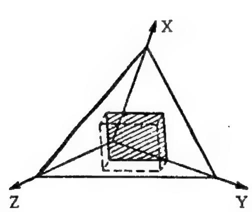


Figure 112. The L-cut plate

This section has recently found rather wide use (it is called an L-cut).

The fundamental frequency of a 45° X-cut plate of Rochelle salt 1 mm thick is 1200 kc, i.e., about half the frequency of an X-cut quartz plate of the same thickness. Plates of Rochelle salt cannot be made thinner than 1 mm; thinner plates break easily. In ultrasonics, Rochelle salt plates are used for frequencies up to several hundred kc.

Ultrasonic Application of ADP, KDP, and Barium Titanate Crystals. Recently an intensive search has been conducted for new piezoelectric materials, which might possess better physical properties than Rochelle salt. Up to 1200 various crystals have been tested for piezoelectric effect*. It has been found that some crystals, while possessing a sufficiently great piezoelectric effect, have at the same time several advantages as compared with Rochelle salt (greater mechanical and dielectrical strength, moisture resistance, etc.). Among the new piezoelectric crystals are ADP (ammonium dihydrogen phosphate) and KDP (potassium dihydrogen phosphate). These crystals are grown artificially, like Rochelle salt. At present they are widely used in hydroacoustic apparatus.

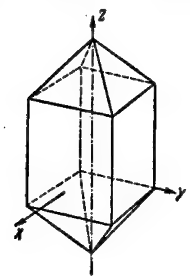


Figure 113. Shape of ADP and KDP crystals and the directions of the crystallographic axes

Figure 113 shows the shape of the ADP and KDP crystals and the directions of their crystallographic axes X, Y, Z.

The cut of the ADP crystal most widely used in ultrasonics is the so-called 45° Z-cut. It corresponds to the 45° X-cut of Rochelle salt, the only difference being that in this case the Z-axis is substituted for the X-axis. The 45° Z-cut plate performs longitudinal vibrations, just as a 45° X-cut Rochelle salt plate. By virtue of its high piezoelectric effect, Rochelle salt takes first place among all the piezoelectric crystals which have been investigated up till now; quartz is rated 28th, barium titanate is rated 2nd, while ADP and KDP are rated among the first ten crystals.

At present, along with quartz, tourmaline, and Rochelle salt, barium titanate ($BaTiO_3$) has come into wide use in ultrasonics. In its physical properties, barium titanate has much in common with Rochelle salt and is classified in the so-called Rochelle salt group. Barium titanate crystals, unlike those of Rochelle salt, are insoluble in water; they have a faint coloration, which depends mainly on the impurities present and which varies from light yellow to red-orange. Barium titanate possesses piezoelectrical properties, its piezoelectric effect being 20-30 times that of quartz. It is extremely difficult to grow barium titanate crystals large enough to cut plates suitable for ultrasound generators. For this reason the usual procedure is different. Small crystals are grown, a few millimeters in size; these are then baked together with minute quantities of a cementing substance. Samples of such polycrystalline barium titanate (plates, rods, etc.) are often referred to simply as barium titanate ceramics; its density is 5.4-5.5 g/cm^3.

In order to acquire piezoelectrical properties, a ceramic plate must first be subjected to a constant electric field of several kilovolts per cm, i.e., a preliminary polarization is necessary. As a result of this polarization, the ceramic retains residual piezoelectrical properties, which are somewhat inferior to those of monocrystalline barium titanate. When an alternating electric field is applied in the direction of the preliminary

* According to estimates made by V.A. Koptsik at the Crystallography Institute of the U.S.S.R. Academy of Sciences, 450 of these crystals were found to possess a piezoelectric effect; 500 did not possess any; and 25 of the remaining 250 did not give definite results. Beside these, 1600 more crystals are known which, although not yet tested in practice, may possess piezoelectric properties depending upon their symmetry conditions.

polarization, longitudinal vibrations arise both in the direction of polarization, and perpendicular to it; when an alternating field is applied normal to the direction of preliminary polarization, shearing vibrations are generated.

In its elastic properties, barium titanate ceramic resembles quartz most of all. For longitudinal vibrations in the direction perpendicular to the axis of polarization, the fundamental frequency of a plate of thickness l is given by

$$f \sim \frac{2500}{l} \text{ kc/mm.}$$

For longitudinal vibrations in the same direction as the preliminary polarization, the fundamental frequency is

$$f \sim \frac{2800}{l} \text{ kc/mm.}$$

The elastic, piezoelectrical, and dielectrical properties of barium titanate ceramic depend on temperature, intensity of the polarizing field, and external pressure. Especially great is the temperature effect upon the electromechanical properties of a ceramic. If a ceramic generator is used in a medium in which the temperature varies a few degrees, then the variation in the resonance frequency of the generator may amount to 2-3 %, i.e., it will be of the order of the width of the resonance curve; as a result, the amplitude of the ultrasonic vibrations generated decreases sharply. In order to eliminate this undesirable phenomenon, it is possible to regulate the temperature of the medium by means of a thermostat, which will prevent temperature variations; but this is not always convenient and feasible.

Barium titanate ceramic also suffers from another disadvantage. If an electrostatic field is applied to a polarized ceramic plate in the direction opposite to that of the polarization, and if this electrostatic field is gradually increased, then the piezoelectrical properties of the ceramic will grow weaker and weaker until, at some value of the field intensity, they will totally disappear. This critical value of the field intensity is called the coercive intensity. A ceramic generator will become "fatigued"* under the action of an alternating electric field with an intensity which is near the coercive field intensity. In order to eliminate this fatigue, it is necessary to apply, in addition to the alternating field, a constant polarizing field in the direction of the preliminary polarization. If the intensity of such an "auxiliary" polarizing field is greater than that of the alternating field, the generator will not become "fatigued". In this way, the electromechanical properties of a ceramic can be improved "from outside"; at times, however, this is not only inconvenient, but even impossible. As shown by investigations, the electromechanical properties of ceramics can be greatly influenced "from inside" by introducing small quantities of admixtures of various substances into the composition of the ceramic. Introducing 4-8 % of lead titanate ($PbTiO_3$) increases the temperature stability at the resonance frequency of the plate and greatly increases the value of the coercive field.

Barium titanate ceramic, due to its piezoelectrical properties, can be used as an ultrasonic generator over a wide temperature range, from a

* The "fatigue" manifests itself as a gradual decrease of the amplitude of the ceramic-generator vibrations, as well as causing widening of the resonance curve.

few dozen degrees above absolute zero up to 120°C. Between 10 and 100°C the piezoelectrical properties of the "improved" ceramic change only very slightly with temperature. At temperatures above 120°C, the piezoelectrical properties of barium titanate ceramic disappear (this temperature corresponds to the so-called Curie point).

When an alternating field of frequency f is applied to a ceramic ultrasonic generator without auxiliary polarization, then in the spectrum of the frequencies generated there will appear, in addition to f, its multiple $2f$. The latter appears in the spectrum because, unlike the quartz piezoelectric effect, that of a barium titanate ceramic is not linear, i. e., the deformations are proportional not only to the first degree of the field intensity E, but also to E^2 and to still higher powers of E. A piezoelectric effect which is proportional to E^2 is called a q u a d r a t i c p i e z o e l e c t r i c e f f e c t or e l e c t r o s t r i c t i o n. Hence, barium titanate ceramic possesses electrostrictive properties, and these can be used for frequency multiplication.

Barium titanate ceramics are used mainly as ultrasonic generators; as receivers they are not always efficient because of their high dielectric constant ε (at room temperatures, $\varepsilon \approx 1100$)*.

Ceramic generators possess a very substantial advantage. The fact is that for some purposes, e. g., for concentration of the ultrasonic beam within a certain space, it is necessary either to make use of acoustic lenses, or to make the generator itself of a complicated shape and with a definite orientation of the crystallographic axes (see § 4, Chapter VII). Manufacturing such generators from single crystals (e. g., quartz or Rochelle salt) is extremely difficult. Since the "anisotropy" of the ceramic is "created" by the polarization, the task of manufacturing ultrasonic generators of a complex shape is simplified in cases where a barium titanate ceramic is used.

We have already seen that for a vibrating quartz plate there is a definite limit of mechanical strength. Because of the inverse piezoelectric effect, which leads to the deformation of the plate, it is impossible to apply an electrical potential of unlimited intensity. This limitation is even more essential in the case of other piezoelectric materials, the mechanical strengths of which are less than that of quartz, but the piezoelectric effects of which are greater. For such piezoelectric generators as plates of Rochelle salt crystal, ADP, and barium titanate ceramic, the maximum allowed relative deformation is $\sim 10^{-4}$. In order to obtain greater deformations with such piezoelectric materials as ADP and barium titanate ceramics, an interesting method has been developed using a tapered metal "horn", or "trumpet". To a stack of ADP plates with an area of a few dozen square centimeters (or to one end of a hollow cylinder of a barium titanate ceramic), a metal (e. g., steel) horn (Figure 114) is cemented,

* From the theory describing the functioning of piezoelectric transducers, it follows that when a piezoelectric plate operates as an ultrasonic receiver, its sensitivity is determined by the ratio

$$g = \frac{\xi}{\varepsilon E}\Big|_{\text{for } \sigma = 0},$$

where ξ is the deformation and σ is the mechanical stress. Because of the very high value of ε for barium titanate ceramic, the large electric displacement $D = \varepsilon E$, which is caused by the ultrasonic field, will not bring about a considerable potential difference on the electrical output side of the receiver, and in the final analysis the sensitivity of such a receiver will be low.

tapered according to the exponential law

$$S=S_0e^{-2\beta y},$$

where S_0 is the area of the mouth of the horn, and β determines the law for the change in cross-sectional area of the horn with increasing y. The stack generates piston vibrations at its own fundamental frequency f; in this case, as we know, the length of the stack in the y-direction is $\frac{\lambda_{cr}}{2}$ (a half-wave stack), where λ_{cr} is the length of a longitudinal elastic wave in the crystal at the given vibration frequency f. If the total length l of the horn is equal to an integral multiple of half-waves (resonance system), then the cemented point is at an antinode (a mechanical stress node) and is subject to no tensile strain. At the distance $\lambda_{st}/4$ (where λ_{st} is the length of an elastic wave in steel at the given frequency f), i.e., at the displacement node, the holder of the horn is placed.

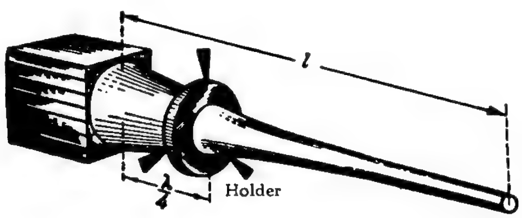


Figure 114. Obtaining large displacements by means of a stack of 45° Z-cut ADP plates and a tapered metal horn

Since the horn becomes narrower, the particle velocity changes in inverse proportion to the horn diameter. For instance, if the diameter decreases from 50 mm to 1 mm, then the particle velocity increases by a factor of 50, and at the end of the horn the relative deformation will be*

$$\xi_{st}=50\frac{c_{cr}}{c_{st}}\xi_{cr},$$

where c_{cr} is the velocity of sound in the piezoelectric crystal, and ξ_{cr} is the deformation of the piezoelectric crystal. For an ADP crystal (45° Z-cut) $c_{cr} = 3.3 \cdot 10^5$ cm/sec, and for steel $c_{st} = 5.05 \cdot 10^5$ cm/sec, so that for a relative deformation in the crystal of the order of 10^{-4}, the deformation in steel is of the order of $3 \cdot 10^{-3}$. In this way a relative deformation may be obtained in the steel of the order of 10^{-2}, which exceeds the elastic limit of steel and leads to a plastic deformation of the steel; in the presence of such deformations it is easier to investigate metals for fatigue.

The rather large piezoelectric effect of barium titanate ceramic was combined with the method for the intensification of deformation described above in order to investigate the wearing down of porcelain insulators and various other materials by attrition. The ceramic generator was made in the shape of a hollow cylinder, from the end of which displacements were transmitted into the "displacement transformer", the latter being a

* This formula holds when $S_0 < \frac{\lambda_{st}}{2}$.

brass cone with an exponential generatrix; at the narrow end of the cone the displacements were considerable, their amplitude being comparable to the amplitude of displacement arising due to the action of wind in places where electric wires are attached to porcelain insulators. Since the generator frequency was much higher than the frequency of oscillation of the wires, a short time was sufficient to check the quality of the insulators under [simulated] conditions of prolonged use.

§ 2. Magnetostriction. Magnetostrictive ultrasonic generators and receivers.

Magnetostriction. Many metals and alloys possess the property of contracting or expanding under the influence of a magnetic field; this phenomenon is called m a g n e t o s t r i c t i o n . Like the piezoelectric effect, magnetostriction is a reversible phenomenon; upon compression or stretching of these metals, a corresponding magnetic field appears.

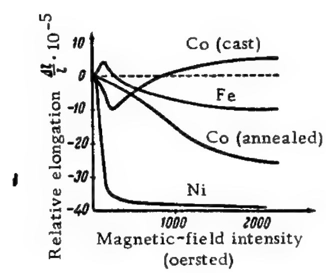


Figure 115. Longitudinal magnetostriction of Fe, Ni, and Co (polycrystalline samples)

Among the metals possessing the property of magnetostriction are iron, cobalt, and nickel, as well as many alloys. Figure 115 shows curves representing the longitudinal magnetostriction of polycrystalline samples of iron, nickel, and cobalt plotted against the intensity of the applied magnetic field*. From the curve representing the longitudinal magnetostriction for iron, it is seen that in a weak magnetic field a sample of iron is elongated (the magnetostriction is positive), while in a strong magnetic field the sample contracts (the magnetostriction is negative). For nickel, a contraction of the linear dimensions of the sample is always observed with an increase in the intensity of the magnetic field. Cast and annealed cobalt always have longitudinal-magnetostriction curves different in form from each other. Appreciable magnetostriction is observed in certain alloys, such as iron-cobalt, nickel-iron-palladium, etc. In the manufacture of magnetostrictive ultrasonic generators and receivers, the most frequently used materials are pure nickel and nickel-cobalt alloys.

Magnetostricive Generators and Receivers. The simplest magnetostrictive generator is a rod made of a metal possessing the property of magnetostriction; about this rod is placed a coil. When an alternating current is passed through the winding of the coil, an alternating magnetic field is generated within it, and the rod contracts and expands periodically, in time with the current oscillations, i. e., it performs mechanical vibrations, mostly in its longitudinal direction**. These vibrations cause the end of the rod to generate audio-frequency or ultrasonic waves.

* See, for instance, Belov, K.P. Uprugie, teplovye, i elektricheskie yavleniya v ferromagnitnykh metallakh (Elastic, Thermal, and Electrical Phenomena in Ferromagnetic Metals). –Gostekhizdat. 1951.

** In order to prevent doubling of the frequency, an auxiliary magnetization or preliminary magnetization is necessary (see p. 68).

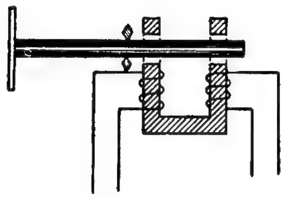


Figure 116. Generation of vibrations in a magnetostrictive rod placed in a hole in a transformer core

The rod is attached at its center point. In order to increase the radiation, a disk is attached rigidly to the end of the rod.

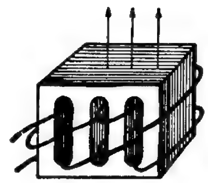


Figure 117. A magnetostrictive vibrator—a transformer composed of sheets of a magnetostrictive material

In order to produce vibrations in a magnetostrictive rod, the rod may also be placed in a hole made in the core of a transformer, while an alternating current is passed through the transformer winding (Figure 116).

To increase the amount of power generated, a plate is rigidly attached to the end of the rod.

If a sound wave falls on the rod, then as a result of the oscillating sound pressure the rod begins to vibrate, expanding and contracting in turn. The consequent alternating magnetic field induces in the winding of the coil a corresponding e. m. f.

The fundamental frequency of a rod attached at its center point is given by the formula

$$f=\frac{n}{2l}c,$$

where $n = 1, 3, 5 \ldots$ represents the harmonic number; l is the length of the rod; and c is the velocity of sound in the rod. Hence, a rod, like a piezoelectric quartz plate, can vibrate only in odd harmonics. For nickel, the fundamental frequency ($n = 1$) is given by the formula

$$f=\frac{237}{l}\ \mathrm{kc/cm};$$

a rod 1 cm in length has a fundamental vibration frequency of 237 kc. For 50 kc, the rod length must be about 5 cm, and for 6 kc it must be 40 cm.

The most frequently employed type of magnetostrictive vibrator is represented schematically in Figure 117. The vibrator is a transformer, the sheets of which are made of a magnetostrictive material, most often of pure nickel. The vibrator radiates in the direction shown by the arrows in Figure 117. An approximate estimation of the fundamental frequency of the vibrator is given by the same formula as the rod. For example, if a resonance frequency of 40 kc is desired, the thickness of the stack must be*

$$l=\frac{237}{40}\approx 5.9\ \mathrm{cm}.$$

* This is a very rough estimate. The real thickness should be somewhat less.

§ 3. The propagation of ultrasound in air and in other gases

Measurement of the Velocity of Ultrasound and of its Absorption. We have already noted that the absorption of sound due to the viscosity and heat conductivity of the medium is proportional to the square of the frequency. Hence, the attenuation of ultrasonic waves increases quickly with an increase in frequency. In air, ultrasonic waves travel only a short distance, so that ultrasound of frequencies above 25-30 kc is not used for purposes of signaling and communication. Ultrasonic waves between 16 and 25-30 kc are still capable of being transmitted over a few hundred meters if a sufficiently powerful ultrasonic generator is used, although even this is not a simple problem. The velocity of ultrasound in air is the same as the velocity of audio-frequency vibrations.

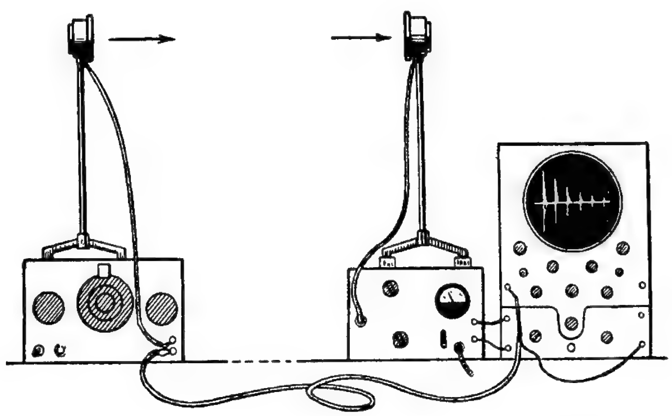


Figure 118. Equipment for measuring the velocity of ultrasound and its absorption using the pulse method

Left—pulse generator and radiator; right—receiver, amplifier, and electronic oscilloscope

In Figure 118 a sketch is given representing the equipment with which, by means of the so-called pulse method, it is possible to measure the velocity of propagation of ultrasound in air and to determine its absorption. A generator of electrical oscillations supplies voltage pulses, the frequency of repetition of which is a few dozen cycles (for example, 50 cycles). Hence the time between these pulses is 1/50 sec (or 1/50 = 0.02 sec = 20 msec), while the duration of the pulse itself amounts to 5-10μ sec (microseconds). These voltage pulses excite the radiator of ultrasonic vibrations. A circular plate, 3 mm thick, of Rochelle salt (L-cut) serves as the radiator. The plate is covered by electrodes; on one side it is glued by means of shellac to a textolite disk, while its other side serves as the radiating surface (Figure 119). The fundamental frequency of the combination of plate and textolite disk is 60 kc.

If we simply strike the plate, it will perform diminishing vibrations at its fundamental frequency, similar to a tuning-fork. An electrical pulse

arriving at the plate, due to the inverse piezoelectric effect causes a mechanical strain and thus serves just as well to make the plate vibrate with a frequency equal to its fundamental (60 kc). Due to its bond with the textolite disk, the vibrations will fade quickly. As a result, the voltage pulses supplied by the generator will lead to the radiation of fading ultrasonic pulses with a frequency of 60 kc. Since the dimensions of the radiating plate are much greater than the length of the ultrasonic wave (the plate diameter is 6 cm, while the wave length $\lambda = \frac{34\,000}{60\,000} = 0.5$ cm), therefore the directional distribution will be quite sharp; the ultrasound will be propagated as a narrow beam, like a beam of light from a searchlight.

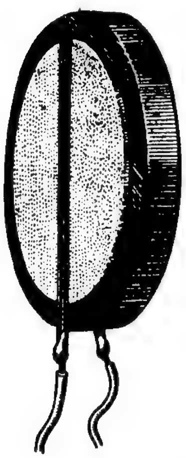


Figure 119. A very simple type of radiator made of an L-cut Rochelle salt crystal

At some distance from the radiating plate another similar plate is placed, which serves as a receiver of the ultrasonic vibrations*. Due to the direct piezoelectric effect, an electrical potential arises between the electrodes of the receiving plate upon arrival of the ultrasonic pulse; and this potential reproduces the shape of the pulse. Since the value of the potential generated across the receiving-plate electrodes is very small, it is amplified by an amplifier, the output pulses of which are already several volts. If the amplifier output is connected to the vertical deflection plates of an electronic oscilloscope, while a saw-tooth (sweep) potential is applied to the horizontal deflection plates, then we shall observe on the screen traces, two photographs of which are given in Figures 120 and 121. In order to make the image on the electronic oscilloscope screen stand still, the sweep frequency is synchronized by means of pulses fed from the generator causing the oscillation of the radiating plate. In this way the beginning of the sweep coincides with the instant of transmission of the ultrasonic signal. The first pulse is the ultrasonic pulse arriving through the air to the receiving crystal and amplified by the amplifier.

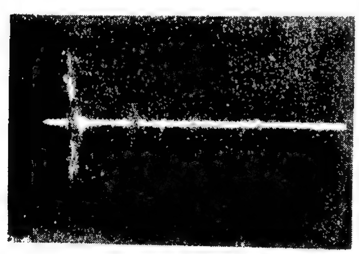


Figure 120. Distance between the radiating and receiving plates is 72 cm; sweep time is 20 msec. Time taken by the ultrasonic pulse to travel from one plate to the other is 2.1 msec

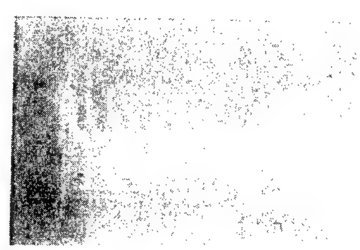


Figure 121. Distance between the radiating and receiving plates is 26.5 cm; sweep time is 20 msec. Time taken by the ultrasonic pulse to travel from one plate to the other is 0.77 msec

* To avoid stray induction, it is necessary to place the receiving plate in a metal vessel filled with castor oil.

If the planes of the radiating and receiving crystals are arranged strictly parallel to each other, then the arriving pulse will be reflected from the surface of the receiving plate in exactly the opposite direction and, on reaching the surface of the radiating plate, will be again reflected, and so on. The motion of the pulse from the radiating plate to the receiving plate and back again will result in the appearance on the oscilloscope screen of a series of pulses of diminishing amplitude, situated at equal distances from one another (Figures 118, 120, and 121). The sweep frequency is such that the whole sweep period from far left to far right occupies the same time interval as that between two electrical pulses, i.e., 20 msec. Since we know the frequency of the transmission of pulses (or, the sweep frequency, which is equal to it), and since the distance between the radiator and the receiver can be measured, the velocity of propagation of the ultrasound may be determined. For this we must only take into account that the ultrasonic pulse travels the distance twice — from the radiator to the receiver and back — and then we must measure on the oscilloscope screen the time between two successive pulses, taking into account that the total sweep time is 20 msec. As a result of such measurement, we find that the velocity of propagation of 60 kc ultrasound in air has the same value as the velocity of propagation of audio-frequency vibrations. If we were to perform similar measurements using higher ultrasonic frequencies, the result would be the same. By studying the decreasing amplitude of the pulses, it is possible to determine the amount of absorption of the ultrasound.

Beside this briefly described pulse method for measuring the velocity and absorption of ultrasound, interference methods have become of great significance for the same purpose. We have already dealt with one such method in a preceding chapter (the traveling-wave interferometer). Another method, proposed by the American physicist Pierce, which is based on the interference between the direct and reflected waves and plays a great part in modern ultrasonic research, consists of the following.

An X-cut quartz plate is excited by an electrical oscillator, the frequency of which is adjusted to the fundamental frequency of the plate. The plate, during vibration, continuously radiates ultrasonic waves in the form of a beam which propagates through the medium in which it is desired to measure the velocity of ultrasound and its absorption. Arriving at the metal reflector-plate, which is aligned parallel to the plane of the quartz plate, the ultrasonic waves are totally reflected and, proceeding in the opposite direction, return to the surface of the radiator. Due to the superposition of the incident and reflected waves, interference occurs, and the formation of standing waves. The latter have a certain effect (cause a reaction) on the radiating quartz plate, and this effect is greatest when the distance between the planes of the receiver and reflector is equal to an integral number of half waves. This reaction is recorded by means of electrical gauges [showing the load on the pulse generator].

The distance between the quartz plate and the reflector can be changed continuously with a micrometer screw.

By moving the reflector, we can in this way measure half of the wave length, and since the frequency f of the generator is known, the ultrasonic velocity may be determined from the formula $c = \lambda f$. By means of the standing-wave interferometer it is possible to measure the ultrasonic

propagation velocity with a very high degree of accuracy, better than 0.1 %. In addition to velocity measurements, the absorption of ultrasonic waves can also be determined by this method.

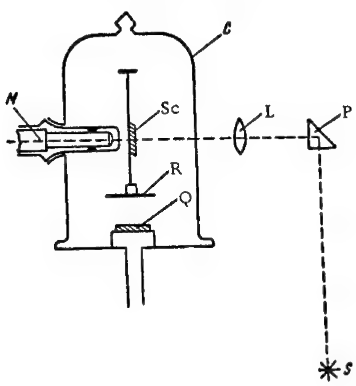


Figure 122. A diagram of the ultrasonic interferometer using standing waves (Pierce's interferometer)

C–container (glass or metal), Q–quartz plate, R–reflector, Sc–scale, M–microscope, L–lens, P–prism, S–light source

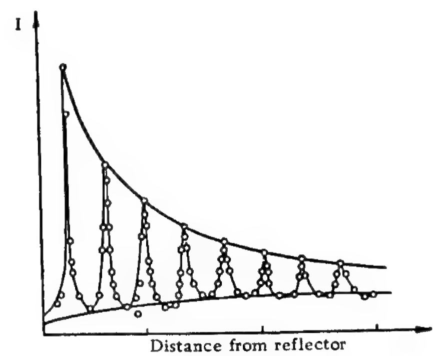


Figure 123. Curve describing the change in current in the quartz-plate circuit as the reflector is moved

Figure 122 shows a diagram of an ultrasonic standing-wave interferometer. Usually, in measuring the velocity of ultrasound in gases at different pressures and temperatures, the shifting of the reflector is brought about from outside of the vessel, containing the gas, the quartz plate, and the reflector with the scale; this is not shown in Figure 122. Observation of the position of the reflector is made with the help of the microscope.

In Figure 123 a curve is given representing the effect (the change in the anode current of the generator causing vibration of the quartz plate) of shifting the reflector. Current maxima correspond to nodes, while the minima correspond to antinodes, of the pressure at the point where the reflector is located. As the reflector is moved further from the quartz plate, the values of these maxima decrease. Using these values of the maxima which decrease with increasing distance, it is possible to determine the coefficient of absorption of ultrasound in the medium between the radiator and the reflector.

Coefficient of Absorption of Ultrasound in Air. Figure 124 gives the curves for the coefficient of absorption of audio and ultrasonic waves for room air as a function of frequency, obtained chiefly by means of the ultrasonic standing-wave interferometer. The curves correspond to a pressure of 760 mm of mercury and a temperature of 26.5°C; under these conditions, room air contains about 0.03 % CO_2 by volume and a humidity such that the water-vapor molecules constitute 1.26 % of all the air molecules (relative humidity of 37 %). The theoretical curve was calculated from the formulas of § 5, Chapter II, taking into account the viscosity and heat conductivity of the air. As seen from Figure 124, at frequencies below 100 kc the absorption in air is much greater than that calculated theoretically. More detailed investigations show that this discrepancy is caused by the presence of water vapor in the air. However, also at frequencies higher than 100 kc

there is appreciable disagreement between theory and experiment (a difference factor of about 1.5); at these frequencies, besides humidity, the presence of carbon dioxide also has an effect (see next section).

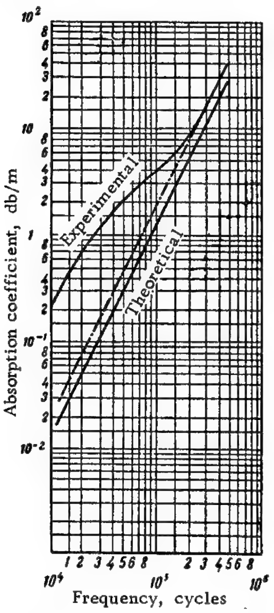


Figure 124. Coefficient of absorption of sound and ultrasound in air as a function of frequency

The given data for the absorption of ultrasound in air indicate that it is impossible to transmit ultrasound in air across large distances (of the order of 1 km or more)*. Indeed, even if ultrasound (for instance, with a frequency of 50 kc) is traveling in still air, its absorption, in accordance with the above data, will be 2 db/m. This means that in traveling a distance of 1 m the amplitude of the acoustic pressure developed by the wave is diminished by a factor of 1.26 (see Table on p. 45). It is easy to calculate that in traveling a distance of 50 m the attenuation will amount to 100 db, i.e., the pressure amplitude will be diminished 10^5 times; over a distance of 100 m, the absorption will already amount to 200 db, and the pressure amplitude will already be diminished 10^{10} times; and so on. Hence it will be seen that practically no increase in power can help to transmit ultrasound through air, even for comparatively short distances. Ultrasound of higher frequency has even greater attenuation (since absorption is proportional to the square of the frequency, see p. 48); besides, under actual atmospheric conditions there are important effects, as we shall see in the next chapter, due to several other factors, which, in general, cause an attenuation of ultrasound which is much greater than that caused by the viscosity and heat conductivity of the air.

§ 4. Molecular absorption and dispersion of ultrasound

Dispersion and Molecular Absorption of Ultrasound. As long ago as 1910, N.P. Neklepaev (a pupil of P.N. Lebedev), while seeking experimental verification of the formula for the coefficient of absorption, discovered that the absorption in air at a frequency of 400 kc was approximately twice as high as that calculated from the theoretical formula. At that time, Lebedev suggested that at high frequencies, where the length of the

* Even at the present time, suggestions continue to come in about arranging a secret (inaudible) communication in air by means of ultrasound transmitted across distances of the order of 1 km and more, whereby ultrasound would be sent in a concentrated beam from the radiator to the receiver. These suggestions arise, it would seem, because popular science books often state that high-frequency ultrasonic waves behave in many respects similarly to light waves, but the basic difference between them is not enough emphasized. Such an analogy suggests to the inexperienced reader the idea that ultrasound is capable of traveling distances as light waves do, which is quite wrong, for the nature of elastic waves is quite different from that of light.

ultrasonic wave is very small, the molecular nature of the gas must be taken into account*.

PETR NIKOLAEVICH LEBEDEV
(1866-1912)

Accurate measurements of the velocity of ultrasound in gases brought about the discovery of an extraordinarily interesting phenomenon. Kneser discovered in 1931 that in polyatomic gases at sufficiently high ultrasonic frequencies the velocity of the ultrasound undergoes a change, i.e., a dispersion of the ultrasound takes place for such gases. Moreover, simultaneously with the change in the velocity of the ultrasound, its absorption also increases. It is true that the change in velocity is, generally speaking, not great, but still it is considerably greater than the measurement error. Thus, for instance, it has been found that for carbon dioxide gas (CO_2), the molecules of which consist of three atoms, the velocity of sound up to a frequency of 10^5 cycles is constant, being equal to 258.9 m/sec, which value agrees with that calculated from Laplace's formula. As the frequency increases, this velocity increases too, by 12 m/sec approximately, reaching a constant value again (271 m/sec) at a frequency of 10^6 cycles. The absorption of ultrasound at 277 kc turns out to be approximately 20 times

* Lebedev's interest in acoustics is worth mentioning. Much was done by his group at the Moscow University in this important branch of physics. Beside the well-known work by Neklepaev on the measurement of the attenuation of ultrasound, the works by A.B. Al'tberg and V.D. Zernov (mentioned elsewhere) are worthy of note at this point, as is the work by A.B. Mlodzeevskii, who demonstrated that the velocity of ultrasound in air does not differ from that of sound.

greater than it should be according to the classical theory of absorption, which takes into account the losses of energy due to the viscosity and heat conductivity of CO_2. At frequencies above 10^6 cycles, the absorption once more agrees with that given by the classical theory. How is this phenomenon to be explained?

The Physical Nature of Molecular Absorption. Relaxation Time. In order to make the following discussion clearer, we have to remind the reader briefly of some fundamental facts of the molecular-kinetic theory. The gas pressure in a closed vessel, and also the pressure of the gas layers upon each other, are caused by collisions of the gas molecules with the vessel walls and also with each other. Hence, this pressure is directly proportional to the energy of translational motion of the molecules, i. e., to their kinetic energy. This energy is greater for higher gas temperatures, since the higher the temperature, the greater is the velocity of motion of the gas molecules.

If a gas molecule were a mass point, it would possess, in the language of mechanics, three degrees of freedom of motion, corresponding to three mutually perpendicular directions. Any of its movements could be resolved into components in these three directions. We may call these three degrees of freedom the external, or translational, degrees of freedom of the molecule; molecules of monatomic gases—helium, neon, argon—under certain conditions may be considered to be mass points. A compound molecule, however, does not present such a simple system. Roughly speaking, it may be represented as a combination of separate balls, tied together with small elastic springs. For example, in a molecule of carbon dioxide (CO_2), C and O_2 may be thought of as such small balls. This model is certainly extremely simplified, but it is sufficient to explain the cause of dispersion and of the anomalous absorption. Beside its three external (translational) degrees of freedom, every compound molecule also possesses internal degrees of freedom. The atoms making up the molecule may undergo vibrations relative to each other—these represent the vibrational degrees of freedom. In addition, every molecule may rotate about its center of mass, i. e., it may have rotational degrees of freedom.

Let us now imagine that in a polyatomic gas, e. g., carbon dioxide, ultrasonic waves are spreading. For the sake of simplicity, let us consider the wave to be rectangular, not sinusoidal (curve a in Figure 125). If the ultrasonic wave causes rapid (adiabatic) compression of the gas at a moment t_0, then at first the energy E_k of translational motion increases and there is a corresponding increase in the pressure p (curves b and e in Figure 125).

What will happen after compression? A part of the energy of translational motion of the molecules, following a series of mutual collisions, will be transferred from the external degrees of freedom to the internal degrees of freedom of the molecules. Let us denote the internal energy of the molecules by E_i; after compression E_i will increase (curve d in Figure 125), while E_k will decrease. The total energy E is the sum of the energy E_k of translational motion and the internal energy E_i:

$$E = E_k + E_i.$$

The total energy remains constant until a new change in volume (curve b in Figure 125).

Since the pressure p arises because of the effect of E_k, the pressure will also decrease after compression (curve e in Figure 125); of course, the pressure will be greater than that at the moment just preceding compression, but it will be less than that at the moment of compression. After a certain interval of time, a new state of equilibrium will be established in a gas that has undergone compression; its temperature will rise somewhat on account of the compression, and there will be established a new distribution of energy between the external and the internal degrees of freedom of the molecules. In the second half-period of the wave, during rarefaction, the situation will be the opposite: at first the energy E_k of translational motion will decrease sharply compared to its equilibrium value; then, as a result of a series of mutual collisions, a part of the internal energy E_i will be transformed into energy possessed by the external (translational) degrees of freedom, and E_k will be increased. The change in pressure will have a similar pattern. Immediately at compression the pressure falls abruptly, and then increases gradually. After a certain time, the equilibrium state corresponding to rarefaction will re-establish itself.

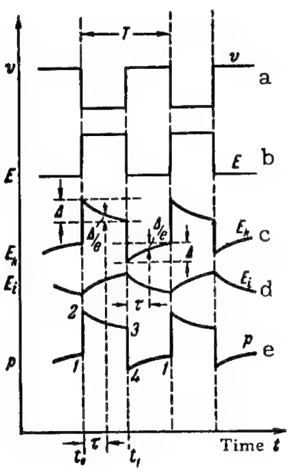


Figure 125. a) periodic abrupt changes in volume v; b) the corresponding change in the total energy E; c) the change in the kinetic energy E_k; d) the change in the internal energy E_i; e) the change in pressure p.

Here we have an example of a so-called relaxation process, a type of process which plays an important part in physics. Relaxation processes tend to transfer a system into an equilibrium state. As a very rough example of a relaxation phenomenon we may consider a light pendulum immersed in a very viscous liquid. If the pendulum is moved out of its equilibrium position, then due to gravity it will, after some time, return to this position. The deflection of the pendulum is said to undergo "relaxation".

The case under discussion, namely, the transfer of energy from the external degrees of freedom in a polyatomic gas to the internal degrees of freedom, under the action of a propagating ultrasonic wave, is also an example of a relaxation process. Later we shall become acquainted with other similar processes while discussing the propagation of ultrasonic waves in liquids.

The time during which the deviations of E_k, E_i and p from their equilibrium values increase or decrease by a factor of e (i.e., ~ 2.7) is called the relaxation time; we shall designate it by τ. This important quantity denotes the time necessary for restoration of the state of equilibrium after compression, as well as after rarefaction, of the gas, i.e., the time required for redistribution of energy between the external and internal degrees of freedom of the gas molecules.

The meaning of τ is evident from Figure 125c. If the total decrease in E_k after compression is equal to Δ, then the time for a decrease of E_k equal to Δ/e is the relaxation time τ. It is also evident that after rarefaction

(for example, at the moment t_1) the relaxation time will be the time which it takes for E_k to increase by the amount $\Delta - \frac{\Delta}{e}$. The maximum change in the velocity of sound takes place when the period T of the sound wave equals $2\pi\tau$ (i.e., at an angular frequency $\omega = \frac{1}{\tau}$). The upper curve in Figure 126 shows the relation between the square of the sound velocity and the frequency (the logarithm of the angular frequency $\omega=2\pi f$ along the abscissa). This relation is derived from the theory of sound propagation in polyatomic gases, and it is verified by experimental data. For carbon dioxide, dispersion occurs at an angular frequency $\omega = \frac{1}{\tau}$ corresponding to approximately 10^5 cycles per second at $t = 18°C$ and normal atmospheric pressure. The relaxation time for carbon dioxide is approximately $5 \cdot 10^6$ sec. In the lower curve of the same figure the sound absorption is shown as a function of frequency. Instead of the absorption coefficient α (see Chapter II, § 5), the quantity $\alpha\lambda$ is plotted on the ordinate axis; this quantity characterizes the attenuation of the amplitude over a distance equal to one wave length.

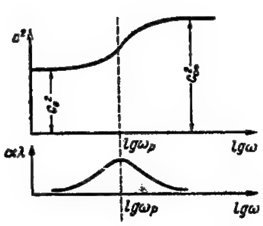


Figure 126. The theoretical curves representing dispersion and relaxation absorption of sound in polyatomic gases

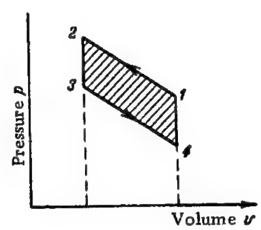


Figure 127. The pv diagram

How, then, is it possible to explain the anomalous absorption of ultrasonic waves at the frequencies where dispersion occurs?

On the basis of curves a and e in Figure 125, we can construct the pv diagram represented in Figure 127. Points 1, 2, 3, 4 in Figure 127 correspond to points 1, 2, 3, 4 of curve e in Figure 126. It is evident that during the total period T of the wave a closed cycle will be completed on the pv diagram. This means, however, that a volume element of gas will perform some work, which can only be used for heating the gas. Indeed, from mechanics we know that when a mass point acted upon by a force F travels a small distance l in the direction of the force, the work done by the force is Fl. In the case at hand, the force is the pressure acting on the surface area S of a gas volume element:

$$F = pS.$$

If, as a result of this pressure, the surface S of the volume element is displaced a distance l, then the work A is

$$A = Fl = pSl = pv.$$

In the pv diagram, the work done is represented by the area of the parallelogram 1-2-3-4. The area of the closed cycle therefore represents the work performed by a volume element of the gas.

This work is performed at the expense of the energy of the sound wave; since this energy is used up in heating the gas, additional absorption of the sound is caused.

Thus, due to the redistribution of energy between the external and internal degrees of freedom of polyatomic gas molecules, occurring because of the compressions and rarefactions caused by the sound waves, absorption of sound takes place. This absorption is called molecular absorption. Maximum molecular absorption occurs at conditions of maximum dispersion, i. e., it occurs at an ultrasonic frequency $\omega = \frac{1}{\tau}$ (when the period of the sound wave = $2\pi \times$ the relaxation time).

Dispersion of Ultrasound in Polyatomic Gases. As mentioned previously, the kinetic energy of the gas molecules is proportional to the temperature; the higher the temperature of the gas, the greater the velocity with which the molecules move.

The specific heat c_v at constant volume is the quantity of heat necessary to raise the temperature of a molar volume v of gas by 1°C, while the volume is held constant*. Hence c_v is nothing more than the increase in the energy of the gas volume v as a result of the 1°C change in temperature. We have defined the total energy E as the sum of the energies E_k of the external degrees of freedom (the energy of translational motion of the molecules) and of the energies E_i of the internal degrees of freedom (the energy of vibrational and rotational motion of the molecules); in the same way the specific heat c_v will be the sum of the specific heats c_{vk} of the external, and c_{vi} of the internal, degrees of freedom of the molecules in a volume corresponding to one mole of gas:

$$c_v = c_{vk} + c_{vi}.$$

For low-frequency sound waves, the compressions and rarefactions within a volume element of gas occur so slowly that the attainment of equilibrium between the excited and the unexcited molecules manages to follow the fluctuations of the pressure in the sound wave; the relaxation time τ is much shorter than the period T of the sound wave ($\tau \ll T$). In this case the sound velocity is determined by the well-known formula:

$$c = \sqrt{\frac{p}{\rho}\gamma} = \sqrt{\frac{p}{\rho}\frac{c_p}{c_v}}.$$

There is an important relation between c_p and c_v:

$$c_p - c_v = R,$$

where R is a constant known as the gas constant. Hence, the formula for the velocity of sound can be written

$$c = \sqrt{\frac{p}{\rho}\left(\frac{c_v + R}{c_v}\right)} = \sqrt{\frac{p}{\rho}\left(1 + \frac{R}{c_v}\right)}$$

* Let us recall that a molar volume is the volume occupied by one mole of the gas. If the molecular weight of a gas is μ, then one mole contains μ grams of the gas.

or, inserting the above expression for c_v,

$$c_0=\sqrt{\frac{p}{\rho}\left(1+\frac{R}{c_{vk}+c_{vi}}\right)}.$$

(Here the velocity is denoted by c_0 instead of c in order to emphasize that this formula is true for low frequencies).

In the case where the frequency of an ultrasonic wave is very high, there is not enough time for equilibrium to be attained between the external and the internal degrees of freedom of the molecules. The relaxation time τ is much greater than the sound wave period $T(\tau \gg T)$, so that the internal degrees of freedom of the molecules will not become excited. Then $c_{vi}=0$, and the velocity of sound will be

$$c_\infty=\sqrt{\frac{p}{\rho}\left(1+\frac{R}{c_{vk}}\right)}.$$

(Here the velocity is denoted by c_∞ in order to emphasize that this velocity refers to very high frequencies). Comparing the formulas for the velocity of sound c_0 at low frequencies with the velocity c_∞ at very high frequencies, we see that c_∞ is greater than c_0 (see Figure 126).

The expression for the velocity of sound may be written

$$c=\sqrt{\frac{1}{k\rho}},$$

where $k=\frac{1}{\gamma p}$ is the adiabatic compressibility. Since

$$\gamma_0=1+\frac{R}{c_{vk}+c_{vi}} \quad \text{and} \quad \gamma_\infty=1+\frac{R}{c_{vk}},$$

therefore γ_∞ is greater than γ_0, and it may be said that the velocity of sound increases at very high frequencies, because then the compressibility of the gas decreases. The more rapid the compression, the less compressible is the gas.

Consequently, the velocity of sound in polyatomic gases changes from c_0 at low frequencies to c_∞ at very high frequencies. The region of this change is the dispersion region (Figure 126).

Anomalous Absorption of Sound in Moist Air. The attenuation of sound waves in air has been seen to depend, to a very great degree, on the humidity. The explanation of this phenomenon is a matter of taking into account the relaxation absorption connected with the presence of water vapor. The absorption coefficient α, according to experimental data, depends upon the frequency of the sound and the humidity of the air. Figure 128 gives the experimental curves for absorption at different sound frequencies at a temperature of 20°C, as a function of the relative humidity of the air*. The curves were obtained by the American acoustician V.O. Knudsen. As seen from the figure, maximum

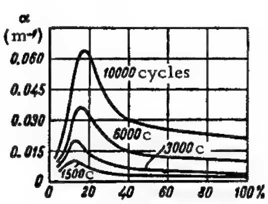


Figure 128. Sound absorption as a function of relative humidity

* The relative humidity of a gas is the ratio between the quantity of water vapor present and the quantity required to saturate the gas at the same temperature. The expression "quantity of water vapor required to saturate the gas" refers to the maximum quantity of water vapor per unit volume which can exist over a plane water surface at a given temperature.

absorption takes place at a very low relative humidity (10-20%); and the absorption increases with frequency. It should be noted that the influence of humidity on sound propagation must be taken into account in architectural acoustics, since it diminishes the reverberation time in buildings (see Chapter VI).

Investigations of molecular absorption and dispersion in polyatomic gases are a part of the important branch of modern sound science—molecular acoustics—and are of great significance in the study of molecular structure.

Many Soviet physicists, both experimental and theoretical, have been engaged on problems of dispersion and molecular absorption of ultrasound in polyatomic gases. Especially important work was performed by L. I. Mandel'shtam and M. A. Leontovich.

Chapter VI

SOUND PROPAGATION IN BUILDINGS AND IN THE OPEN AIR

§ 1. Filling a room or building with sound. Reverberation

Filling a Room or Building with Sound. When sound propagates in a closed area (a room or building), the sound waves, as they strike obstacles or walls, are reflected in various directions, whereby they are partially absorbed, and partially transmitted through the walls. Figure 129 shows how a sound pulse of 1/100 sec duration has spread within a building 20 m long and 10 m wide, 1/50 sec after leaving point O. The time required to

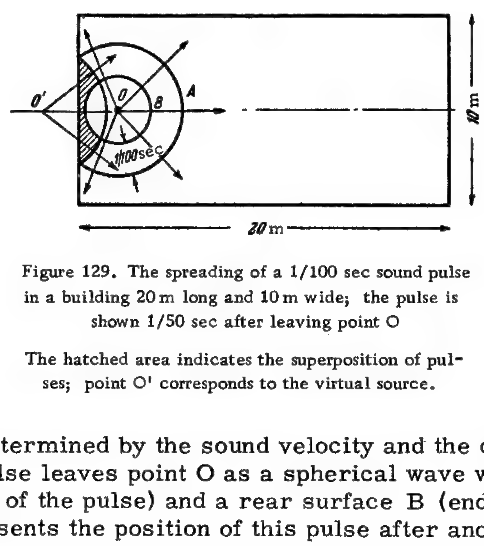


Figure 129. The spreading of a 1/100 sec sound pulse in a building 20 m long and 10 m wide; the pulse is shown 1/50 sec after leaving point O

The hatched area indicates the superposition of pulses; point O' corresponds to the virtual source.

fill the area is determined by the sound velocity and the dimensions of the building. The pulse leaves point O as a spherical wave with a forward surface A (beginning of the pulse) and a rear surface B (end of the pulse). Figure 130 represents the position of this pulse after another 1/25 sec; the increased number of reflections and the superposition of waves traveling in various directions are evident. After some time, due to multiple reflection from walls, ceiling, and floor, the sound will have filled the whole building, and the sound waves will be traveling in all directions, producing a mixed, or "diffused", sound. At every point within the building the flux of sound energy will be approximately equal in all directions. Figure 131 shows three photographs, obtained by the dark-field method, of the reflections of a sound pulse; the pulse was produced by an electric spark and was caused to spread throughout a model shaped like a hall. The photographs were taken 1/40, 1/25, and 1/18 sec after formation of the pulse.

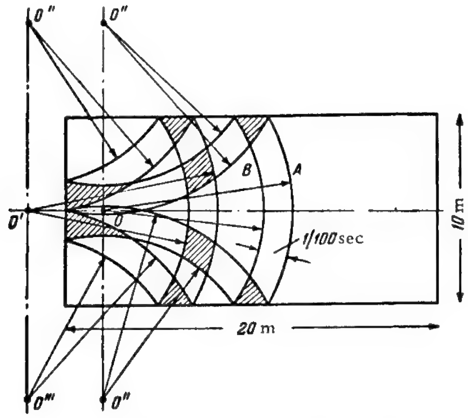


Figure 130. The same conditions as Figure 129, but 3/50 sec after the sound pulse has left point O

The hatched areas indicate the superposition of pulses; points O', O'', O''' correspond to virtual sources.

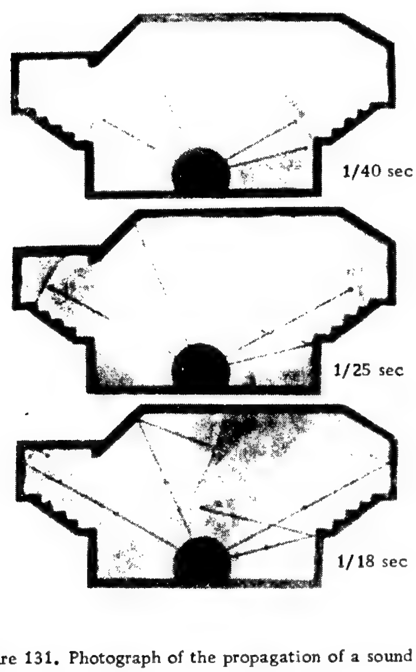


Figure 131. Photograph of the propagation of a sound pulse, obtained from an electric spark, in a model of a theater hall

Photographed by the dark-field method at intervals of 1/40, 1/25, and 1/18 sec after formation of the pulse.

In the photographs it may be seen how the sound pulse, reaching the walls of the hall and being reflected in different directions, after some time spreads throughout the whole building. On the basis of such experiments, using models of buildings, it is possible to explain the mechanism of sound propagation in real architectural structures (halls, lecture-rooms, etc.). Such models are often used to investigate the propagation of sound in buildings and rooms of different shapes.

When sound is reflected from smooth, hard walls, very little energy is lost; therefore, in buildings and rooms with walls of this kind a large number of reflections take place before the sound stops altogether. If we shout or clap our hands in an empty building with smooth walls, the sound will not die out at once, as it does out-of-doors, but will fade gradually. This gradual dying away of sound in buildings (residual resounding) is called reverberation, or repercussion.

The phenomenon of reverberation should not be confused with echo. The ear is capable of distinguishing one sound pulse from another, similar pulse, when the interval between them is not less than 1/15 sec. If sound pulses arrive more frequently, they merge into one continuous sound. The sequence of pulses reaching a listener as a result of a single sound pulse reflected from the walls of a building usually have intervals much less than 1/15 sec; therefore, reverberation does not have the properties of an echo, but of a continuous residual ringing. In an empty building or room with smooth, hard walls sounds do not die out for a long time, words mix with each other, and speech becomes indistinct; the sound in such places is said to be too "hollow".

On the contrary, in rooms with soft furniture, with walls of irregular shape, or with hangings and draperies, sound waves lose a considerable part of their energy, when they are reflected [and absorbed], and so the sound dies away very quickly; the sound in such places is said to be too "muffled". In this case a speaker has difficulty in addressing the public; music loses its richness, becoming empty and lifeless; and the same impression is produced as that when a band plays out-of-doors.

Hence, both overly strong and overly weak reverberation spoils the acoustic quality of a building. It is evident how important it is to ensure that a building is provided with the best acoustic qualities when designing structures of large dimensions: concert and theater halls, cinemas and clubs, and lecture halls. The acoustics of a hall will be good if every listener is reached by sounds of sufficient loudness, without interfering echoes and distortions. The sound reflection must be neither too strong nor too weak. These requirements must be satisfied by the room or hall over a rather wide frequency range, between approximately 100 and 3000-4000 cycles.

In the case of a sound source radiating spherical waves, the intensity of the sound diminishes inversely with the square of the distance from the source. A listener situated 25 meters from the speaker will receive 100 times less sound energy than one situated 2.5 m from the speaker. If this difference in sound energy is expressed in decibels, then the sound intensity at the first listener will be less than that at the second listener by $K = 10 \lg 100 = 20$ db. In the presence of reverberation in a building with good acoustic

properties, this difference in sound intensity can be reduced, for these same distances from the speaker, to approximately 5 db*.

The acoustic properties of halls and lecture-rooms have received a great deal of attention for many centuries. As a result of practical experience in erecting buildings with good acoustic qualitites, certain rules and methods have been developed in architecture, and these rules are followed today. For instance, it has been established that a building of moderate dimensions (average-sized theater) will possess good acoustic properties if its length, width, and height have a proportion of 8 : 5 : 3 (the so-called "golden section" [roughly]). These relations remained incomprehensible and mysterious; and if an architect constructed a building with good acoustics, it was considered a matter of mere chance. If the new building turned out to be acoustically bad, there was no way to remedy the situation.

The development of acoustics has indicated, however, not only ways of designing buildings intelligently, but improving buildings with bad acoustics as well. At present, the building shape, architectural style, finish, coverings, and a whole series of other factors are chosen on a scientific basis; this practice has been (and is being) further developed into an important division of modern acoustics—architectural acoustics. One of the basic factors determining the acoustics of enclosed spaces is known as the reverberation time.

Reverberation Time. This is defined as the time required for the sound intensity to decrease to one millionth of its original value. In order to obtain a more tangible concept of this time interval, let us study the sound attenuation occurring in a building after a sudden interruption of the action of the sound source.

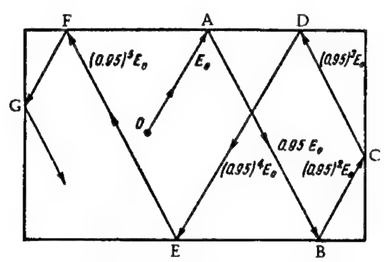


Figure 132. Diagram of the consecutive reflections of sound from the walls of a building; at each reflection, the sound intensity decreases by 5%.

Figure 132 shows a sound wave produced at point O, which strikes the wall at point A, is reflected, reaches point B, is reflected again, arrives at point C, and so on. Assume that the sound intensity decreases by 5% at every reflection. If the energy of the sound wave originating at point O is E_0, then after reflection at point A its energy will be

$$E_0 - 0.05\,E_0 = 0.95\,E_0.$$

* In a building filled with people, sound spreading through the crowd is subject to strong attenuation (see § 2 of this chapter); hence, in this case the decrease in sound intensity over a distance R is not proportional to $1/R^2$, but is more rapid.

After reflection at point B, the wave will lose another 5% of its energy, and on leaving this point its energy will be

$$0.95\,E_0 - 0.05 \cdot 0.95\,E_0 = 0.95\,E_0\,(1 - 0.05) = (0.95)^2\,E_0.$$

After the third reflection (at point C), the wave energy will amount to $(0.95)^3 E_0$. Thus, after n reflections, the sound wave will possess an energy equal to $(0.95)^n E_0$.

It is desired to calculate how many reflections must take place before the energy of the wave diminishes a million times. Obviously, for this to be true,

$$(0.95)^n\,E_0 = 0.000001\,E_0.$$

Using logarithm tables, it is easy to calculate that $n = 269$. Hence, if the walls of a room or hall absorb 5% of the sound energy, then this energy will be diminished by a factor of 10^6 after 269 reflections.

The less the absorption at the walls of a room, the more reflections are required for the sound energy to diminish 10^6 times. Thus, if 3%, instead of 5%, of the energy is lost during each reflection, then the required number of reflections is 461. Only a single sound beam has been considered so far, but it is not difficult to extend the discussion to include all the beams traveling in all directions from point O.

The above reasoning is certainly quite simplified. Actually, in order to calculate the number of reflections more precisely, it should have been considered that sound waves also undergo absorption in the course of their propagation through the air. The effect of this absorption becomes quite pronounced beginning with frequencies of 2000-3000 cycles. Moreover, the absorption of sound during reflection depends upon the angle of incidence between the wave and the plane of the wall, but that has not been taken into account either.

Now, having determined the approximate number of reflections necessary to diminish by a factor of 10^6 the energy of a sound wave, the reverberation time may be found. For this, obviously, it is necessary to know the total distance traversed by the sound during the given number of reflections. This distance, divided by the sound velocity, is the reverberation time. However, from one reflection to another, the sound wave covers different distances, which is evident from Figure 132. Therefore, in order to find the total distance traveled by the sound during the time required for n reflections, it is necessary to employ statistics, i. e., to compute the average distance [mean free path] traveled by a wave between successive reflections. This mean free path, denoted by s, can be calculated from the formula:

$$s = \frac{4V}{S}.$$

In this formula, V is the volume of the room or hall, while S is the total area of walls, ceiling, and floor. For example, if the dimensions of a building are $24\,\mathrm{m} \times 15\,\mathrm{m} \times 9\,\mathrm{m}$, then $V = 3240\,\mathrm{m}^3$, while $S = 1422\,\mathrm{m}^2$. By the formula given above, the mean free path is

$$s = \frac{4 \cdot 3240}{1422} \approx 9.1\ \mathrm{m}.$$

Now the reverberation time for such a building may be calculated, provided the percentage of sound energy absorbed by the walls when they are struck by sound waves is known. Let this loss be 5%; as already calculated,

269 reflections must take place before the original energy is diminished 10^6 times. If the mean free path s is multiplied by this number, the average distance traveled by a wave before its energy diminishes a million times will be found:

$$s \cdot 269 = 9.1 \cdot 269 \approx 2450 \text{ m}.$$

If this distance is divided by the velocity of sound, a reverberation time equal to $\frac{2450}{340} \approx 7.2$ sec is obtained. A building with such a reverberation time will sound too "hollow". In order to obtain a shorter time of residual resounding, it is necessary for the walls of the building to absorb a greater quantity of sound energy than the 5% assumed above. If the absorption amounts to 25%, then after 48 reflections the sound energy will decrease by a factor of one million, and so the reverberation time will be

$$\frac{48 \cdot 9.1}{340} \approx 1.3 \text{ sec}.$$

As a result of a great number of experiments, it has been established that if the reverberation time for a building is greater than 5 sec, it has very poor acoustics—it sounds too "hollow". If this time is from 3 to 5 sec, it has poor acoustics; and for a time from 2 to 3 sec, the acoustics are good. The optimum reverberation time is from 0.5 to 1.5 sec, depending on the purpose of the room or building, whether it is designed for speech or for music.

The following table gives acoustic data for some well-known auditoriums.

	Volume of hall m^3	Number of seats	Reverberation time (sec)		Acoustic quality
			empty hall	full hall	
Small hall of Moscow Conservatory	2550	550	3.46	1.3	Very good
Hall of Columns in the House of Soviets	12,500	1600	3.55	1.72	Very good
The Bolshoi Theater	13,800	2300	2.06	1.55	(Opera house) good
Great hall of Moscow Conservatory	18,400	2400	3.6	2.3	(Symphony hall) somewhat "hollow"

The simplified reasoning which enabled us to calculate the reverberation time has been based on the so-called statistical approach to the explanation of reverberation, whereby the mean free path of the wave between two reflections was calculated. Moreover, in all these calculations laws describing the rectilinear propagation of sound were employed, and the sound wave was taken to follow ray-paths. In other words, geometrical acoustics was used, and nowhere was the wave nature of sound propagation considered. Such an approach to the discussion of sound propagation in buildings proves to be very valuable for designing structures with good acoustic properties and it serves as the basis of architectural-acoustic engineering. However, as mentioned previously, the concept of a sound ray and the use of purely geometrical acoustics to investigate wave propagation is useful only within certain limits. When the wave length becomes comparable to the size of obstacles in its path, then the fundamental properties of wave motion become influential, and diffraction and interference must be considered.

Recently, a new method, based on wave concepts, has been developing successfully in architectural acoustics, the wave theory of sound propagation in closed spaces. This theory defines more precisely the limits for the application of geometrical acoustics to studies of sound propagation in closed spaces; and it has proved very useful in practice.

However, in the majority of cases of practical importance the length of the sound waves is considerably less than the size of the area. It is therefore quite permissible to employ geometrical acoustics, but its limitations should, of course, be kept in mind.

§ 2. Sound absorbers

Sound Absorbers. To provide good acoustics in halls and lecture-rooms, great importance must be attached to proper choice of the shape of the hall, as well as to the use of various arrangements preventing the normal reflection of sound—columns, convex surfaces, etc. In some cases the walls are not built parallel to each other, or else the ceiling slopes. Sometimes, however, all these measures prove inadequate. The common building materials—wood, glass, and plaster—do not absorb more than 3 % of the energy of the incident sound. Therefore, in a hall where no special measures are taken to increase absorption, the optimum reverberation time cannot be achieved, and the sound in the hall is too "hollow".

When sound waves are propagated in such materials as wool, felt, cotton batting, or asbestos, the vibrating air particles experience great friction, leading to considerable sound absorption. There is a sharp difference between the reverberation time for an empty hall and a hall full of people. Clothes are a good sound absorber; hence, in a full hall the reverberation time is considerably less. One person's clothes, on the average, absorb the same amount of sound energy as $20\,m^2$ of ordinary plaster. At present, a great number of porous materials have been found with high sound-absorption properties. To be usable in the construction of buildings such materials, in addition to their good absorption properties, must also possess such qualities as fireproofness, durability, lightness, appealing finish, and, finally, economy. Moreover, the frequency characteristics of the sound-absorbing materials, i.e., the relation of absorption to sound frequency, must be such that absorption over a frequency range from 300-400 cycles to 3-4 kc remains approximately the same.

It is very difficult to satisfy all these requirements at once, and it is especially difficult to obtain high absorption at low frequencies. As yet, a sufficiently developed theory of porous sound absorbers does not exist, and in the manufacture of such materials the method of trial and error is used, i.e., during production the composition of the porous substance is varied and the absorption at different frequencies is measured. Usually, porous absorbers have a sound-absorption coefficient between 0.4 and 0.5, i.e., they reflect 50-60 % of the incident energy.

Measuring the Sound-Absorption Coefficient of Materials. One of the most widespread methods of measuring the absorption coefficients of different sound-absorbing materials in the case of normal incidence of the sound waves is the method of the acoustic standing-wave interferometer. A dynamic loudspeaker, placed over the upper end of a long (3 or 4 m) metal

pipe (see Figure 133), produces flat waves with wave fronts perpendicular to the axis of the pipe (for this purpose, the wave length must be at least twice the pipe diameter). If there is an acoustically rigid wall at the other end of the pipe, the sound waves are totally reflected from it. The combination of incident and reflected waves results in standing waves, with nodes at which the acoustic pressure is equal to zero; the same will also be true of the amplitude of the acoustic velocity, with the difference that a velocity node will correspond to a pressure antinode, and vice versa. If, instead of a rigid wall, the waves strike a sound-absorbing material, which partly absorbs the sound, then the standing waves formed in the pipe will no longer have sharply defined pressure nodes (minima). If the sound-absorbing material could absorb the sound completely, only traveling waves would be propagated in the pipe.

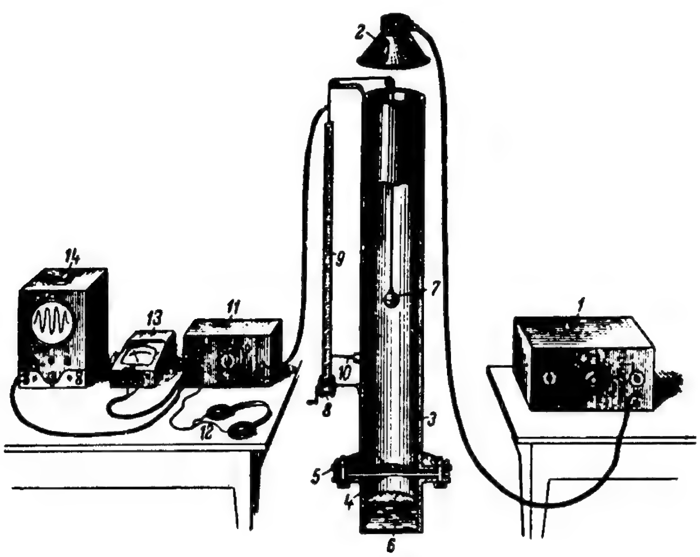


Figure 133. Diagram of equipment for measuring the coefficient of absorption of materials

1) audio oscillator, 2) dynamic loudspeaker, 3) pipe, 4) sample being tested, 5) flange, 6) water, 7) piezoelectric microphone, 8) winch, 9) steel tape with divisions, 10) indicator, 11) measuring amplifier with filter, 12) earphones, 13) meter with pointer, 14) electronic oscilloscope.

From theory, it follows that the absorption coefficient of a material, i.e., the ratio of absorbed to incident energy, is given by the formula

$$\eta = \frac{4}{2 + \frac{p_{max}}{p_{min}} + \frac{p_{min}}{p_{max}}},$$

where $\frac{p_{min}}{p_{max}}$ is the ratio of the minimum amplitude of the sound pressure (or velocity) to the maximum amplitude.

The values of p_{max} and p_{min} are recorded by means of a small piezoelectric microphone which is suspended inside the pipe by a thin, flexible shielded wire and which is able to be moved along the axis of the pipe. The

alternating electrical potential produced by the microphone is amplified by the measuring amplifier and then applied to the indicating instrument—a pointer-type meter or electronic oscilloscope. For a more precise determination of p_{max} and p_{min}, special filters may be employed, which eliminate the effect of the harmonics which may be produced by the dynamic loudspeaker. By readjusting the audio oscillator which supplies the loudspeaker with its audio-frequency signal, measurements may be made over the whole frequency range in which we are interested. It should be noted that in this type of interferometer the wave length of the sound is easily determined by moving the microphone (it is the distance between adjacent p_{min} and p_{max}). If the latter is known and the oscillator frequency is known, it is possible to calculate the velocity of sound in the pipe.

In architectural acoustics, reverberation is often employed in order to measure the absorption coefficients of various sound-absorbing materials. For this, the sample or the object to be tested is placed in a room of irregular shape, with smooth walls, floor, and ceiling which are good reflectors of sound, the reverberation time of the room being well-known for various frequencies (such a room is called a reverberation chamber). Then, by the change in the reverberation time of the chamber, it is possible to determine the sound-absorption coefficient of the material being tested, provided its area is known.

In making precise measurements of the sound absorption of materials in a reverberation chamber, beginning with frequencies of a few kilocycles, it is necessary to take into account the absorption of sound due to the chamber itself, i.e., due to the air. The chamber may also be used to solve a problem opposite to the previous one: if the coefficients of sound reflection from the chamber walls are given, the sound-absorption coefficient in air may be calculated by the decrease in sound level, provided that the attenuation of the sound is not too low, e.g., in measurements of the absorption of sound with frequencies above 1000 cycles in air with various degrees of humidity.

For the achievement of good acoustics in large rooms and buildings, the dimensions of which make it possible for a distinct echo to be heard, the requirements of sound-absorbing materials are much more exacting. Soviet scientists and acoustical experts were confronted with a very difficult task when developing the acoustical design of the Great Hall of the Palace of Soviets. The gigantic cupola of this hall, about 100 m high, was required to achieve complete absorption of sound in the range between 200 and 4000 cycles, for otherwise the sound reflected from it would reach the audience as an echo, distorting the transmission of speech and music.

A multi-layer, porous sound-absorbing material, with a high absorption coefficient, was develpoed by G. D. Malyuzhinets. As a result of numerous experiments, special sound absorbers, known as resonance sound absorbers, were developed by S. N. Rzhevkin and his co-workers, among whom V. S. Nesterov merits special mention.

Resonance Sound Absorbers. Many years ago in the ancient churches of Novgorod and Pskov, and some other old buildings, metal or ceramic resonator-vessels were set into the walls (Figure 134). This vessel was called a "voicer" (golosnik) and served the purpose of augmenting the resounding qualities of the voices of a choir or of an instrument. In the Malyi Theater in Moscow, too, similar vessels were once installed, but

afterwards destroyed during repairs of the building. These resonator-vessels of various sizes and with various fundamentals created, by means of their resonant qualities, an impression of "hollowness", i.e., they

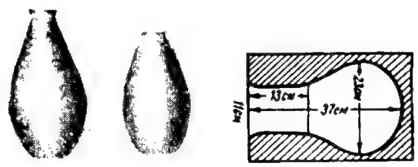


Figure 134. Golosniks ("voicers"), augmenting the sound of a choir or musical instrument (used in ancient buildings)

increased the reverberation time. By accumulating sound energy within their volume, they produced an effect similar to increasing the volume of the building. It turned out that, if porous absorbing material is placed in its neck, for example, several layers of gauze or other loose material, such a resonator will act contrary to the "golosnik", i.e., it will become a sound absorber.

Instead of vessels of different sizes with absorbing material placed in them, present-day resonance sound absorbers are made by making a great number of small holes in sheets of plywood or metal, and then placing layers of loose cloth or gauze over them. Such a sheet is attached to a cross-ribbed framework, and forms a rigid connection with a wall of the building (Figure 135). In order to obtain sound absorption over a wide frequency range, multi-layer resonance absorbers are used, which consist

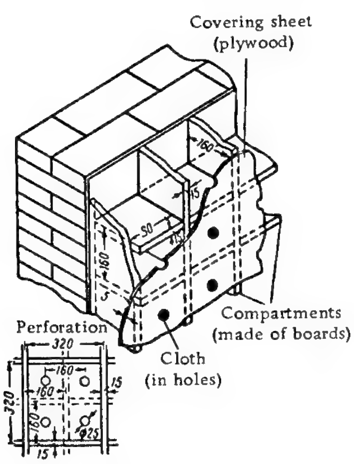


Figure 135. ZP-1 resonance sound-absorbing panel, fastened rigidly to a wall

of several perforated layers with different-sized holes. Such sound absorbers are very simple and very strong; and only a minimum amount of the absorbing material is used. In addition, they make possible the

achievement of absorption coefficients from 0.9 at low frequencies to 0.98 at frequencies of a few kilocycles. Moreover, for these absorbers it is possible to carry out theoretical calculations; this is very important, since the requirements for the frequency characteristics of sound absorption may be different for different interiors.

Resonance sound absorbers developed by Soviet acoustical experts have found several important practical applications in architectural acoustics. Such absorbers, for instance, have been installed in the radio studio of the Sound-Recording House in Moscow.

Sound-Measuring Chambers. For acoustical measurements, rooms are very often necessary whose walls, ceiling, and floor absorb sound totally. Such rooms are known as sound-measuring chambers. In such chambers are performed important measurements of the directivity characteristics of radiators and receivers, of microphone sensitivity, and a great many other problems of architectural and physiological acoustics are solved. A sound-measuring chamber is an acoustical "black body" (by analogy with the term referring to electromagnetic radiation).

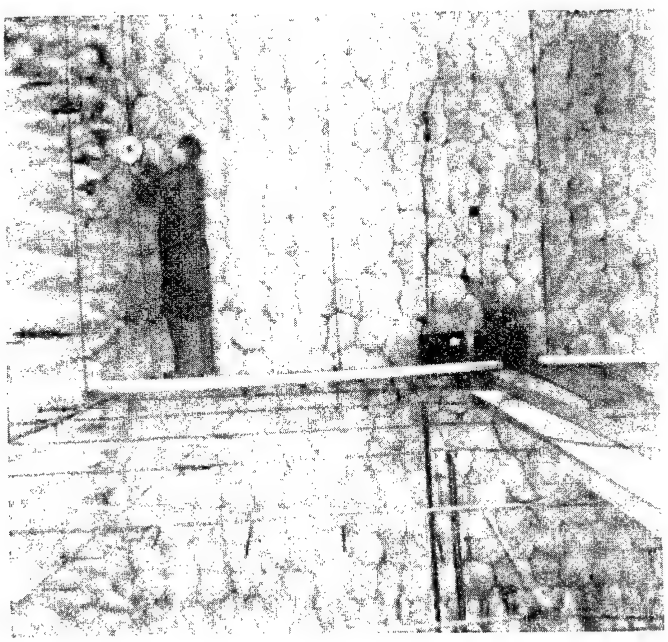


Figure 136. Sound-measuring chamber of the Acoustics Laboratory of Moscow University, with rolls of glass wool

There exist many designs of sound-measuring chambers, and one of them is represented in Figure 136. The walls, floor, and ceiling of such a chamber are covered with special screens, the meshes of which are stuffed with rolls of glass wool (slag wool is also used). The inside of the chamber looks like a "bristling hedgehog". The glass-wool rolls insure

smooth transition of the acoustic resistance ρc from its value for air to its value for the wall, which produces a higher coefficient of sound absorption. Modern sound-measuring chambers are so designed that the sound radiators and microphones may be manipulated from outside the chamber, and all reading taken of the measuring and recording instruments; in this way, the experimenter himself does not disturb the acoustic field of the chamber.

In addition to ensuring optimum reverberation conditions in halls and lecture rooms and obtaining good acoustic conditions in them, architectural acoustics also deals with the soundproofing of buildings from outside noises. The importance of ensuring quietness in hospitals, radio studios, concert halls, libraries, etc., does not require special explanation. Equally important is the softening of working noise in factory shops, of noise in an airplane cabin, and in many other cases. Certainly, the best way of fighting disturbing sounds is to reduce them at the source. However, this is certainly not always feasible. It is not possible to damp completely the noise of an airplane engine, or eliminate external noise coming through the wall from an adjacent room. For this reason it is necessary to adopt special measures of soundproofing buildings. Sounds and noise can penetrate into a building in two ways: through air spaces and cracks in doors, windows, ventilation vents, etc.; and also through the walls and partitions themselves. In order to minimize the amount of outside noise penetrating through the walls, they must be built sufficiently massive. Recently various sound-absorbing materials have come to be widely used for soundproofing buildings from outside noise.

§ 3. Propagation of sound in the atmosphere

Influence of Wind on the Propagation of Sound Beams. In the previous discussion of sound propagation, the air in which it is propagated was considered to be homogeneous and stationary. Such is the case indoors, but in the atmosphere the air is nearly never found in a state of rest. During a year's time, it is only rarely that there is a day of absolutely no wind, a dead calm. Even then, however, there are descending and ascending air currents (convection), due to the uneven heating of the earth's surface. Moreover, even though there may be no wind at the surface of the earth, there is still liable to be wind at some height above it, as may be seen, for instance, by the movement of the clouds. The wind velocity changes frequently during the day and night—in magnitude as well as direction. An average wind velocity usually amounts to a few meters per second, but on windy days it may reach 10 m/sec and more.

Every one of us has certainly had an opportunity to observe that sound traveling with the wind is heard better than sound traveling against the wind. However, it should not be assumed that when a sound wave travels against the wind, the wind interferes with its propagation, or that sound moving with the wind is helped by the wind. The explanation of the phenomenon involves a consideration of sound refraction.

The velocity of sound adds geometrically to that of the wind, and this modifies the shape of the sound-wave front; in the presence of wind, some parts of the wave front will move faster, while others will move slower.

In the vicinity of the earth's surface, movement of the air layers is retarded due to the friction caused by the unevenness and roughness of the surface; hence, the wind velocity usually increases with height. Sound beams, issuing from a point and at an angle to the surface, will therefore pass into regions of constantly increasing wind velocity. If the beams travel with the

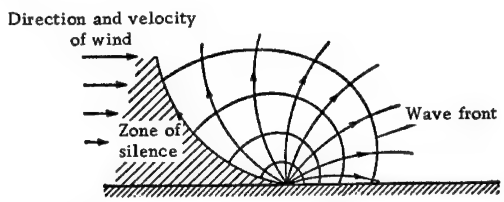


Figure 137. The effect of wind on the propagation of sound beams

wind (Figure 137), then, as a result of refraction, the sound will gradually be bent toward the surface of the earth, in a manner similar to the bending of water waves approaching a shallow shore. In this case the wind appears to bend the sound down toward the earth. On the contrary, sound beams traveling obliquely against the wind will be bent upward, receding even farther away from the surface of the earth. For this reason, in a strong wind a loud sound is often heard at great distances downwind from the source, while upwind it may not be heard even over a short distance.

Influence of Temperature. The propagation of sound in the atmosphere is influenced not only by wind, but also by temperature. Layers of air situated at different heights above the ground have different temperatures; the changes are especially sharp near the surface. The variation in air temperature with altitude may differ in its nature. Usually the average temperature of the air decreases with altitude, approximately 1°C for every 200 m, but quite often there are cases when it decreases more sharply. In the vicinity of the earth's surface the opposite phenomenon often takes place; the temperature increases until a certain altitude, and only then begins dropping. This is called temperature inversion. Such a phenomenon is observed, for example, on frosty nights and also on still summer nights.

It is known that the velocity of sound depends on the temperature. If the temperature rises 1°C, the velocity of sound increases by about 0.5 m/sec. Hence, the presence in the atmosphere of air layers of different temperatures leads to the refraction of sound beams.

If the temperature decreases with altitude, which usually is the case during the daytime, sound beams from a source situated near the ground

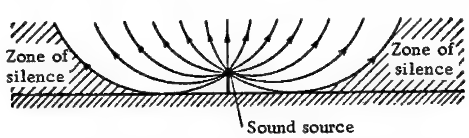


Figure 138. Temperature decreasing with altitude. The diagram represents bending of sound beams from a nondirectional source, situated above the earth's surface

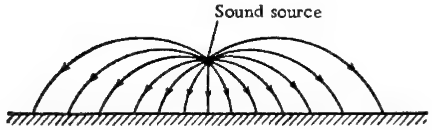


Figure 139. Temperature increasing with altitude. No silent zones

are bent upward (Figure 138). At some distance from the source, the sound is no longer heard. But if the temperature increases with altitude (temperature inversion), the sound beams are bent downward (Figure 139), and the sound reaches more distant points along the surface. This explains the often observed fact that at night sounds are heard over longer distances than in the daytime. At a very pronounced temperature inversion, sound beams undergo strong reflection and return to the surface, are reflected again, and again travel upward (Figure 140). There may be several such

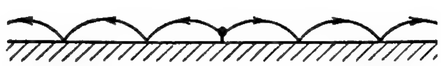


Figure 140. Pronounced temperature inversion: the sound beams return to the earth, are reflected, ascend, and again return to the earth

reflections; in this case, the sound energy will be confined within a certain layer, which serves as a sound channel. The distance of propagation under such conditions is increased greatly. This is especially noticeable on a still night over a river. Due to the smoothness of the water surface, the sound waves are totally reflected from it, and before considerable attenuation of the sound takes place, the number of reflections may become very great. That is why along a river we can sometimes hear, even over several kilometers, sounds of low intensity—singing, a cock crowing, or the creaking of the oar-locks of a rowboat.

If the air temperature changes only slightly with altitude and there is no wind, then a sound from a source spreads without noticeable reflection; for example, on frosty winter days, the whistle of a locomotive may be heard over a distance of several kilometers, and the creaking of a sleigh or the sound of an axe in the forest are heard from far away.

Atmospheric Turbulence and its Effect on Sound Propagation. In the presence of wind, the influence of temperature on the distance over which sound is propagated becomes less noticeable; the air layers are mixed by the wind, and the temperature becomes equalized.

The wind does not constitute air motion at some definite, unchanging speed and in some definite direction. We notice, while watching gusts of wind, that sometimes the wind quiets down, and sometimes it becomes stronger again. This can be observed, for example, in a field of rye or in the treetops, or by the ripples appearing on the smooth surface of a lake. Such air motion is said to be turbulent. This concept will now be defined more exactly and the nature of turbulent motion will be briefly considered.

If water is made to flow slowly through a glass tube, then, if a thin stream of ink is poured into the tube, the ink will move along the tube in a thin, clearly defined filament. This indicates that the water particles

in the tube are moving along definite lines, which are known in hydrodynamics as streamlines. Such a flow of a liquid or gas in streams or layers is called laminar motion (from "lamina", meaning layer).

If the speed of the stream of water in the tube is increased gradually, then, at a certain speed, the nature of the flow will change sharply; all the water in the tube will suddenly become colored, and the streak of ink will no longer be visible. This internal mixing of the liquid can obviously take place only if the particles of water move in directions perpendicular to the direction of flow. This motion of liquids and gases, in which internal mixing takes place, is called turbulent motion; it is extremely widespread in nature. The motion of air in the atmosphere or water in rivers and seas has a distinctly turbulent nature.

Experimental investigation of the motion of liquids and gases in tubes has shown that the transition from laminar to turbulent motion takes place at a quite definite relation between the flow velocity v, the tube diameter d, and the kinematic viscosity ν of the medium; this relation is represented by the number

$$R = \frac{vd}{\nu},$$

known as the Reynolds number (the kinematic viscosity is the ratio of the viscosity of the medium to its density, $\nu = \frac{\eta}{\rho}$). For the flow of water in a tube with smooth walls, turbulence occurs at Reynolds numbers of the order of 10,000 to 20,000.

It turns out that the Reynolds number not only characterizes the motion of liquids and gases in tubes, but it also possesses a more general meaning. In other kinds of liquid and gas flow, instead of the tube radius some quantity representing the geometrical properties of the stream is used. Thus, for an air current above the earth, the characteristic length is a, the height above the earth's surface of the point at which the current is observed. Under atmospheric conditions, due to the low viscosity of air and the large values of a, the Reynolds number becomes very large, and the motion of the air is always turbulent (even for low flow velocities).

If it were possible to trace the motion of a particle in a turbulent stream, it would be seen that this particle, possessing an average motion along the stream, at the same time undergoes very complicated and irregular movements. Moreover, its velocity is not constant in time, but changes in an extremely irregular, haphazard way, varying about some mean value which coincides with the average velocity of the whole stream.

At every point in the atmosphere the velocity of the air particles fluctuates in magnitude and direction about some mean value which is called the wind velocity. Due to the fact that the fluctuations in wind velocity occur in all directions (including the direction perpendicular to that of the mean velocity of the wind), smoke, dust, and other impurities in the atmosphere are rapidly mixed in the wind. For instance, smoke from a factory chimney will often expand in windy weather and turn into a cloud, while on quiet days it produces regular, tall columns of smoke. Figure 141 shows a photograph of the irregular motion of small (of the order of a few mm) particles in a turbulent stream of water, obtained by means of tracking motion-picture photography. The particles consist of a mixture of chlorobenzene, vaseline, and white pigment (for coloring) and have a density approximately equal to that of water.

Figure 141. Photograph of trajectories of small colored particles in a turbulent stream of water, obtained by tracking motion-picture photography (B. A. Fidman)

Let us now turn from the observation of the motion of separate particles of a liquid to a somewhat different study of the nature of turbulent flow. Let us isolate a small elementary volume in the stream of liquid and observe how the flow velocity changes with time. In the first case the observer was considered to move with the particle, but in the present case the observer is stationary, while the particles of liquid pass by. (A more detailed study shows that both methods of regarding the liquid motion are equivalent, the only question being which of them is the more suitable for the actual problem at hand).

Due to irregular, random changes of the velocity of separate particles in the elementary volume, the velocity of motion of the liquid will be constantly fluctuating, both in magnitude and direction, about an average value, which coincides with the velocity and direction of the current as a whole*. The fluctuations, or pulsations, of the velocity can be measured; this measurement will be described below.

We see that turbulent motion is characterized by the presence of velocity fluctuations at every point of the stream. Owing to the random nature and irregularity of this phenomenon, it is not possible to define exactly the velocity or the direction of motion of the liquid, even if at a certain previous moment the velocity and direction were known; for laminar motion such a prediction is possible. It is only possible to speak of a most probable, or average, value of the velocity of the liquid and of the other quantities characterizing the flow (direction of motion, pressure, acceleration, etc.).

This type of statistical study of phenomena has already been encountered in this book during determination of the mean free path of a wave between reflections in a closed space. The probabilistic, or statistical, approach to the study of turbulence made possible a detailed explanation of the nature of the turbulent stream and an intensive study of its inner structure. The mechanism of generation of turbulence has not yet been cleared up completely, but much study has been made of the established state of turbulence; such studies form a new branch of hydrodynamics — the statistical theory of turbulence. Much has been done in this direction by Soviet scientists — A. A. Fridman and L. V. Keller, and recently, by A.N. Kolmogorov and A.B. Obukhov. Interesting experimental study of the internal structure of a turbulent stream has been performed by M. V. Velikanov (on the flow of water in pipes and rivers) and by A. M. Obukhov (on atmospheric turbulence).

* The velocities of fluctuation are, of course, less than the average velocity of the current itself; they can reach a few percent of this average velocity (for example, in the atmosphere they may be from 10 to 15% of the wind velocity).

The internal structure of a turbulent current of air in the atmosphere, according to the proponents of the statistical theory of turbulence (Kolmogorov and Obukhov), may be roughly described in the following way. During the movement of great air masses, due to the unevenness and roughness of the earth's surface there occur considerable fluctuations in velocity, which may be likened to large eddies, drawing their energy of motion from the energy of the air stream as a whole. The usual size (scale) of these very large eddies is of the same order as the stream as a whole (for example, in the atmosphere it may be the distance from the observed point to the earth's surface). These eddies cannot exist as stable formations and decay incessantly into smaller ones. The "degeneration" of turbulence and the transfer of energy from larger eddies to smaller ones continues down to the formation of very small eddies and is finally stopped by the action of viscosity; the energy of the smallest eddies is converted into heat.

The lowest values of the velocity fluctuations measured in the atmosphere are of the order of 1 cm/sec. The stream as a whole does not exert a directing influence on all the eddies, but only on the largest ones; the movement of the small eddies can therefore be regarded as homogeneous and isotropic*.

Thus, for the turbulent motion of the air there is a continuous spectrum of velocity fluctuations (gusts): there are large (in amplitude), slow fluctuations, as well as small, fast fluctuations.

Wind velocity is usually measured with a device called an anemometer. This is a rotator turned by the wind; the number of anemometer revolutions per unit time is proportional to the wind velocity. However, an anemometer cannot record rapid changes of the velocity of the air movement, but only the average wind velocity; also, due to inertia, the instrument only gradually attains its velocity of rotation. For the detection of more rapid fluctuations of the wind velocity another device is employed, which is called a hot-wire anemometer. This is an electrical bridge, one of the arms of which includes a thin platinum wire (a few microns in diameter), heated by current (Figure 142). With the bridge current at a determined level, regulated by the resistor R, the current passing through the indicating instrument (galvanometer) is adjusted to zero by means of the resistor R_2. When the platinum wire is blown upon by a stream of air, it is cooled and its electrical resistance is decreased. The bridge becomes unbalanced, and current flows through the galvanometer. The electrical current will change with the velocity of the air stream. Such an instrument can be calibrated in advance, i.e., every given deflection of the galvanometer correlated to a definite wind velocity. This is usually done in a wind tunnel, which can produce a desired velocity of the air stream. Due to the thinness

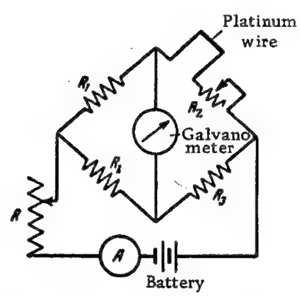


Figure 142. Diagram of a hot-wire anemometer

* As a result, the "microstructure" of a turbulent stream at large Reynolds numbers is subject to definite statistical laws, which do not depend upon the geometry of the flow or upon the properties of the current as a whole.

of the platinum wire, the hot-wire anemometer has very low thermal inertia and is capable of recording, without distortion, very rapid fluctuations in the wind velocity.

Because of the turbulent nature of the motion of the air in the atmosphere, thermal inhomogeneities, occurring in the air due to different rates of heating of the earth's surface, are stirred by the wind. Therefore, the temperature at every point of space also undergoes random, irregular deviations from its average value.

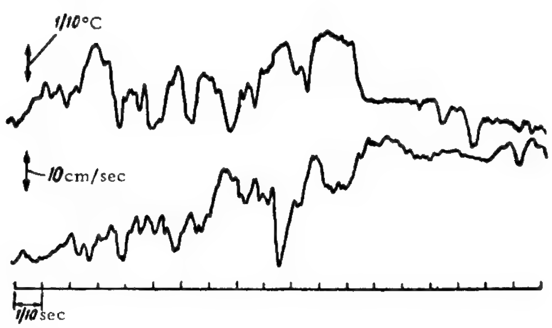


Figure 143. Simultaneous recordings of the fluctuations of the vertical component of the wind velocity (bottom) and of temperature (top)

Average wind velocity 4 m/sec, average temperature 16.5°C, height of observed point 3 m above the earth's surface

Figure 143 shows a recording of the fluctuations of the vertical component of the wind velocity (lower curve) and of the temperature fluctuations (upper curve)*, obtained for an average wind velocity of 4 m/sec and a temperature of 16.5°C at a height 3 m above the earth's surface; the time marks on the recording correspond to intervals of 0.1 sec; the vertical segments marked correspond to a change of 10 cm/sec in wind velocity on the lower curve and to a change of 0.1°C in temperature on the upper curve. The velocity fluctuations were recorded by means of the hot-wire anemometer; the temperature fluctuations, by means of a thermometer capable of registering rapid changes in air temperature (a microthermometer); the latter works on a principle similar to that of the hot-wire anemometer.

It is very important not to make measurements of the wind-velocity and temperature fluctuations just at a single point, but rather to measure the difference of the fluctuations at two points. If measurements of the fluctuations in velocity (or temperature) are taken at a single point, then large inhomogeneities will play a greater part than small ones, while the results of the measurements will be materially dependent upon the period of time during which the measurements are made. This difficulty is eliminated if the difference in velocity at two neighboring points of the stream is measured, i.e., if the relative motion of two elementary volumes of air situated close to each other are followed. This difference is not influenced by the large eddies, the dimensions of which are much larger than the distance between the two points.

* These recordings (as well as the two following ones) were obtained by A.M. Obukhov and S.I. Krechmer.

In studying molecular diffusion it is assumed that the movement of a molecule does not depend upon the other molecules in its immediate vicinity. In turbulent flow, the situation is different. The neighboring elements of the liquid (or air) approach the same velocity as the element under consideration, provided the distance between them is small. If a turbulent flow is regarded as a superposition of eddies (fluctuations) of various scales, the distance between two neighboring elements will at first be altered only through the action of the smallest eddies. The large eddies will simply transport the pair of points (elements) under consideration as a whole, without any tendency to separate them. But as soon as the distance between the elementary volumes of fluid is increased, larger eddies become involved, in addition to the small ones. Therefore, in a turbulent stream of liquid, the motion of a liquid element itself is not as important as the changes in its distance from neighboring elements. A.N. Kolmogorov was the first to propose a mathematical treatment of these general considerations and a description of turbulent flow by means of statistical factors based on mean values not of the velocity at one point, but rather of the difference between the velocities at two points in the stream*.

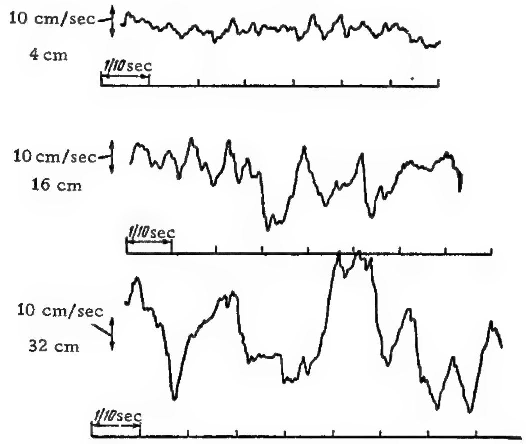


Figure 144. Record of fluctuations in velocity difference

The distances between the anemometer wires are 4, 16, and 32 cm; the mean wind velocity is 5 m/sec; and the temperature is 8.5°C at a height 3 m above the ground. The segments along the ordinate correspond to a velocity difference of 10 cm/sec

* For the mean-square value of the velocity fluctuations at two points 1 and 2 of the stream, Kolmogorov and Obukhov obtained the so-called "2/3 Law":

$$\overline{(v_1 - v_2)^2} = C^2 r_{1,2}^{2/3},$$

where C is a constant (the "turbulence characteristic", of the order of 1 $\mathrm{cm}^{2/3}\cdot\mathrm{sec}^{-1}$ under atmospheric conditions); $r_{1,2}$ is the distance between points 1 and 2; and the vinculum indicates averaging. As demonstrated by Obukhov, a law of the same form holds for temperature fluctuations in a turbulent stream:

$$\overline{(T_1 - T_2)^2} = B^2 r_{1,2}^{2/3},$$

the value of B under atmospheric conditions being of the order of 10^{-1} to 10^{-2} degree $\cdot\,\mathrm{cm}^{-1/3}$.

Figures 144 and 145 represent three recordings of the fluctuations in velocity difference and three recordings of the fluctuations in temperature difference for three separations of measurement points (distances between platinum wires): 4, 16, and 32 cm. The recording of the fluctuations in the velocity difference (Figure 144) was obtained at a height 3 m above the ground, with a mean wind velocity of 5 m/sec and a mean air temperature of 8.5°C. The recording of the fluctuations in the temperature difference (Figure 145) was obtained at the same height, with a mean wind velocity of 0.8 m/sec and a mean temperature of 17°C.

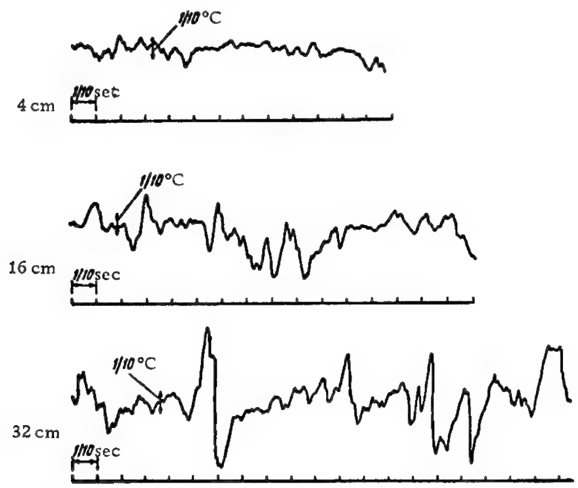


Figure 145. Record of fluctuations in the temperature difference

Distances between the anemometer wires are 4, 16, and 32 cm; the mean wind velocity is 0.8 m/sec; and the mean temperature is 17°C at a height 3 m above the ground.

From these records it is obvious that, as the distance $r_{1,2}$ increases, the difference of the velocity fluctuations grows in amplitude. The statistical treatment of such records, obtained for different distances between wires, is in accordance with the Kolmogorov-Obukhov theory.

Nonuniformities in wind velocity or in atmospheric temperature lead to a series of interesting phenomena when such a turbulent medium is traversed by sound waves. First of all, the turbulent state of the atmosphere leads to high attenuation of sound. We have already mentioned that the attenuation in quiescent, homogeneous air depends upon its viscosity and heat conductivity. It turns out, however, that during its propagation through the atmosphere sound undergoes far greater attenuation than predicted by the corresponding theoretical considerations. There is no doubt that such high attenuation occurs because of the turbulent condition of the atmosphere, but until now there has been no good explanation of the nature of this attenuation. It seems that the greater part occurs due to the scattering of sound on nonuniformities in velocity and in temperature; these are of the order of magnitude of the wave length of the sound. Such a result is obtained from the theoretical discussion of the problem of sound propagation in a turbulent stream of air.

Another phenomenon caused by nonuniformities in the wind and the atmospheric temperature is indicated by the fact that at the point where the receiver is situated the sound intensity does not have a constant value, but fluctuates in time. The sound grows alternately stronger and weaker, as if the sound source were unsteady, like the flickering of distant lights in summer over a water surface or the twinkling of the stars*. In Figure 146

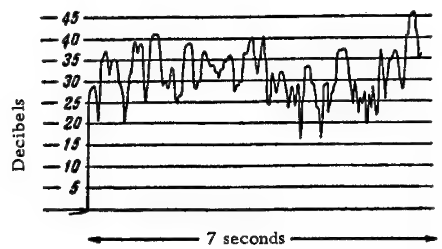


Figure 146. Recording of fluctuations in the intensity of 5000-cycle sound

The distance between the loudspeaker and the microphone is 67 m; the loudspeaker and microphone (suspended from poles) are located 8 m above the ground; the mean wind velocity is 4 m/sec

the curve shows the amplitude of sound with a frequency of 5000 cycles; the wind velocity is 4 m/sec, and the distance between the loudspeaker and the microphone is 67 m.

This phenomenon, known as acoustic fading, has an effect proportional to the wind velocity, the sound frequency, and the distance between the radiator and receiver of sound. Roughly, its explanation is as follows. Due to turbulence in the atmosphere, the entire space between the radiator and receiver of sound is filled with nonuniformities in wind velocity and temperature. If the magnitude of these nonuniformities is comparable with the sound wave length, then the sound waves are scattered in all directions. But if the nonuniformities are much larger than the wave length of the sound, then the sound beams are refracted by the nonuniformities. The wind nonuniformities may be likened to a great number of convex and concave lenses of various sizes, distributed in a random manner; and the sound beam may be compared to an [elastic] tube of given cross section. When the "tube" encounters a concave "lens" in its path, it expands; when it encounters a convex "lens", it contracts. The tube cross section fluctuates. As a result, the amount of sound energy passing through a unit cross-sectional area normal to the direction of the beam becomes greater or smaller, causing fluctuations in the sound intensity at the point where the receiver is located.

The same phenomenon is observed for sound propagation through a thermally nonhomogeneous medium.

* The explanation of these phenomena has much in common with that of the fluctuation of the intensity of sound propagating through the atmosphere; it is also caused by atmospheric turbulence, which leads to fluctuations in the air density, and consequently to changes in the refractive index of light waves. Recently it has also been established that atmospheric turbulence causes a whole series of interesting phenomena accompanying the propagation of radio waves in the centimeter range (scattering, fluctuations in signal level, and fluctuations of the propagation velocity of radio waves).

Not only the intensity, but also the velocity of sound undergoes fluctuations during sound propagation through a turbulent atmosphere. Due to the presence in the atmosphere of nonuniformities of wind and temperature, the sound velocity, which depends both upon wind speed (and direction) and air temperature, will be changed somewhat. Since the changes in wind velocity and temperature are irregular, the sound velocity will experience irregular changes, or fluctuations, of the same kind.

Let us suppose that, in a turbulent current of air (the atmosphere in the presence of wind), a loudspeaker and a microphone are separated by a distance D. If this distance is equal to exactly N wave lengths (for the given sound frequency), then the phase difference between the electrical potential driving the loudspeaker and the output voltage of the microphone amplifier will be zero. Because of irregular fluctuations in the wind velocity, the sound velocity will also change; and, accordingly, the number N of waves (λ cm each) which could be placed in the distance D will also vary somewhat. As we know, this will lead to a change in the phase difference between the loudspeaker-supply voltage and the microphone-amplifier output voltage. The fluctuations in phase difference caused by the internal structure of the wind can be recorded by means of a recording phasemeter and a loop oscillograph, in the same way as in sound-velocity measurements (see Chapter IV, § 2). Figure 147 shows a sample recording of the fluctuations in phase difference between a loudspeaker and microphone separated by a distance of 67 m (both being suspended from poles 8 m high), with a wind velocity of 5 m/sec and a wind direction perpendicular to the direction of sound propagation; the sound frequency is 3000 cycles. Above the record of the fluctuations in phase difference, time markings are shown; every spike pulse is 2/3 sec from the next one.

Figure 147. Record of fluctuations in the phase difference between a loudspeaker and microphone separated by a distance of 67 m (height above ground = 8 m).

Sound frequency, 3000 cycles; mean wind velocity, 5 m/sec; wind direction, perpendicular to the direction of sound propagation

Measurements show that the higher the wind velocity and sound frequency, the greater are the changes in phase difference and the fluctuations in the sound velocity c.

With such experiments as a basis, using an acoustic interferometer, it is possible not only to determine the principal properties of the propagation of sound waves through a nonhomogeneous, moving medium (such as the

atmosphere), but also to make a series of interesting inferences concerning the internal structure of the flow*.

Scattering of Sound by Atmospheric Turbulence. The nonuniformities in refractive index (fluctuations in wind velocity and temperature) caused by turbulence must lead to scattering of the sound encountering them. The two phenomena discussed above—the fluctuations in the phase and amplitude of the sound—can be treated as evidence of sound scattering.

The scattering of light is a familiar phenomenon. We observe it visually every day, and in fact we see surrounding objects almost exclusively due to scattering and reflection. On a cloudy day, when the sun is covered, all objects are lighted only by scattered light.

Sound scattering cannot be seen, and it is impossible to hear sound scattered by such a "turbid" medium as a turbulent air current. It is only by means of very ingenious and delicate experiments that the scattered sound can be detected. The problem is that the nonuniformities in refractive index in the atmosphere are very small. The relative change in sound velocity $\frac{\Delta c}{c}$ amounts approximately to a few parts per thousand or ten thousand. Hence, these nonuniformities do not represent solid bodies, but very weak nonuniformities in refractive index, which lead to very weak sound scattering. One possible method of detecting sound scattering due to turbulent nonuniformities will now be described.

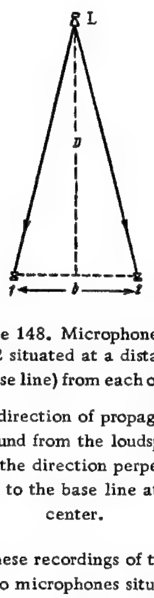


Figure 148. Microphones 1 and 2 situated at a distance b (base line) from each other

The direction of propagation of sound from the loudspeaker L is the direction perpendicular to the base line at its center.

Consider a sound radiator located on the ground, which has a sharp directivity characteristic and which sends sound upward at a certain angle to the surface (Figure 149). In order to obtain sufficient directivity and to create a sound "searchlight", the dimensions of the radiator, as we know, must be considerably larger than the wave length of the sound. In order to keep these dimensions from becoming too large, a higher sound frequency must be used.

Let us also consider a receiver similar to the radiator, and located at a distance (30-40 m) from the latter. The sending and receiving directions of the radiator and receiver respectively will then

* These recordings of the fluctuations in phase difference between a loudspeaker and microphone (or between two microphones situated at a distance—the base line—from each other, this base line being perpendicular to the direction of sound propagation; see Figure 148) for various wind velocities and temperatures, and for various sound frequencies and loudspeaker-to-microphone distances, can be treated statistically. As a result of such treatment, the mean-square value of the phase difference may be obtained.

The "2/3 law" for turbulent flow may be used, for example, to obtain theoretically the mean-square value $\overline{(\varphi_2-\varphi_1)^2}$ of the phase difference for two microphones (Figure 148):

$$\overline{(\varphi_2-\varphi_1)^2} = \mathrm{const}\left(\frac{f}{2\pi c^2}\right)^2 C^2 D b^{5/3},$$

where 'const' is a constant of the order of unity, f is the sound frequency, C is the turbulence characteristic, D is the distance between the loudspeaker and the base line center, and b is the distance between the microphones.

This formula agrees sufficiently well with experimental data. On the other hand, from the formula the turbulence characteristic C may be obtained, the order of magnitude of which is in good agreement with the value for C determined by means of the hot-wire anemometer.

intersect, the intersection region having a certain volume V. Just as light from automobile headlights is scattered in all directions by fog droplets, so also sound which passes through a "turbid" atmosphere must be scattered in every direction, including that of the receiver. The sound

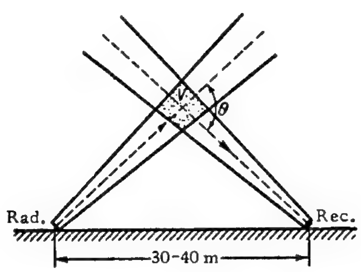


Figure 149. Diagram for detection of sound scattering

approaches the receiver at an angle θ [with respect to the original direction], known as the scattering angle. If the radiator and receiver did not exhibit directivity, then direct sound would reach the receiver, and with this intense sound as a background it would have been impossible to isolate the weak scattered sound. But, even with the sharp directivity characteristic shown schematically in Figure 149, a direct radiator-receiver signal, though a very weak one, will still reach the receiver due to the diffractive properties of the directional pattern (see Figure 70). Hence, it is clear that in order to detect scattering, provided there is any, sound pulses should be used, rather than a continuous signal. In this case, the arrival at the receiver of the direct signal causes no problems, because it is always possible to separate the moment of arrival of this signal and that of the sound scattered by the volume V.

Figure 150. Electrostatic radiator for experiments on sound scattering

Such experiments on the direct detection of sound scattering in a turbulent atmosphere have been performed, and scattering was actually observed*. The radiators and receivers, which were identical, operated on the electrostatic principle (the same as in the condenser microphone, see Figure 47). The radiation was in the form of pulses with a carrier frequency of 11.0 kc. Figure 150 shows a photograph of the electrostatic radiator employed; the sides of the rectangular membrane are 80×90 cm, and the angle of opening of the central lobe of the directivity diagram was 1.5 to 2°. Figure 151 shows oscillograms of the sound pulses received. In the first photograph one pulse is seen, corresponding to the direct signal

* Sound scattering by nonuniformities of the refractive index, due to turbulence, was predicted and discussed theoretically by Obukhov (1941). The experiments described above were performed at the Institute for Atmospheric Physics of the Academy of Sciences U.S.S.R. by M. A. Kalistratova (1958).

(the beginning of scanning does not correspond to the first pulse, a certain time delay having been introduced). In the second photograph a second pulse is observed, which in the first photograph coincided with the direct signal. In this case, the angle θ was changed, and therefore the volume V changed its position. In the third photograph, separation of the pulses is still more noticeable.

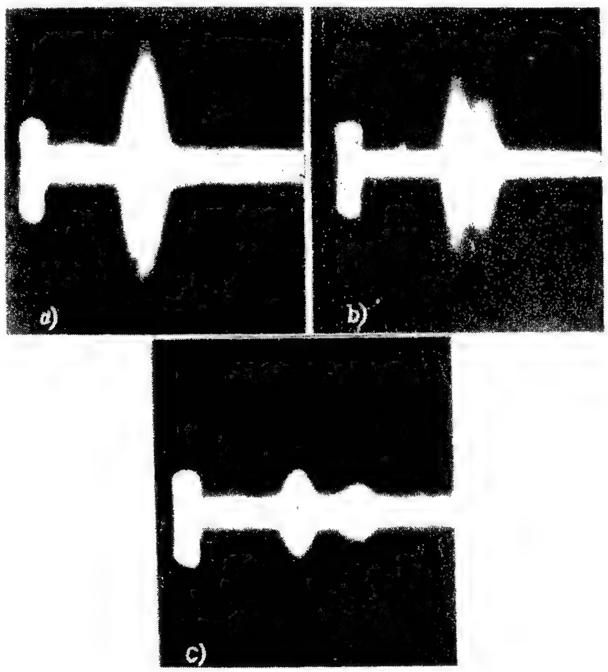


Figure 151. a) $\theta = 20°$, the pulse is received directly from the radiator, b) $\theta = 25°$, a pulse has appeared which is due to scattering, c) $\theta = 40°$, the pulse coming directly from the radiator is separated from the scattered sound

The scattered pulse arriving at the receiver is in itself very weak, and great amplification is required to detect it (high sensitivity of the whole equipment). In the observation of scattering its nature and intensity were found to depend upon meteorological conditions; this is to be expected, since the same conditions determine the intensity of turbulence. In a turbulent atmosphere, as we have already noted, there are various scales of nonuniformity of the refractive index. However, for scattering which takes place at a certain angle , not all scales of nonuniformity are possible, but only those satisfying specific conditions. The whole spectrum of nonuniformities in a volume V can be represented as a set of interlacing three-dimensional lattices; to every scale l there corresponds a three-dimensional lattice. Only those lattices the distance between the "lines" of which is equal to l and which will satisfy the so-called Bragg condition will

participate in the scattering of the sound reaching the receiver. The Bragg condition will be necessary later, so that here it is appropriate to discuss it briefly.

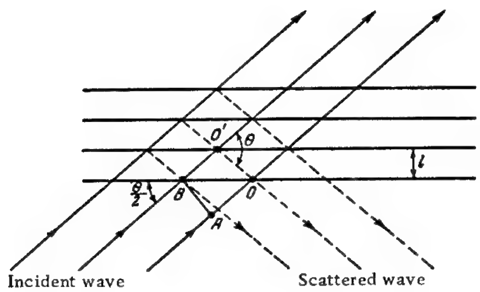


Figure 152. The Bragg condition for scattering

Consider a series of parallel layers (which may have different refractive index), at an equal distance l from each other (in the case under discussion of scattering by nonuniformities, this represents the scale of the nonuniformity). Let us further assume that a plane wave is incident upon the layers at an angle $\frac{\theta}{2}$, where θ is the scattering angle (Figure 152). This wave will be partially reflected from the first layer; the part which is transmitted is partially reflected from the second layer, then from the third layer, and so on. The wave reflected from this series of layers will be built up as a result of the interference of waves reflected from each of the layers. Let us neglect the attenuation of the incident wave in passing through these layers. Now it becomes clear that mutual reinforcement of these beams will take place at such angles for which the difference in path length between the beams BO' + OO' and AO represents a whole multiple of the wave length λ; it is not difficult to show that this difference is $2\sin\frac{\theta}{2}$. The same will hold for all other beams striking the layers at an angle $\frac{\theta}{2}$. Hence, the general condition for reinforcement of the vibrations will be the condition known as the Bragg condition*:

$$2l\sin\frac{\theta}{2}=m\lambda \qquad (m=1,\ 2,\ldots).$$

If the volume occupied by the layers has a refractive index n, differing from the refractive index of the surrounding medium, then the Bragg condition may be written:

$$2l\,n\sin\frac{\theta}{2}=m\lambda \qquad (m=1,\ 2,\ldots).$$

Thus, the sound scattering taking place in the volume V at a scattering angle θ is caused only by nonuniformities of scale l, the Bragg condition being fulfilled. Studies of scattering at different angles and at different frequencies, if this circumstance is kept in mind, represent an acoustical method of spectral analysis of the microstructure of a turbulent stream. By means of studies of sound scattering, in view of the Bragg condition,

* The Bragg condition was introduced by William H. Bragg; in particular, it plays an important part in the determination, by means of X-ray analysis, of the distance l between the reflecting crystal planes, where λ and $\frac{\theta}{2}$ are known.

the discrimination of the corresponding nonuniformities is possible, in the same way as a filter with variable sharp tuning is a spectral device discriminating discrete frequencies in a complex spectrum. In particular, choosing high "sound" frequencies of the order of 20-30 kc and corresponding scattering angles, it is possible, by observing the sound scattering, to evaluate very minute nonuniformities, which have, as already mentioned, dimensions of the order of 1 cm in a turbulent current of air.

The experiment described above, as already noted, serves as a direct proof of sound scattering by nonuniformities of the refractive index in a turbulent atmosphere.

A similar phenomenon related to the propagation of radio waves was discovered at the end of the forties. It was found that, using ultrashort waves (meter and centimeter bands) which follow rectilinear paths only, reception of a signal was possible far beyond the limits of direct visibility. Such reception has no connection with the formation of refractive-index layers for radio waves, which might serve as channels or waveguides leading to "superdistant" propagation of the radio waves. It was assumed later, the assumption being to a large degree based on both theory and experiment, that such signal reception beyond the radio horizon becomes possible due to radio-wave scattering in the intersection region of the directivity diagrams of the transmitter and receiver. This scattering, just as sound scattering, is caused by nonuniformities in the refractive index for radio waves. However, as distinguished from sound (where fluctuations in the refractive index are due to fluctuations in velocity and temperature), these nonuniformities, also caused by atmospheric turbulence, consist of fluctuations in temperature and humidity. Temperature and humidity may be regarded as passive factors, distributed about by the field of velocity fluctuations of the turbulent stream. In themselves, the relative deviations of the refractive index from the mean value are extremely small; under normal atmospheric conditions, they are just a few units per 10^6; nevertheless, they are sufficient for the scattered signal to be detected far beyond the horizon, with a sufficiently powerful radio transmitter and a sufficiently sensitive radio receiver. Such radio-wave scattering (known as tropospheric scattering) makes it possible to bring about radio communication (although not always stable) over distances of hundreds of kilometers. A similar scattering of radio waves by means of nonuniformities of the refractive index in the ionosphere (known as ionospheric scattering), due to the position of the [scattering] volume V being at a greater height above the ground, makes it possible to bring about radio communication over distances greater than 1000 km. The importance of these scattering phenomena is obvious; they may make it possible to transmit television programs and radio communication with ultrashort waves far beyond the limits of direct visibility.

Although the majority of experts on radio-wave propagation believe that the propagation of meter and centimeter waves beyond the radio horizon in the absence of waveguide properties of the atmosphere may be explained by scattering of these waves by nonuniformities of the refractive index under turbulent conditions, nevertheless it should be mentioned that all these problems require further study.

§ 4. Sound locators. Sound ranging

Sound Locators. Knowledge of the laws of sound propagation in the atmosphere is very important for the solution of a whole series of practical problems. Among such problems, for example, is the planning of acoustics for large areas (open-air theaters, squares, stadiums, etc.), signaling in a fog, and also sound location. Recent successes in radio engineering have led to a vigorous development of radio methods of location, known as radio location. During the First World War, in the absence of radio location, great importance was attached to sonic locating devices. We shall mention these briefly, since several modern submarine acoustical devices work on a similar principle.

When a sound reaches us, it is possible to determine the direction of its source; the accuracy of the determination of direction is about 4°. This property of hearing depends on the so-called binaural (two-ear) effect. A man who has lost the power of hearing in one ear is almost incapable of finding the direction of a sound source without turning his head.

The binaural effect is explained by the fact that one ear is nearer to the source than the other, and so the sound reaches it earlier than it does the other ear. The brain center is able to take into account the difference in time of the effect of the sound on each ear, and this difference in arrival time is transformed by the brain into a sensation of direction. Besides, the intensity of sound is greater for the ear which is nearer to the sound source than it is for the other ear; the head plays the part of an obstacle to the sound waves, and screens off one of the ears (however, this holds true only for high frequencies; for low frequencies the sound is diffracted around the head, and there is no noticeable difference in the sound intensity). This circumstance increases the ability to determine the direction of the sound source. It is obvious that the difference in the arrival times and therefore the accuracy of the determination of the direction, will be greater as the distance between the ears is greater. For a man, this distance is about 18 cm. By increasing the base line between two sound receivers, it is possible to increase the accuracy of the determination of direction, and this is made use of in sound locators working on the principle of the binaural effect. Sound locators make it possible to determine the direction of a flying airplane with an accuracy of about one degree. However, they are not capable of detecting a plane by the sound of its propeller and engine over distances greater than 10-15 km; and this distance is far too short, considering the modern speeds of aircraft. Besides, the wind in the atmosphere and the atmospheric turbulence cause large errors and general unsatisfactory functioning of sound locators under poor meteorological conditions.

Sound Ranging. With very loud sounds, for instance powerful explosions, a very interesting phenomenon is observed.

Sound traveling along the ground is greatly absorbed and scattered due to unevenness of the surface, and also to inhomogeneities in temperature and wind velocity. For this reason, the sound of a powerful explosion can be heard only to a distance not exceeding 20-30 km. However, at greater distances the sound again becomes audible.

This phenomenon is explained by the fact that an altitude of 50-70 km there are layers of atmospheric ozone, with a temperature of + 50 to + 70°C.

The sound velocity here is greater than in the lower layers, and sound traveling at an angle to the surface of the earth is gradually bent, describing an arc and again returning to the ground (Figure 153). This explains the fact that beyond the silent zone, at a distance of 150-200 km and more, it is again possible to hear the sound of a powerful explosion. There may sometimes be more than two zones of audibility, because the sound beams which come from above are again reflected by the ground, go upward, and again return to the ground after a similar course of travel. In Figure 154, the map shows the silent zone and the annular zone of anomalous audibility for a powerful explosion in Moscow on 9 May 1920.

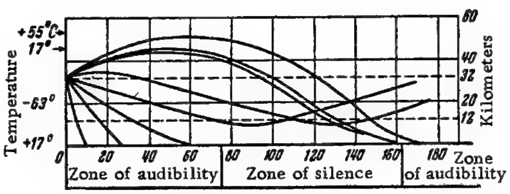


Figure 153. Path of sound beams in the atmosphere. The abscissas represent distances in km

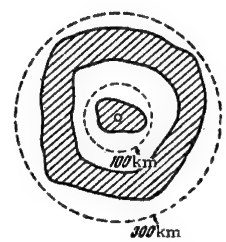


Figure 154. Audibility zones (hatched) and silent zones after a powerful explosion in Moscow on 9 May 1920

As a result of the catastrophic eruption of the volcano Krakatau (near the island of Sumatra) on 27 August 1883, which was accompanied by a very powerful earthquake, there arose in the atmosphere pressure waves of such force (infrasonic waves with a period of fractions of a minute or more) that they are considered to have gone around the earth several times.

For nuclear explosions, the resulting shock waves (for shock waves, see below, Chapter VIII, §6) also travel great distances, being transformed at some distance from the source into powerful infrasonic waves, owing to damping effects. Such waves can easily be recorded by infrasound receivers; and in this way, by means of these waves, a nuclear explosion can be detected which has taken place in the air, at a great distance from the observation point. Several methods are known for detecting nuclear explosions taking place at great distances, depending on where the explosion has taken place—in the air, under water or under ground. Acoustical methods of detecting nuclear explosions are quite reliable, especially when the explosion takes place in the air or in the water. In the case of an underground explosion, seismic methods (see Chapter X) also make it possible to determine reliably the location of the explosion.

Since low-frequency sounds are propagated in the atmosphere over comparatively great distances, it is possible to take the bearings of artillery guns. Such sound-intelligence service is called sound ranging. By the sound of the firing, it is possible to locate a gun with sufficient accuracy to shell it and hit it.

When an artillery piece is fired, a powerful sound pulse is generated in the form of a spherical wave, which bears the special name of a muzzle

wave (see Figure 155). The center of this spherical pulse is located a bit ahead of the muzzle of the gun. Near the gun, the velocity of the muzzle wave is somewhat greater than the velocity of sound; but after a few tens of meters this velocity is already the same as the normal velocity of sound.

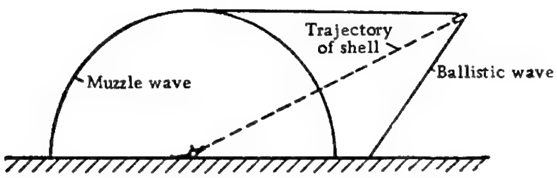


Figure 155. Muzzle and ballistic waves formed when a gun is fired

In addition to the muzzle wave, a gunshot is accompanied by still another wave, called the ballistic (or shock) wave, caused by the motion of the shell (Figure 155). Since a shell (or rifle bullet) travels at 1000 or more meters per second, which is a supersonic velocity, it is always ahead of the waves which it causes; for this reason, the latter are propagated within a certain conical envelope (Figure 156). The greater the ratio of projectile velocity to sound velocity, the smaller the angle of the cone. The ballistic wave is propagated in the direction normal to the cone generatrix.

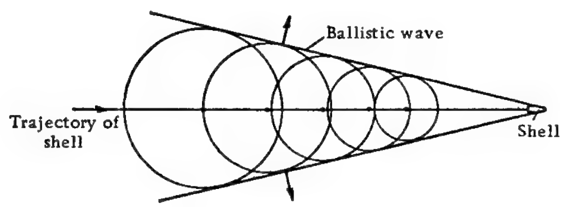


Figure 156. Formation of ballistic (shock) wave during the motion of a projectile

A picture which is superficially similar to the formation of a ballistic wave during the motion of a projectile can be observed during the motion of a ship. From the bow and the stern of the ship, waves spread out, which are known as bow and stern waves. However, while the angle of opening of these waves is independent of the speed of the ship, the cone angle of the ballistic wave depends upon the velocity of motion of the projectile.

And so, for a gunshot there are two sound phenomena—the formation of a muzzle wave and a ballistic wave. When a projectile travels past an observer, a strong, sharp clap is heard; the sound reaches the observer from the direction normal to the cone of the passing ballistic wave, where at that moment a real source of sound is no longer to be found. By the way, it should be noted that, beside the ballistic wave, a projectile in flight generates a characteristic whistling, shrieking, or whizzing, usually caused by rotation of the projectile about its axis and by the unevenness of the projectile surface.

How, then, is it possible to make use of the sound of a gunshot in order to locate a firing gun? Let us consider two sound receivers, situated at some distance (the base line) from each other. This base line is taken to be rather large (1-2 km). If the direction of the gun coincides with the perpendicular drawn to the midpoint of the base line (Figure 157), then it is obvious that the sound of the shot will reach both receivers 1 and 2 simultaneously.

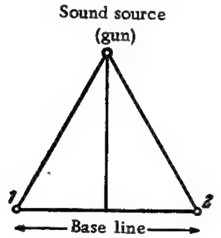


Figure 157. For a base line between the receivers perpendicular to the direction of propagation of the wave, the sound reaches both receivers 1 and 2 simultaneously

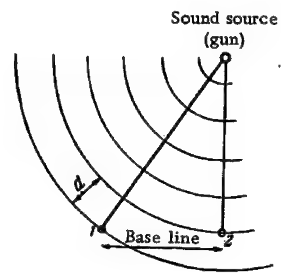


Figure 158. Sound reaches receiver 2 before receiver 1

d is the difference in the paths traveled by the beams reaching the second and first receivers

We can also arrive at the converse conclusion. If the sound reaches both receivers 1 and 2 simultaneously, it means that the gun lies on the perpendicular drawn to the midpoint of the base line.

In case the gun is not situated on this perpendicular, the sound will always reach one receiver earlier than the other; the difference in the arrival time depends upon the difference in the path length of the sound (Figure 158). The latter depends upon the length of the base line and the angle between the direction of the latter and the direction of the sound source. According to Figure 158, the difference in arrival time will be

$$t=\frac{d}{c},$$

and hence the difference in the paths traveled by the beams reaching the two receivers is:

$$d=t\cdot c.$$

If t is given, then in this case the direction of the sound source may be determined. For this purpose, it is only necessary to draw from point 1 a circle with a radius equal to the path difference d (Figure 159). Then the normal to the tangent drawn to the circle from point 2 will represent the desired direction. In this way, by means of two receivers placed on a base line, the direction of a sound source may be determined. In order to determine not only the direction, but also the position of the source, two groups of receivers must be used, each of which will determine by this method its own direction with respect to the sound source. The intersection of these directions will be the point where the firing gun is located. It is possible,

though, to use only three receivers, instead of four (Figure 160), if one of the three lies on both base lines.

Instead of such geometrical constructions, special tables, prepared beforehand, may be used, making it possible to locate the firing gun with sufficient speed. In these tables, corrections are introduced for meteorological conditions (temperature, wind velocity, etc.). The basic initial data for the difference in the arrival time of the sound at the different receivers are determined by a special installation, known as the sound-ranging station, consisting of sound receivers and sound-recording equipment. The receivers have their maximum sensitivity in the infrasonic range, for frequencies of a few cycles, which makes it possible to record only the muzzle wave, and to neglect the interfering ballistic wave; for it is only the muzzle wave that gives data for the location of the gun. This is possible since, in the spectrum of the muzzle wave, infrasonic frequencies (below 16 cycles) are very strong, while in the ballistic wave they are much weaker, sometimes being entirely absent. For greater sensitivity, the receivers usually employ carbon microphones. From these microphones, reacting only to infrasonic frequencies, the signals are transmitted to the observation point, where the recording equipment is situated. The tape-driving mechanism of the latter moves a paper tape at a strictly fixed speed. In order to ensure exact knowledge of the speed, time markers are made on the tape by means of an electrically excited tuning fork with a known period of vibration. The sound vibrations are recorded by means of a thin capillary tube, with one end in an ink reservoir and the other pressed lightly against the tape. The tube is attached to a coil placed in a magnetic field. In the absence of current, the tube is stationary, tracing a straight line on the tape. When the sound of firing reaches the microphone, current appears in the coil circuit, moving the coil, and also the glass tube. The latter traces vibrations on the tape, which characterize the arriving sound of a gunshot. Each tube, or pen, together with its coil, is connected to its respective microphone, and the paper tape records simultaneously the sounds of gunshots arriving at the different receivers.

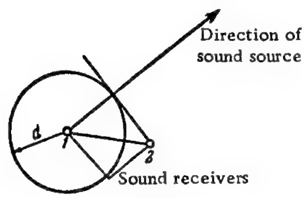


Figure 159. Determination of the direction of a sound source

From point 1 a circle is drawn, the radius of which is equal to the path difference d; the direction of the gun will coincide with the normal to the tangent drawn to the circle from point 2.

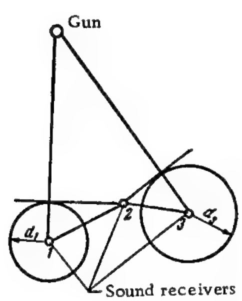


Figure 160. The sound source is located at the intersection of the two straight lines drawn from points 1 and 3, each of which is perpendicular to the tangents to circles with radii equal to the respective differences in path length

Figure 161 gives a sample recording of the firing of a 27 cm gun at a receiver situated 4 km from the gun. From this recording it is evident that first the receiver is reached by the ballistic wave, the amplitude of which is not large; and then the muzzle wave arrives.

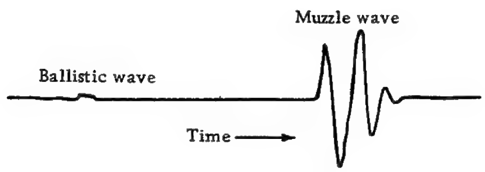


Figure. 161. A recording of the firing of a 27 cm gun, situated 4000 m from the receiver

From the nature of the recording, an experienced sound ranger is not only able to locate exactly the firing gun, but also to determine the type of gun (howitzer, cannon, mortar, etc.). However, when several guns are firing at different intervals, it is very difficult, sometimes quite impossible, to decipher the recordings of a sound-ranging station. This is one of the disadvantages of sound-ranging. Considerable error is also introduced by the changing conditions of sound propagation in the atmosphere, especially the refraction of sound beams.

§ 5. Aerothermoacoustics

In this section an important branch of modern acoustics will be discussed briefly, a branch which has gained special importance recently in connection with the rapid development of supersonic aviation and jet-propulsion engineering. It is called *aerothermoacoustics*, although the term has not yet become well established. Aerothermoacoustics is the branch of acoustics studying noises and sounds of aerodynamic origin, and also the interaction of sound and turbulence. In many cases thermal phenomena may play an essential, if not a determining, role. In order to acquire an idea of the scope of aerothermoacoustics, some of its tasks and problems are listed below.

Some aerothermoacoustic problems are: sound generation during flow around solid bodies (aerodynamic, or vortical, sound); generation of sound (noise) during turbulent conditions, in particular by the turbulent jets leaving the exhaust nozzle of a jet engine, including cases with a Mach number $M = \frac{v}{c}$ exceeding unity; radiation of sound by a pliant solid wall in a turbulent stream; damping of all these noises; sound (noise) generation as a result of turbulent conditions, taking into account thermal phenomena (this especially refers to jets); autovibrations in combustion chambers of jet engines, leading to instability in the functioning of such engines; sound propagation in a turbulent stream (mentioned above); and so on.

We will limit ourselves to only some of the problems of this vast field of diverse phenomena, being mostly interested in problems of sound generation.

Aerodynamic Sound. The formation of elastic waves results not only from an explosion or shot, but also from the motion of bodies at supersonic speeds (see Chapter VIII). However, elastic waves may also be produced by bodies traveling at subsonic speeds, or (which is the same thing) in cases when the velocity of flow around a body is less than the speed of sound.

Generation of sound in such cases is a very common phenomenon, to be encountered at every turn. Such phenomena include: the sounding of wires and strings when the wind blows upon them; the whistling sound as air goes around corners; the singing of a ship's rigging or airplane struts and so on. In all these cases we are dealing with a e r o d y n a m i c n o i s e, generated by the eddies breaking away from a body in a flow. The sound is generated not because the body (wire, strut, rope, etc.) vibrates, although there is some sound radiated for this reason too, but because vortexes in the medium break away from the body. For this reason, the sound generated in such cases is itself called vortical sound (or aerodynamic sound).

It is known that vortexes break away from bodies which are placed in a stream of liquid; this phenomenon is especially noticeable in water flowing around the piers of a bridge. Behind the piers, eddies may be seen to detach in turn to the left and to the right of the piers, forming a row of eddies, known as the Kármán vortex street (Figure 162). Every eddy which leaves the surface of a body creates a certain pressure pulse and thus becomes a sound source. The vortexes, which are periodically breaking away from the surface of a body in a flow, lead to the generation of aerodynamic sound, the fundamental frequency of which coincides with the frequency of breaking-away of vortexes. The frequency f of the aerodynamic sound is determined by the formula:

$$f = A(\mathrm{R})\frac{u}{d} = A\left(\frac{ud}{\nu}\right)\frac{u}{d},$$

where $A(\mathrm{R})$ is a numerical coefficient, depending on the Reynolds number R (see § 3 of this chapter); u is the flow velocity; and d is a characteristic dimension of the body. For a sphere or cylinder, d is the diameter.

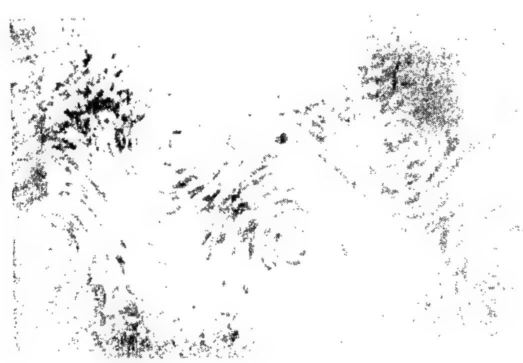


Figure 162. Kármán vortex street

For flow around a long cylinder situated at right angles to the stream, and with Reynolds numbers from $1 \cdot 10^3$ to $3 \cdot 10^4$, the quantity $A(\mathrm{R})$, according to measurements, has a constant value of 0.2. Hence, for the cylinder the frequency of the aerodynamic noise is

$$f = 0.2\frac{u}{d}.$$

This formula was first obtained by Strouhal in studying the sounding of a string in a stream of air (the "Aeolian harp"), and the number 0.2 (0.185 according to Strouhal) is known as Strouhal's coefficient.

As shown experimentally, the intensity of aerodynamic sound depends to a very great degree upon the flow velocity u, being, in fact, proportional to the sixth power of u.

While an airplane propeller rotates, aerodynamic noise is also generated; as the blades cut through the air, vortexes break away from them. The frequency of the aerodynamic noise changes from the center of the propeller to its ends, because of the changing geometrical configuration of the blades. The aerodynamic noise of the propeller is heard mainly near the rotating blades, and because the frequency of this sound is comparatively high, it is rapidly absorbed by the air. It should be noted that, beside aerodynamic noise, the propeller also generates rotation sound*. The frequencies of the latter are much lower than those of vortical sound.

There is one factor which is very important in the functioning of a sound receiver placed on a body moving in a stream. Vortexes periodically break off the body, and they not only lead to the appearance of aerodynamic sound, but also create acoustic disturbances, which interfere with the operation of the sound receiver. These acoustic disturbances are especially great in the case when the eddies break away from the surface of the receiver membrane itself; in such a case the breaking away of an eddy causes the membrane to vibrate at its resonant frequency. In order to eliminate the eddy disturbance, it is necessary to install the sound receivers on well-streamlined profiles, where vortex generation is less pronounced, or to employ anti-wind protection. The simplest such protection consists in wrapping the microphone in a ball of gauze.

The "Voice of the Sea". A very interesting phenomenon was discovered by V.V. Shuleikin** and subsequently explained by N.N. Andreev. It consists in the generation of infrasonic waves ("the voice of the sea") when the undulating surface of the sea is swept by a current of air. Andreev suggested that "the voice of the sea" is generated over the waves due to the breaking away of vortexes from the wave crests; from this point of view, the infrasonic waves generated are nothing else than aerodynamic sound. Consider the wave to be a long cylinder with a characteristic dimension d (its height) which is 100 cm, and a wind velocity $u \approx 3 \cdot 10^3$ cm/sec; then, in this case, according to the formula given above, the frequency of the aerodynamic sound is $f = 6$ cycles. This value for the frequency is in good agreement with experimental data.

Infrasonic waves, propagated at the velocity of sound, and subject to only very slight absorption in the atmosphere owing to their low frequencies, move far ahead of the sea waves and spread over long distances. When the sea near land is still and there is no wind over the shore, "the voice of the sea" originating in a distant storm may reach the shore and may be detected by suitable instruments. It is interesting to learn that many sea creatures, for instance, sea fleas, leave the pebbled shore for drier land, while jelly-fish leave the shallows for the deeper sea, so as not to be ground

* Every rotating body (including a propeller) causes periodic changes in the pressure near the rotation plane. These periodic pressure changes (compressions and rarefactions) are propagated as sound waves; such sound is called rotation sound.

** The reader may find more particulars about "the voice of the sea" in: Shuleikin, V.V. Ocherki po fizike morya (Essays on the Physics of the Sea). -Izdatel'stvo Akademii Nauk SSSR, 1949.

into pulp by pebbles near the shore. Possibly nature has enabled these creatures to sense the coming storm; apparently, their bodies are capable of reacting somehow to the warning "voice of the sea". There may exist a possibility of utilizing the "voice of the sea", detected by special infrasound receivers, for storm warning.

It should be mentioned that, besides such aerodynamic sound as the "voice of the sea", the undulating surfaces of seas and oceans serve as gigantic generators of infrasonic waves of low frequency (fractions of a cycle per sec). The resulting radiation is in principle similar to that from any oscillating surface. Recently geophysicists have been showing interest in such infrasound, because it may play some part in processes taking place in the terrestrial atmosphere.

Pseudosound. Let us assume a sound receiver to be placed in an unsteady stream of air. It may be, for instance, the wind, which represents air motion at an average velocity, on which velocity fluctuations are superposed—fluctuations both in magnitude and in direction. Let us consider what influence such a stream will have on the sound receiver.

From the foregoing discussion, it follows that, first, the breaking away of vortexes from the body of the receiver may lead to the appearance of sound disturbances; the receiver will react to these, and its vibrating surface may become excited at one of its natural frequencies. Second, in the flow around the receiver aerodynamic sound may appear, which will to some extent be picked up by the reciever. However, this does not complete the picture; there are two other effects acting upon the receiver. One of them is the so-called pseudosound (this is the accepted term, proposed by D.I. Blokhintsev).

In a nonstationary (for example, turbulent) stream, characterized by the presence of random fluctuations of velocity, according to Bernouilli's equation, the pressure fluctuates, too. In fact, let us write Bernoulli's equation in the following form (see Chapter II, § 4)

$$p+\frac{\rho u^2}{2}=\text{const}.$$

Let us assume that

$$u=\bar{u}+u',$$

i.e., that the flow velocity u is the sum of the average velocity $\bar{u}$ and the small fluctuating velocity u'. The velocity fluctuations lead to pressure fluctuations, since Bernouilli's equation shows they are interrelated; and it may be assumed that

$$p=\bar{p}+p'.$$

Substituting these expressions for u and p into Bernouilli's equation:

$$\bar{p}+p'+\frac{\rho(\bar{u})^2}{2}+\rho\bar{u}u'+\frac{\rho u'^2}{2}=\text{const}.$$

But for the average values $\bar{p}$ and $\bar{u}$ themselves, Bernouilli's equation must also hold:

$$\bar{p}+\frac{\rho(\bar{u})^2}{2}=\text{const}.$$

The quantity $\frac{\rho u'^2}{2}$ may be neglected, since u' is small compared with $\bar{u}$, and its square even smaller. Hence, the pressure fluctuation p' must be approximately $\rho\bar{u}u'$,

$$p'\cong\rho\bar{u}u'.$$

Frequencies of turbulent fluctuations in the air reach up to a hundred cycles (atmospheric turbulence); the same frequencies characterize the pressure fluctuations*.

* In a turbulent stream, the acceleration fluctuates, too.

Obviously, a sound receiver placed in a stream with pressure fluctuations will pick up the latter, and an output signal will be transmitted. However, pressure fluctuations in a nonstationary stream do not represent normal sound, the origin of which lies in compressions of the medium. On the contrary, such fluctuations have no connection with compressions, and are propagated not with the velocity of sound, but at the flow velocity. Nevertheless, the sound receiver picking up these fluctuations does not know whether they represent sound or not. It was for this reason that this effect upon the receiver was named "pseudosound". Obviously, pseudosound causes acoustic disturbances.

One more distinction should be noted between sound and pseudosound. As we know, for small sound amplitudes the principle of superposition holds true, i.e., sound waves are propagated in a medium irrespective of any other sound waves which may be propagating in the medium at the time (the laws of propagation are linear). This is not true for pseudosound, since the superposition principle does not hold true for pressure fluctuations.

Finally, it is found that a sound receiver placed in a turbulent stream will detect still another sound—and this time it is a "real" sound, or rather a noise—one which is caused by the turbulence, which itself proves to be a sound source. The sound caused by the wind is very small and is difficult to detect in its pure form, because it is difficult to eliminate the other effects upon the receiver. However, in many cases, for instance for a turbulent jet issuing from a jet-engine nozzle, a continuous source of sound exists, the result of turbulence, which is the most powerful of all sound sources so far created by man. This will be discussed now.

Sound Caused by Turbulence. Since in a turbulent stream, due to velocity fluctuations, the pressure fluctuates too, it is not difficult to understand how a solid, though pliant, wall placed in the turbulent stream will vibrate under the action of the pressure fluctuations and will therefore radiate sound like any vibrating body. The sound thus radiated will have a very complex spectrum, determined by the spectrum of the turbulent pressure fluctuations and the nature of the natural frequencies of vibration of the wall.

It is more difficult to form a graphic concept of sound radiation by a turbulent stream in the absence of any limiting surfaces. It is considered that the radiation of sound by a homogeneous turbulent stream in the absence of either pliant or rigid walls may be explained by quadrupole radiation. The quadrupole nature of the sound radiation due to turbulence follows from a general theoretical discussion, first given by the English physicist Lighthill in 1952. According to one conclusion of the theory, a homogeneous isotropic turbulent stream radiates as a system of quadrupoles, which are randomly distributed in space. For the sake of simplicity, the whole stream may be considered to be divided into separate similar cubes with a side l; the quantity l represents the scale of the inhomogeneities in the flow velocity. None of these elements is connected with the others and each is considered to be isolated (in reality, of course, there exist various scales, while the separate cubical elements are interrelated in a definite manner). Such an element may be represented as an isolated, longitudinal quadrupole (see Chapter III, § 3), all the quadrupoles being equal with respect to the intensity of the sound radiated by them, while their orientation is random. It is possible to calculate the intensity of the sound radiated by a single quadrupole; and, given their total number, we can calculate the intensity of the sound radiated by all the quadrupoles, i.e., by the whole volume occupied by the turbulent stream.

The theory based on these assumptions and applicable only to the simplest case of a homogeneous, isotropic stream gives the following relation between the sound intensity (or acoustic power) I due to the turbulent stream, and the physical characteristics:

$$I \approx \rho_0 (u')^8 c^{-5} l^2,$$

where ρ_0 is the density of the undisturbed stream, i.e., such a stream as would have been laminar at the

same velocity as the mean turbulent flow velocity; u' is the fluctuating velocity; and c is the velocity of sound.

Here, the most remarkable fact is that

$$I \approx (u')^8,$$

i.e., the intensity of the sound generated by turbulence proves to be proportional to the eighth power of the fluctuating velocity of the stream. Assuming that the fluctuating velocity is in turn proportional to the mean flow velocity (this being approximately true for a sufficiently wide range of fluctuations of the mean flow velocity), we arrive at the conclusion that I is proportional to the eighth power of the mean flow velocity, i.e., it increases very quickly as the latter increases. This theoretical conclusion, corroborated by experiments with turbulent jets (see below), nevertheless cannot be extrapolated to any flow velocities, however great. In the latter case, we would arrive at the nonsensical conclusion that the sound intensity I is greater than the mechanical power producing the jet. In reality, it is found that at high flow velocities, when the Mach number $M = u/c$ approaches unity, and when finally the critical region $M = 1$ is passed (i.e., when the flow becomes supersonic (see Chapter VIII, § 6), then the power becomes less than 8 (from experimental data). Physically, it means that at large values of u, when M is near, or greater than, unity, the sound becomes so intense that it causes a reverse effect on the turbulent stream which generated it. However, this problem has not yet been sufficiently worked out theoretically.

While correctly describing the general aspects of sound generation by a turbulent stream, the existing theory suffers from several serious defects.

In order to determine the strength of the sound generated by a certain turbulent region, it is necessary to know the spectral distribution of intensities among the individual sound sources (quadrupoles), i.e., among their scales in space; different scales, generally speaking, must radiate different intensities. For such a spectral distribution, the corresponding theoretical formulas could be used to perform a summation over all the frequencies, obtaining the total sound intensity. However, for this purpose it is necessary to know the relation between the distribution of velocities with respect to frequency and the distribution with respect to the scales in space. But this problem for the case of turbulence has not yet been fully solved.

For this reason, the frequency spectrum of sound is not actually considered in the present-day theory of sound generation due to turbulence. The turbulent region is considered to be divided into noninteracting, isolated inhomogeneities (quadrupoles) with a characteristic scale l, and the radiation of the whole region is taken as just the sum total of the radiations of separate inhomogeneities of the given scale.

The situation is rather curious, since previously the main subject of interest was the frequency spectrum of the aerodynamically caused sound (noise) (for example, as described by Strouhal's formula), paying practically no attention to the problem of the sound intensity; at present the reverse is, in a sense, true.

Some qualitative considerations concerning the spectral composition of the noise radiated may be stated in the following way. It may be assumed that the time τ during which the velocity u changes over a distance corresponding to the characteristic scale l must be of the order of the period T of the sound (radiated by the inhomogeneity of the velocity of this scale l) with a wave length λ:

$$\tau = \frac{l}{u'} \approx \frac{\lambda}{c} = T,$$

i.e.,

$$l \approx \frac{u'}{c}\lambda.$$

It should be mentioned that for turbulent jets (see below) the experiment at $M < 1$ gives

$$l \cong \text{from } 0.01\lambda \text{ to } 0.05\lambda.$$

This result is more or less in agreement with the above formula, assuming that the fluctuating velocity u' can be 10 to 15% of the mean flow velocity $\bar{u}$.

The wind, which is a form of atmospheric turbulence, must in principle generate noise of aerodynamical nature, and the reason why it is difficult to detect such noise is connected with the fact that wind velocities are comparatively low (it should not be forgotten that the intensity of such noise is proportional to the eighth power of the flow velocity).

It is interesting to note in this connection that, for example, in the solar atmosphere, where there are incandescent gaseous streams traveling at tremendous velocities and characterized by extremely turbulent motion, the aerodynamic noise must be of such intensity that it should be expected to play a considerable role in the physical processes taking place in the

solar atmosphere. There exist astronomical theories to the effect that the solar corona, which is easily observed during solar eclipses and stretches over a distance of several solar diameters, is heated to a considerable extent by this acoustic noise. The intense noise, spreading out from the sun, is transformed during its propagation into powerful shock waves, which, when they are absorbed, heat the solar corona. There is also an opinion that the same shock waves are the cause of the flares in the solar corona.

It is interesting to see how modern acoustics is penetrating into such a seemingly distant field as astrophysics.

Noise of a Turbulent Jet. The development of the theory described in the preceding paragraphs was stimulated greatly by the necessity for studies of aerodynamic jets. A jet issuing from a nozzle (for example, from the exhaust nozzle of a jet airplane) represents a complex type of aerodynamic flow.

Figure 163 represents schematically a jet issuing from a nozzle with an orifice of diameter d. In the figure, the structure of various parts of the jet is shown.

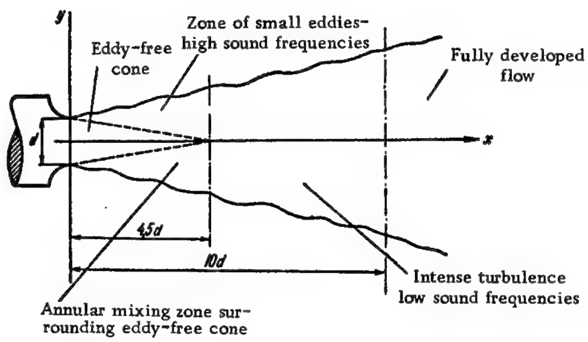


Figure 163. Turbulent jet from a nozzle

For jet engines of great power, the acoustic power produced by the jet reaches some tens of kilowatts, and the level of the noise may be over 160 or 170 db above the response threshold — power amounting to 1% of the total output of the engine. In such engines, the sound begins to exert a reverse effect on the jet causing it.

Experiments with jets lead to the following basic results.

1. The acoustic power generated by the jet is proportional to the eighth power of the mean flow velocity of the gas in the jet ($\sim(\bar{u})^8$), where $\bar{u}$ is the gas-flow velocity at the nozzle orifice. The result agrees with theory (which gives $I \sim (u')^8$), if we assume that $u' \sim \bar{u}$.

2. The ratio of acoustic power to the kinetic-energy flux density (mechanical power):

$$K = \frac{\text{acoustic power}}{\text{mechanical power}} \approx 10^{-4} M^5,$$

where $M = \frac{u}{c}$ is the Mach number.

3. The frequency spectrum of the jet noise is very wide, occupying

over seven octaves. This spectrum is more or less regular, with a small peak at the frequency $\frac{\bar{u}}{2d}$.

4. The high-frequency sound is radiated by the region of mixing. The low frequency sound, responsible for the main bulk of the acoustic power and audible at large distances from the nozzle (for example, the sound of a jet airplane), is radiated by the region of turbulent flow, at distances of approximately 4 to 10-15 orifice diameters from the nozzle.

5. The sound radiated by a jet is directed in such a manner that the radiation makes an angle with the axis of the jet which depends on the Mach number.

All the above results refer to jets with $M<1$. For supersonic jet flow, the intensity of the sound radiated by the jet proves to be proportional to a lower power of the velocity than that for subsonic jets.

Damping (muffling) the noise of a turbulent jet is an extremely difficult task, but is important for ramjet and turbojet airplanes, and for jet-propulsion engineering in general. It should be noted that for definite damping of the jet noise it is first of all necessary to decrease $\bar{u}$, but this decreases simultaneously the engine thrust and is therefore not suitable.

While the engine is idling on the ground (during warm-up and adjustment of the engine, and preparation for flight), silencers may be used. But even this is a complicated task, for the silencer must not interfere with the functioning of the engine. For an engine during flight, there have been tests made using several nozzles with smaller orifices than that of a single large nozzle, i. e., it has been attempted to divide the flow into several flows and to shift the maximum of the noise spectrum into a region of higher frequencies. However, such measures are very inconvenient, and at present other methods of silencing the noise are being sought diligently. It may be noted that employing sound-absorbers with pliant walls may not bring about silencing but on the contrary may cause more intense sound radiation, because the pseudosound may be transformed into a real sound (see above).

It must be added that a hot jet (jets issuing from the nozzles of engines may reach temperatures of thousands of degrees) generates more intense sound (noise) than a cold one.

Sound can be generated not only by a jet of gas, but also by a water jet discharging at a high speed into water. This noise has not yet been studied as completely as the noise from an air jet.

It should be noted that a water jet, in addition to radiating hydrodynamic noise, the physical nature of which is the same as that of aerodynamic noise, must also radiate noise due to cavitation.

Aerothermal Phenomena. Among those problems of aeroacoustics in which thermal phenomena play a part, one will be discussed briefly—a phenomenon discovered by Rijke in 1858 and which is acquiring considerable importance at present. This phenomenon is a good example leading to an understanding of the specific characteristics of thermoacoustical phenomena in general.

Rijke discovered that if metal gauze is placed in an open [vertical] tube (of glass, metal, or some other material), at about one quarter of the way up from its lower end, and if the gauze is heated from the lower end of the tube by means of a burner, then, after the burner has been removed, a

rather loud sound is produced in the tube, which gradually dies away. The sound has a wave length equal to approximately half the pipe length. The pipe may be made to sound continuously by making the gauze of nichrome or constantan and heating it with electric current. The sounding of such a tube, known as a Rijke tube, is an impressive lecture-hall experiment; but its explanation is involved.

The generation of sound in a tube with a heated gauze was first explained by Rayleigh, as follows. Consider a vibrating mass of air, for example, in a cylinder with a piston. If in any way heat is imparted to the air at the moment of maximum compression (when the temperature rises due to the compression) and is taken away from it at the moment of maximum rarefaction (when the temperature falls due to the rarefaction), then it is clear that the air vibrations will be reinforced (or maintained). If at some moment a sound originates in the tube containing the heated gauze, then its maintenance depends on the transfer of heat from the hot gauze to the standing wave. The transfer is effected by the alternating motion of the air up and down through the gauze (acoustic shift) and the constant (upward) motion due to convection. When the acoustic shift is directed downward, the convection is decreased; therefore, the air near the gauze is heated more and the pressure (which is already greater than atmospheric by the amount of the sound pressure) is increased. On the contrary, half a period later, the acoustic shift is directed upward and now it aids convection, the gauze heating the medium less than if there were no sound. Hence, during rarefaction less heat is transferred. As Rayleigh writes: "The alternating heat transfer depends upon the vibratory motion, while the effect of the transfer depends upon the changes in pressure. Therefore, the gauze should be placed at a point where both effects are noticeable, i.e., neither at a node, nor at an antinode, the most favorable position being at a distance one quarter of the way up the tube from its lower end".

A tube may also be made to sound by placing the gauze at a distance one quarter of the way down from the upper end of the tube; in this case, the gauze should not be heated, but cooled. A theory strictly explaining the Rijke effect is very complicated and far from being completely developed.

The generation and maintenance of sound vibrations through the use of thermal processes is a very important field, with applications in the study of fuel combustion in the combustion chambers of jet engines. The phenomenon just considered contains to some extent the general features of problems of this kind.

Chapter VII

SOUND AND ULTRASOUND WAVES IN LIQUIDS

In the preceding chapters discussion was limited to waves propagating in the air. In this chapter we will consider the properties of propagation of sound and ultrasound waves in liquids, chiefly in water, as well as some applications of sound and ultrasound which have assumed great importance during the last 30 or 40 years.

§ 1. Velocity of sound in a liquid. Absorption of sound in water

Velocity of Sound. The generation of elastic waves in a liquid does not require special explanation. A liquid, like a gas, constitutes an elastic medium, possessing three-dimensional elasticity; just as gases, liquid media admit the propagation of longitudinal sound waves.

The main distinction between liquids and gases, from the acoustical point of view, is that a liquid medium is much denser, and hence many times less compressible, than a gas. Even at very high pressures it is difficult to detect a change in the volume of water. Yet, accurate investigations show that not only water and other liquids, but even solids (for example, metals), are compressed, however slightly — even during such very small pressure changes as the acoustic pressure. It is due to the compressibility of liquids and solids that elastic waves can be propagated in them.

The propagation velocity of sound waves in a liquid is given by the formula

$$c=\sqrt{\frac{E}{\rho}},$$

where E is the bulk modulus of elasticity of the liquid and ρ is its density.

As already mentioned in Chapter II, the elasticity is the reciprocal of the compressibility; hence, the formula for c can also be written

$$c=\sqrt{\frac{1}{k\rho}},$$

where k is the coefficient of compressibility of the liquid, i.e., the relative change in volume per unit change in pressure*. For water, the compressibility is $k=4.7\cdot 10^{-11}$ (cgs units). The density of water (at 8°C) is 0.998, so that for the velocity of sound in water the formula gives

$$c = 1440 \text{ m/sec}.$$

Thus, the sound velocity in water is about 4.5 times that in air. The

* For air the specific-heat ratio $\frac{c_p}{c_v}=1.43$, but for water it is 1.01; hence for water the difference between the adiabatic and isothermal compressibility coefficients need not be taken into account.

velocity of sound in pure water is independent of frequency up to very high ultrasonic frequencies, i.e., sound is propagated through water without dispersion.

The following table gives the velocity of sound in certain liquids, and also the densities of these liquids and the change in sound velocity for a 1°C change in temperature. The minus sign indicates that the sound velocity decreases with an increase in temperature (negative temperature coefficient of velocity); the plus sign indicates that the sound velocity increases with an increase in temperature (positive temperature coefficient of velocity). For example, the velocity of sound in benzene decreases by 5.2 m/sec for a 1°C increase in temperature, while the velocity in water increases by 2.5 m/sec under the same conditions.

Liquid	Temperature, °C	Density, g/cm^3	Velocity, m/sec	Temperature coefficient of velocity, m/sec·deg
Water	25	0.997	1497	+2.5
Methyl alcohol	20	0.792	1123	-3.3
Benzene	20	0.878	1326	-5.2
Castor oil	19	0.960	1500	-
Glycerin	20	1.261	1923	-1.8
Mercury	50	13.47	1440	-0.7

Ultrasonic Standing-Wave Interferometer. The methods of measuring the propagation velocity of sound and ultrasound by means of interference and pulse methods have already been mentioned in the discussion of the propagation of ultrasonic waves in air. The same methods are also employed for measurements of the sound velocity in liquids, for example, in water. If the sound velocity in a liquid is known, it is easy to determine its compressibility—an important quantity both in scientific experiment and in technology. In addition, the propagation velocity of sound is interesting from still another point of view, since it characterizes the physical properties of the liquid.

Accurate measurements of sound velocity at low frequencies are rather cumbersome, since large amounts of liquid are required. But on the other hand, such measurements can be very accurately performed at ultrasonic frequencies using a small volume of liquid. This is even more the case with respect to absorption measurements, since absorption in liquids is very low at audio frequencies and hence difficult to measure. Moreover, in absorption measurements it is desirable to work with plane waves; and it is impossible, in practice, to obtain these at low frequencies, because the sound source must be very large.

Of course, there does exist one quite accurate method of absorption measurement at audio frequencies, which is free from the above difficulties. This is the "reverberation method". It has already been noted (see the end of Chapter III) that the sound-absorption coefficient for air can be measured with sufficient accuracy by finding the residual sounding time in the reverberation chamber. Similar measurements can also be performed in liquids. For this purpose, a "reverberation tank" is used, which is filled

with the liquid under investigation. It is only necessary to design the tank in such a manner that its walls have the highest possible reflection of sound; for example, they may be bare walls. Absorption measurements made with the reverberation tank, like those made with the reverberation chamber, reduce to a measurement of the residual resounding time (reverberation). If this is known, it is not difficult to calculate the sound-absorption coefficient for a given frequency. Absorption measurement by means of the reverberation tank plays an important part in hydroacoustical measurements, where it is required to determine, under natural marine conditions, the sound-absorption coefficient for different frequencies at various depths of the sea. For frequencies of a few kilocycles (and above), the size of the tank does not have to be too great.

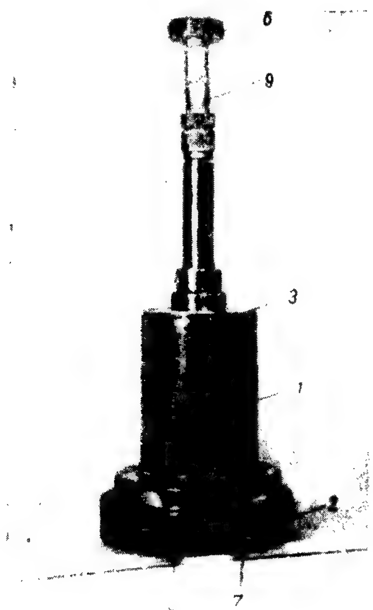


Figure 164. Ultrasonic standing-wave interferometer

1) Large brass cylinder, 2) textolite plate, 3) cover, 6) head of micrometer screw, 7, 8) terminals, 9) reading drum (each division is 0.01 mm)

The ultrasonic standing-wave interferometer (Pierce interferometer, see Chapter V, § 3) is used for velocity and absorption measurements in liquids.

Figure 164 shows a photograph of an ultrasonic standing-wave interferometer for measurements in liquids, and Figure 165 shows the same instrument, dismantled. The piezoelectric quartz plate which is the ultrasound source is pressed against the thin metal undersurface (membrane) of the vessel in which the liquid under investigation is placed.

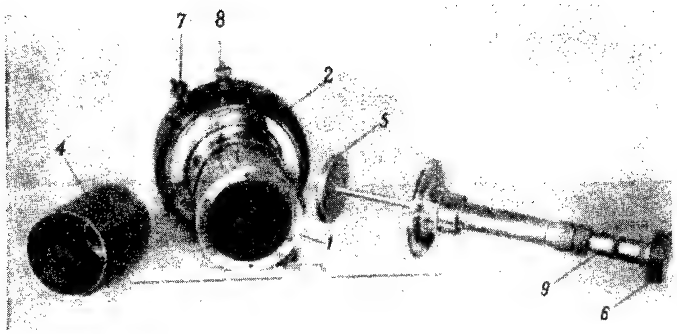


Figure 165. Ultrasonic interferometer, dismantled

For 1-3 and 6-9, see Figure 164, 4) inner cylinder, for the liquid being investigated, 5) piston-rod with reflector

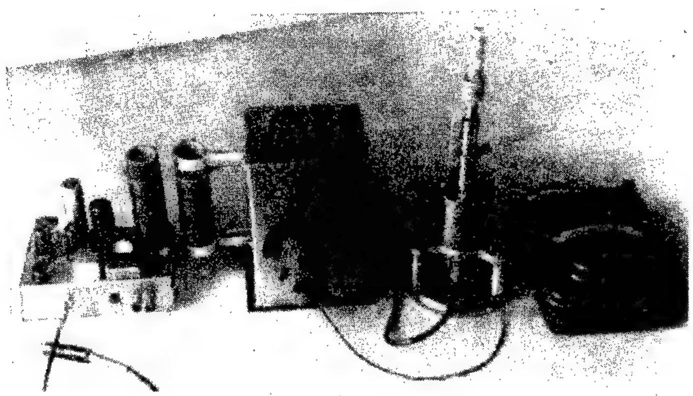


Figure 166. Equipment for ultrasound velocity and absorption measurements in liquids, using a standing-wave interferometer

Figure 166 shows a photograph of some very useful equipment* for ultrasonic velocity and absorption measurements in liquids. An oscillator (not shown in the figure) feeds a sinusoidal voltage of the required frequency through a shielded cable to the voltage amplifier (on the left). The oscillator is known as a standard signal generator and is widely used in radio laboratories for numerous purposes, for example, for tuning and calibrating radio receivers.

Since the maximum output voltage of the standard signal generator is small (about 1 v), the amplifier consists of two preliminary amplification stages and one final (output) stage. The latter has an oscillator circuit at the output, containing a small variable capacitor and a self-inductance coil, inductively coupled with the tunable receiving circuit (for tuning into

* Designed by V.P. Sivov at Moscow State University.

resonance with the frequency of the standard signal generator); the receiving circuit is mounted on a separate chassis (second unit from the left). The inductive coupling between the output coil of the amplifier and the coil in the receiving circuit is easily adjusted simply by moving the amplifier away from the chassis on which the receiving circuit is mounted. From the circuit, the voltage is supplied through a short shielded cable to the quartz plate in the interferometer. Figure 167 shows the connection diagram for the quartz plate and the instrument (microammeter at lower right of photograph) recording the reverse reaction (voltage). The interferometer shown in Figure 166 is of a somewhat different design from that shown in Figure 164. The accuracy of ultrasound-velocity measurement by means of the standing-wave interferometer, if certain precautions are taken, can reach 0.1 %. In absorption-coefficient measurements, the accuracy is considerably less, reaching at best 5-10%.

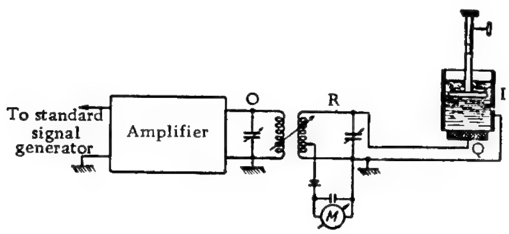


Figure 167. Diagram of the equipment shown in Figure 166.

O–amplifier-oscillator circuit, R–receiver circuit, M–microammeter, I–interferometer, Q–quartz plate

Sound Absorption in Fresh and Sea Water. Water is very different from air in its acoustical properties. The acoustic impedance ρc for air, in cgs units, is 41; for water, $\rho = 1\ g/cm^3$ and c is approximately 1500 m/sec, and hence $\rho c_{water} = 1.5 \cdot 10^5$, which is about 3500 times that for air. The vibration velocity v of the particles in a plane wave is

$$v = \frac{p}{\rho c},$$

where p is the sound pressure. Hence, for the same sound pressure, the vibration velocity of air particles is 3500 times that for water particles. Moreover, sound waves are propagated in water with a much smaller absorption*. The sound absorption in water is approximately 1000 times less than in air.

One reason for the smaller sound absorption in quiescent homogeneous water, compared to that in quiescent homogeneous air, is that the ratio of viscosity to density (kinematic viscosity) for water is much less than that for air. The absorption coefficient is proportional to the kinematic viscosity.

The low sound absorption in water explains the widespread use of sound

* The absorption coefficient for sound in water is given by the same formula as that for air; but in the case of water, the viscosity and density of water and its sound velocity must be used in the formula.

and ultrasound waves for underwater signaling, communication, and underwater location.

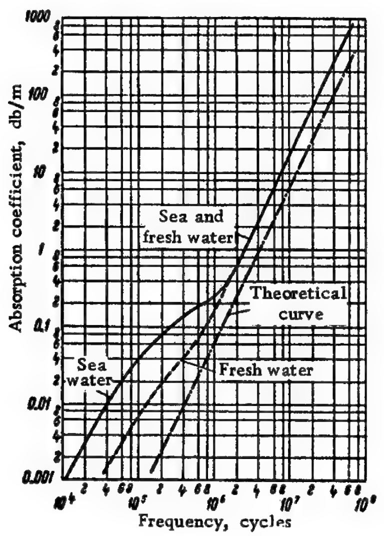


Figure 168. Sound-absorption coefficient in fresh and sea water, as a function of sound frequency

Figure 168 shows the sound-absorption coefficient in fresh and sea water as a function of sound frequency. As seen from the figure, for fresh water, beginning at frequencies of $\sim 6 \cdot 10^5$ cycles, the experimental values of the absorption coefficient are approximately double the theoretical ones, calculated from the viscosity and thermal conductivity of water. The discrepancy between the experimental and the theoretical curves for fresh water can apparently be explained by the difficulty of measuring the very small absorption at these frequencies. For sea water, at frequencies above 10^6 cycles, the discrepancy between theory and experiment becomes the same as for fresh water. Below 10^6 cycles, the sound absorption in sea water proves to be considerably higher than in fresh water. It has recently been established that this is explained by the relaxation processes in sea water, occurring due to the presence of various salts and impurities (chiefly, it seems, magnesium salts).

§ 2. Reflection and refraction of sound at interfaces

Reflection Coefficient. In discussing (Chapter II) the reflection of sound from a solid wall, only the basic law of reflection was indicated, referring to the directions of the incident and the reflected waves, and it was assumed that the total energy of the incident wave was transformed into that of the reflected wave.

It is true that almost all the incident energy is transferred to the reflected wave. Nevertheless, a part of it penetrates into the solid body, and in the latter, too, sound waves arise. It turns out that the amount of energy in the reflected wave depends upon the difference between the acoustic

impedances of the two media, at the interface of which the incident waves arrive. The greater the difference, the greater the energy transferred to the reflected wave.

For the case of perpendicular (normal) incidence, the ratio of the acoustic particle velocities (or acoustic pressures) of the incident and reflected waves, called the coefficient of reflection r, is determined from the formula

$$r = \frac{R_1 - R_2}{R_1 + R_2},$$

where $R_1 = \rho_1 c_1$ is the acoustic impedance of the first medium, and $R_2 = \rho_2 c_2$ is that of the second medium. For example, consider sound from the air striking a steel plate. For air, $R_1 = (\rho c)_{\text{air}} = 41$; for steel the density, ρ, is $7.6\ \text{g/cm}^3$, and the sound velocity, c, is 5500 m/sec, so that $R_2 = (\rho c)_{\text{st}} = 41.8 \cdot 10^5$.

Thus,

$$r \approx -0.99998,$$

i. e., the sound is almost completely reflected from the steel plate. For sound arriving from the air onto a water surface, $R_2 = (\rho c)_{\text{water}} = 1.5 \cdot 10^5$, and $r \approx -0.9993$.

Since the energy of a sound wave is proportional to the square of the amplitude of the acoustic particle velocity, the coefficient of energy reflection is r^2; for an air-water interface, $r^2 = 0.9986$. Thus, only about 0.1 % of the sound energy passes from the air into the water (or, conversely, from water into air). In spite of the fact that 99.9 % of the energy of the sound waves is reflected from the water surface, it is still possible to detect the sound of an airplane flying near the surface of the sea by means of sound detectors mounted in a submarine.

Normal Incidence of Sound at an Interface. During the perpendicular incidence of sound from air onto a water surface, the waves, as we have seen, are almost completely reflected, and a standing wave is produced in the air. At the air—water interface there will be a node in the acoustic pressure p. Consequently, the pressure amplitude at the interface (i. e., at the node) will be twice that in the incident wave. The transmission coefficient for the waves passing from the first medium into the second (for example, from air into water) is

$$q = \frac{p_2}{p_1} = \frac{2R_2}{R_1 + R_2}$$

(the subscripts 1, 2 refer to the first and the second media respectively). For $R_2 > R_1$ it is equal to 2; thus, the acoustic pressure for water is $p_2 = 2p_1$, twice that in air, and a pressure gauge placed in water is capable of detecting a sound arrival perpendicular to the water surface. It should be noted that, since the sound intensity of a plane wave is $p^2/2\rho c$, the energy is determined not only by p, but also by the acoustic impedance ρc. The acoustic pressure in water is twice that in air, but, on the other hand, the acoustic particle velocity in water is very small compared to that of air particles.

The sound transmission is much higher when a sound wave traveling in water strikes a solid. For this case it is easy to calculate that, for example, the coefficient of reflection from steel is $r = 0.93$. Since $r^2 = 0.86$, it follows that 14 % of the energy of the incident wave will be transmitted into the steel, while the other 86 % will be reflected.

Reflection and Refraction of a Plane Wave Striking an Interface at an Oblique Angle. If a wave strikes an interface at an oblique angle, then, when it is transmitted into the second medium where the sound velocity is different from that in the first, it deviates from its original direction. Figure 169 shows a wave approaching an interface between two media (I and II) at an angle α and reflected at the same angle α. The angle β between

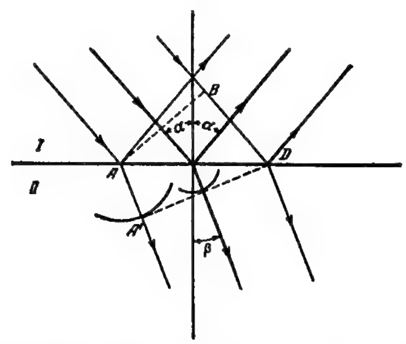


Figure 169. Reflection and refraction of a plane wave striking an interface between two media (I and II)

the normal to the interface at the point of incidence and the direction of propagation of the refracted wave is called the angle of refraction.

The change in the direction of propagation of sound waves during transition from one medium to another is easily explained with the aid of Huygens' principle. When a plane wave AB (Figure 169) reaches an interface at point A this point becomes a source which propagates secondary waves into the second medium with the velocity of sound there. After an interval t, point B of the incident wave will reach the interface at point D; during this time, the wave starting from A will reach A'. Every point on the wave front AB will pass over to a corresponding point on A'D, the front of the refracted wave*.

§ 3. Ultrasonic waves in liquids

As mentioned above, ultrasonic waves in air have comparatively little application, because of their high absorption. In water, however, ultrasound has several important applications, perhaps even more important than the applications of low-frequency sound waves. The properties of sound propagation and some practical uses of sound in water will be discussed below, but at present ultrasonic waves will be considered.

Photography of Ultrasonic Waves. In Chapter II methods of photographing sound waves were described. In the same way, ultrasonic waves in liquids can be very successfully photographed. Several such photographs, obtained by S.N. Rzhevkin and S.I. Krechmer, are shown in Figures 170-175**.

* From Figure 169 it is not difficult to arrive at the law of refraction:

$$\frac{\sin\alpha}{\sin\beta}=\frac{c_1}{c_2},$$

where c_1 is the sound velocity in the first medium and c_2 is that in the second medium.

** Obtained by stroboscopic methods.

Figure 170. Ultrasonic waves radiated by an X-cut quartz plate into vaseline

The plate is vibrating at its fundamental frequency, and its thickness is 2 mm. Schlieren photography.

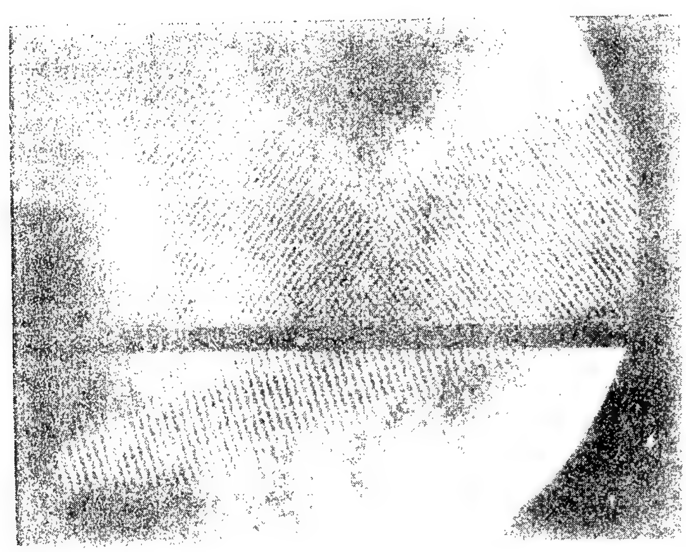


Figure 171. Reflection and refraction of ultrasonic waves incident upon the interface between vaseline and a solution of table salt

The waves strike the interface from the right.

The photographs present a graphic physical picture of the wave propagation. In them the phenomena most typical of wave motion are seen: diffraction, scattering, interference, and the basic geometrical laws for

Figure 172. Focusing plane ultrasonic waves by means of cylindrical mirror

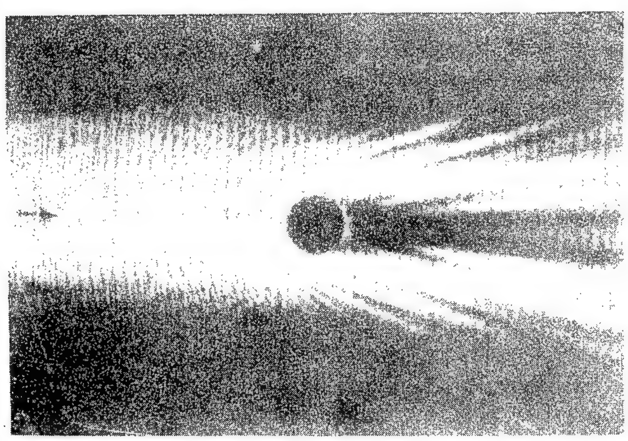


Figure 173. Diffraction of plane ultrasonic waves at a cylinder.

The cylinder diameter is 10 mm, and the ultrasonic wave length $\lambda = 1.8$ mm. The arrow indicates the direction of propagation of the waves

the incidence and reflection of waves at obstacles. Moreover, if the vibration frequency of the quartz plate (the ultrasonic radiator) is given and if, on the photograph, the distances between adjacent compressions and rarefactions are measured (the ultrasonic wave length), it is easy to determine

the propagation velocity of ultrasound in a liquid. Figure 170 shows a photograph of ultrasonic waves produced in vaseline by a quartz plate. The plate is seen at the left of the photograph. It emanates plane waves; in the photograph the interference bands are evident, apparently formed as a result of the superposition of plane waves and oblique waves occurring because of flexural waves of short length produced in the plate*.

Figure 171 shows a photograph of the reflection and refraction of ultrasonic waves at the (lower) interface between the vaseline and the table salt solution; the interface corresponds exactly to the lower edge of the dark band. In this photograph waves reflected from the interface are evident, as well as refracted waves passing from the vaseline into the salt solution. The photograph in Figure 172 shows the focusing action of a cylindrical mirror upon ultrasonic waves. Since the ultrasonic wave lengths are considerably smaller than the mirror dimensions, in this case the geometrical concept of a sound ray may be used. However, a more accurate observation shows the presence of secondary rays originating due to the interference of incident and reflected waves.

Very interesting photographs, demonstrating the diffraction and scattering of ultrasonic waves at a cylinder, are shown in Figures 173 and 174. In the first photo, a system of parabolic interference lines are seen, which originate as a result of interference of the incident and reflected (scattered) waves in front of the cylinder. It is clearly seen how, as a result of diffraction at the cylinder, waves travel along the axis into the shadow zone, whereupon this zone is almost completely absent in the second photograph, where the cylinder diameter is approximately equal to the ultrasonic wave length. The diffraction also causes a system of hyperbolic interference lines to appear behind the cylinder. The photographs have much in common with photographs of wave diffraction at a slit or at an obstacle lying on a water surface (see Chapter I).

If, instead of a single cylinder, a number of cylinders or wires (similar to a fence) are placed at equal distances in the liquid, then as ultrasonic

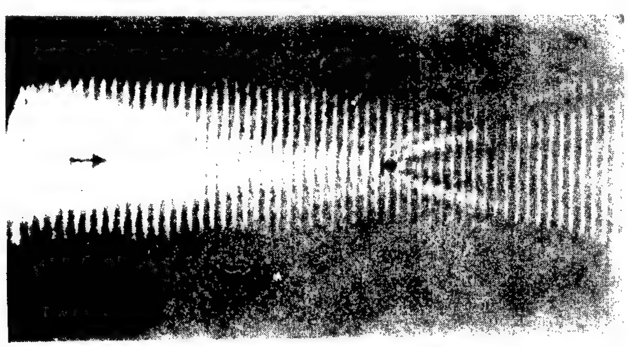


Figure 174. Diffraction of plane ultrasonic waves at a cylinder

The cylinder diameter is 1.8 mm, and the ultrasonic wave length $\lambda = 1.6$ mm.

* For flexural waves, see Chapter IX.

waves pass through the system a diffraction pattern is obtained, the photograph of which is shown in Figure 175. Such a system of cylinders represents a special type of diffraction grating.

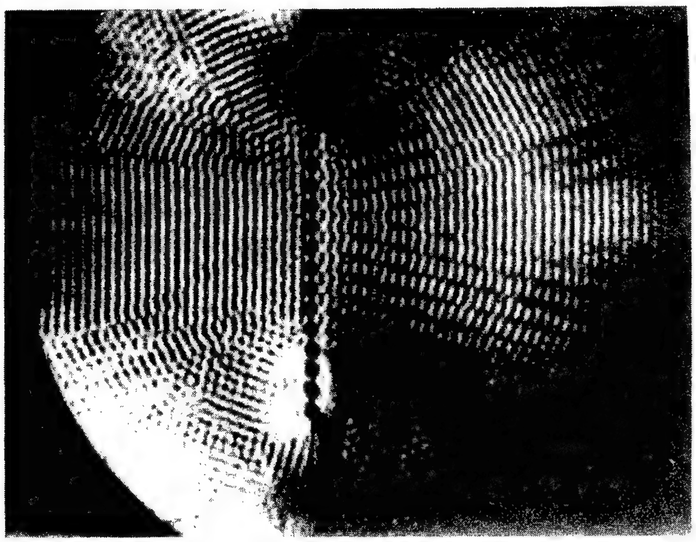


Figure 175. Diffraction of plane ultrasonic waves passing through a series of equidistant thin wires (diffraction grating)

The wire diameter is 2 mm, and the ultrasonic wave length $\lambda = 1.45$ mm.

Figure 176 shows a photograph of an ultrasonic beam in water, obtained by the schlieren method without a stroboscope; the beam is produced by a plane quartz plate. The photograph indicates that the beam does not represent a uniform ultrasonic field. It should be borne in mind that different

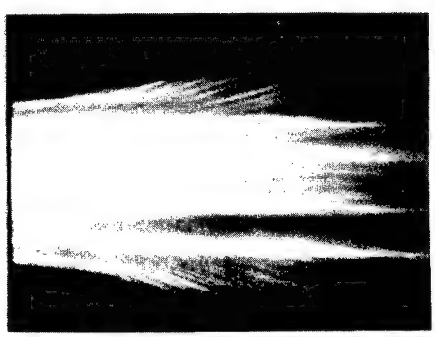


Figure 176. Ultrasonic beam in water, produced by a plane quartz plate

Frequency 1.5 mc, and ultrasonic intensity 2-3 watt/cm^2.

quartz plates give considerably different homogeneities of the sound field; hence, in investigations where homogeneity of the field is especially required, it is necessary to select a suitable plate. The photograph in the figure refers to a plate producing comparatively high acoustic power (several watts/cm^2). At high intensities the inhomogeneities in the beam become more pronounced, because changes in the density of the medium are greater, causing deviations of the light beam which were previously unnoticeable at low intensities (for the given sensitivity of the experimental equipment) to become visible at the higher ultrasonic intensities. For this reason the lateral beams due to diffraction are evident in the photograph.

Light Diffraction by Ultrasound. While working with ultrasound in liquids, Debye and Sears in Germany and Lucas and Biquard in France discovered an extremely interesting phenomenon. It was found that if ultrasonic waves are produced in a transparent liquid and white light is passed through the vessel containing the liquid (e.g., sunlight or light from an incandescent lamp), then, after the light has passed through a lens with a long focal length, a colored spectrum appears on a screen placed behind the vessel. With an increase in the intensity of the ultrasound, additional spectra appear beside the principal one.

From optics, it is known that when a beam of white light falls on a glass covered with a great number of very thin opaque rulings — known as a diffraction grating — the light is separated into its component colors. On a screen placed behind the grating, a series of colored lines is observed, representing all the colors of the rainbow. The phenomenon is explained by the wave nature of light.

It has been observed that a wire grid placed in the propagation path of ultrasonic waves causes diffraction, the diffraction pattern (Figure 175) varying with the ratio between the ultrasonic wave length and the distance between the wires. Similarly, an optical diffraction grating causes the diffraction of light waves, where the diffraction pattern is different for different wave lengths. For white light, a combination of the diffraction patterns for each separate light wave composing it is produced; and a resultant diffraction pattern is formed behind the grating as a series of colored spectral lines.

The diffraction grating is a device enabling us to perform a spectral analysis of light, just as sound analyzers (discussed previously) enable us to examine the sound spectrum.

A liquid in which ultrasonic waves are propagated behaves like an ordinary diffraction grating. To the lines of the grating correspond the periodical variations in the refractive index, the result of periodic changes in the density of the liquid as the ultrasonic waves pass through it.

Figure 177 is a diagram of the equipment used to observe light diffraction by ultrasound. A quartz plate, excited to vibrate at its natural frequency by an electronic oscillator, produces ultrasonic waves in a vessel containing a liquid. An electric lamp sends a plane-parallel beam of light, formed by a slit in a diaphragm and a collimator lens, through the vessel in a direction perpendicular to the propagation of the ultrasound.

When no ultrasound is present, a single band of light is observed on screen S — the image of the luminous slit R. But when ultrasonic waves travel through the vessel, then, in addition to the transmitted light, colored spectral lines also appear on the screen. If a beam of some definite color

is passed through the liquid, then on the screen, alongside the transmitted, undeviated (zero-order) line, two lateral lines will be seen, which are the lines representing the first-order spectrum. With increased intensity of the ultrasound, there appear, in addition to the first-order spectrum, spectra of the second and third (± 2 and ± 3) orders, and so on.

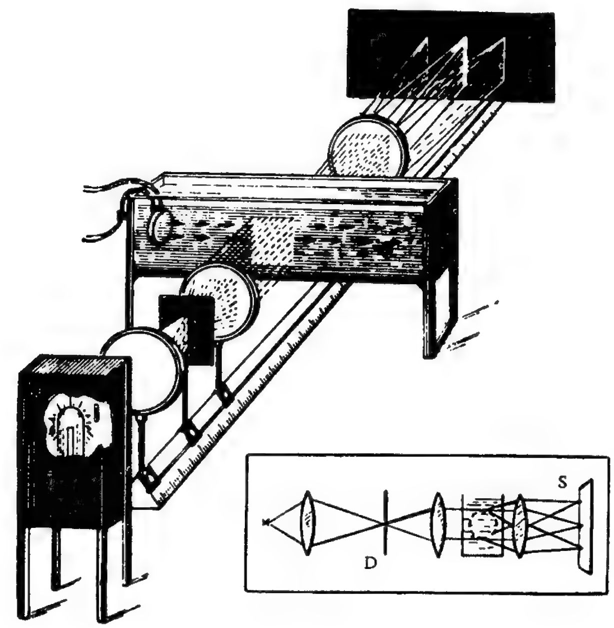


Figure 177. Equipment used for observation of light diffraction by ultrasonic waves

This pattern is similar to the diffraction of ultrasound by a wire grid (Figure 175), where spectral lines of the zero, first, and second orders can also be observed.

It may be asked how a liquid through which ultrasonic waves propagate can serve as a diffraction grating for light, since the liquid is transparent during both compressions and rarefactions.

When longitudinal ultrasonic waves spread through the liquid, the compressions and rarefactions move as layers, parallel to the surface of the vibrating plate, and the light passes along these layers. It has been noted that the refractive index for light is greater in regions of compression than in regions of rarefaction; hence, the compression and rarefaction layers are also layers with maximum and minimum values of the refractive index.

The velocity of light is many thousands of times greater than that of sound in liquid; therefore, during the time the light passes through the whole vessel the compression and rarefaction layers remain practically in the same position. To the light, they appear stationary, though they actually move with the velocity of sound. As they propagate along the ultrasonic

wave fronts, the light beams are concentrated near the axes of compression layers, where the velocity of light is a minimum. The layers serve as "corridors" for the light beams. Figure 178 represents the path of light rays in such a "corridor"; they bend from rarefaction regions into compression regions, so that the maximum light intensity occurs along the axes of compression layers, and minimum intensity occurs along the axes of rarefaction layers. Therefore, in spite of the fact that compression regions and rarefaction regions are both transparent to light, a liquid in which ultrasonic waves propagate behaves like a diffraction grating, with rarefaction layers playing the part of the rulings or grooves, and compression layers corresponding to the intervals between them. The distance between the

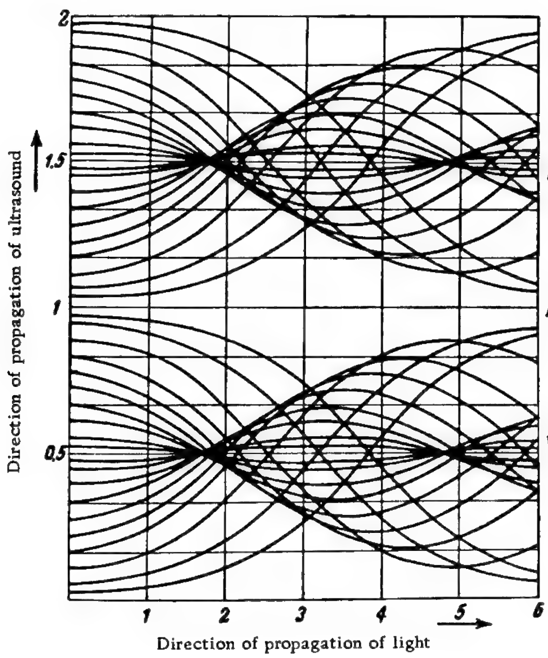


Figure 178. Path of light beams in a liquid within which ultrasound waves of length Λ are spreading

grooves in an ordinary diffraction grating is called the grating constant. Hence, for traveling ultrasonic waves, the grating constant equals the ultrasonic wave length. If standing waves are formed in the vessel containing the liquid, the diffraction pattern is not much different from the diffraction pattern for traveling ultrasonic waves. The constant of such a grating is also equal to the ultrasonic wave length.

The angle φ_n between the central ray and the ray forming the diffraction maximum of the nth order is determined by the relation

$$\sin \varphi_n = n \frac{\lambda}{\Lambda},$$

where λ is the wave length of the light, Λ is the ultrasonic wave length (serving as the grating constant), and n is an integer. If the wave length λ of the light is known, and φ is measured, the ultrasonic wave

length Λ may be calculated. Finally, the velocity of ultrasound in the liquid may be found, provided the signal frequency of the electronic oscillator exciting the plate is known, since $c = \Lambda f$.

This method of measurement of the velocity of ultrasound is often used, up to very high frequencies, of the order of 10^9 cycles. By measuring the intensity of the spectral lines resulting from the diffraction of light by ultrasound, very accurate measurements of the absorption of ultrasonic waves in translucent liquids have been successfully performed.

The diffraction of light by an ultrasonic "grating" differs very little from light diffraction by a plane ruled grating, provided the following ratio remains much less than unity:

$$\frac{\lambda l}{\Lambda^2},$$

where l is the path length of the light in the ultrasonic field (the width of the ultrasonic beam). If this ratio is greater than unity (a wide beam—a small value of Λ, and therefore a high ultrasonic frequency), then the three-dimensional nature of the ultrasonic diffraction grating becomes apparent. The three-dimensional properties of the grating show themselves in a selective light reflection obeying the Bragg condition*. For small Λ and large l, the number of "rulings" of the ultrasonic grating become so great that it begins to act as a mirror.

In this case, the intensity of light will reach a maximum when the quartz plate is so placed that the angle $\theta/2$ between the direction of incident light and the ultrasonic wave front satisfies the Bragg condition (see Figure 152).

Figure 179 shows photographs of light diffraction by standing ultrasonic waves, for ten different ultrasound intensities; it is seen that as the intensity increases, the number of secondary lines gradually increases. The last photograph already includes spectral lines of the ± 4th order. It is also seen how the intensity of the spectral lines is redistributed; the transmitted ray (central line) gradually becomes weaker as lines of ever higher orders make their appearance.

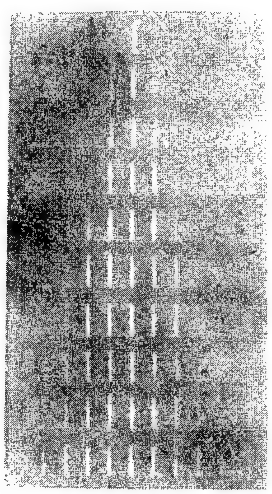


Figure 179. Diffraction of light by standing ultrasonic waves for ten different ultrasound intensities

This effect can be used to construct a so-called ultrasonic light modulator**. Let us block, for instance, the central ray, and collect all the side lines into a single line by means of a lens. The intensity of the light collected by the lens will change simultaneously with changes in the amplitude of the voltage applied to the ultrasound-radiating quartz. Such light modulators were formerly used in television.

Absorption and Dispersion of Ultrasound in Liquids. Relaxation Theory. The propagation of sound, and especially of ultrasound, in liquids is accompanied by various relaxation processes. One type of relaxation process has already been discussed, in connection with the propagation of ultrasound in polyatomic gases, where it served as an explanation for dispersion and anomalous (molecular) absorption; in that case, it consisted in the redistribution of energy, under the action of an ultrasonic wave, between the external and internal degrees of freedom of the molecules. In liquids, the situation is much more complicated,

* This was pointed out by S.M. Rytov.

** First proposed by S.Ya. Sokolov and at the same time, independently, by L.I. Mandel'shtam, N.D. Papaleksi, and G.S. Landsberg.

inasmuch as the structure is much more complex than that of gases, and so in liquids a great variety of relaxation processes are possible.

The absorption of a plane sonic or ultrasonic wave in a gas or liquid usually comprises the absorption of wave energy due to the action of the viscosity and thermal conductivity of the medium. For nonmetallic liquids, the latter plays a minor part and may be neglected. In this case the absorption coefficient is determined from Stokes' formula (see the end of Chapter II):

$$\alpha_{st}=\frac{\omega^2}{2\rho c^3}\frac{4}{3}\eta,$$

where ω is the angular frequency, ρ the density, c the sound velocity, η the shear viscosity. In the case of a compressible liquid, another viscosity coefficient may play a part. This coefficient is connected not with shear effects, but with the over-all change in volume and is therefore called the volume coefficient of viscosity, or the second coefficient of viscosity*. Since sound propagation is made possible by the compressibility of the liquid, this second viscosity (or volume viscosity) must certainly have some role in acoustic phenomena.

If we take into account the second viscosity coefficient (denoted by η'), then instead of Stokes' formula for sound absorption the following formula gives the coefficient of absorption in terms of both the shear and the volume viscosities:

$$\alpha=\alpha_{st}+\alpha_{vo}=\frac{\omega^2}{2\rho c^3}\left(\frac{4}{3}\eta+\eta'\right),$$

where α_{st} is Stokes' absorption coefficient, related to shear viscosity, while α_{vo} is the coefficient of absorption related to the volume viscosity. This formula may be called the corrected Stokes' formula.

There are many methods for measuring the normal, or shear, viscosity (e.g., by means of a viscosimeter). With regard to the volume viscosity, there are no direct methods of measurement; however, it is possible to measure η' indirectly. This indirect method (as yet the only method) consists in measuring the absorption coefficient for ultrasound $\alpha=\alpha_{st}+\alpha_{vo}$, and comparing the coefficient so obtained with the theoretical value of α_{st}. The difference in the values of α and α_{st} is assumed to be the result of the volume viscosity, which is calculated from this. For practically all liquids, the absorption proves to be greater than that calculated from Stokes' formula for α_{st}.

The absorption coefficients, both those values obtained by measurement and those values calculated theoretically from Stokes' formula (i.e., taking into account only the shear viscosity), are given in the following table, for certain liquids. The absorption coefficient $\alpha=\frac{1}{x}$ (where x is the distance in cm over which the wave amplitude decreases by a factor of $e\sim 2.7$) is represented by its ratio to the square of the frequency (because of the quadratic relation between α and the frequency f, the ratio α/f^2 must be constant for different frequencies, provided the corrected Stokes' formula holds true). Note that if α is to be expressed in db/cm the tabulated values of α must be multiplied by 8.68.

As seen from the table, only for mercury is the Stokes' absorption coefficient α_{st} almost the same as the experimental value. For all the other

* [This coefficient is also known as the bulk viscosity, or even the ultrasonic viscosity].

liquids, the absorption coefficient obtained experimentally is considerably higher than α_{st}. Especially large ratios of $\alpha_{measured}/\alpha_{st}$ are observed for such liquids as carbon disulfide, benzene, and acetic acid. If the difference is attributed to the influence of the volume viscosity, then the latter may be calculated. The second table gives values for the volume-viscosity coefficients for several liquids, calculated in this way.

Absorption coefficients (measured and calculated) for certain liquids

Liquid	Temperature °C	f, mc	$\frac{\alpha}{f^2}\cdot 10^{-17}$ sec^2/cm measured	$\frac{\alpha_{st}}{f^2}\cdot 10^{-17}$ sec^2/cm calculated	$\frac{\alpha\ meas}{\alpha\ st\ calc}$
Water	20	7-250	25	8.5	2.95
Mercury	20-25	20- 50	6	5.05	1.2
Carbon disulfide	20	1- 10	6000	5	1200
Benzene	20-25	1-165	900	8.7	103
Toluene	27	0.15	205	7.8	26
	20-25	1- 75	80	7.8	10
Carbon tetrachloride	20	1-100	500	20	25
Acetone	25	1- 4	70	7	10
	20	5- 70	30	7	4.3
Nitrobenzene	25	1- 15	80	14	5.7
Methyl alcohol	20-25	1-250	34	14.5	2.35
Ethyl alcohol	20-25	1-220	54	22	2.45
Ethyl acetate	25	1	516	8.3	62
Acetic acid	18	0.5	90 000	17	5300
	18	67.5	158	17	10.8

Liquid (t = 20°C)	η, poise	η', poise	f, mc
Water	0.0109	0.018	7-250
Ethyl alcohol	0.0032	0.009	20- 25
Toluene	0.0059	0.054	1- 75
Benzene	0.0065	0.64	1-165

From the second table it follows that in all these liquids $\eta' > \eta$; for example, in benzene the volume viscosity is 100 times greater than the shear viscosity.

A number of experiments carried out by different workers have led to the conclusion that, in the ultrasonic frequency range in which measurement is possible (up to approximately 10^8 cycles), the square-law relation between absorption and frequency given by the corrected Stokes' formula holds true for the majority of liquids. There are, however, some liquids for which this square-law relation is not satisfied. Among these are acetic acid, benzene, toluene, carbon tetrachloride, etc. In such liquids α/f^2 decreases as the frequency is increased, and at a certain frequency tends to become equal to α_{st}/f^2, the value calculated from Stokes' formula without taking the volume viscosity into account. It has been discovered experimentally that there are some liquids (e.g., castor oil*) for which α/f^2

* This effect was discovered by P.A. Bazhulin.

decreases with an increase in frequency, but in such a manner that the ratio becomes less than the Stokes value. This means that, with increasing frequency, the influence of both shear and volume viscosities upon the absorption may decrease. An evaluation of the absorption at frequencies of about 10^{10} cycles, made by studying the fine structure of the spectral lines for the Rayleigh scattering of light by hypersonic waves in liquids (see below), also shows the absorption at these frequencies to be considerably lower than the value calculated from the corrected Stokes' formula (with volume viscosity taken into account).

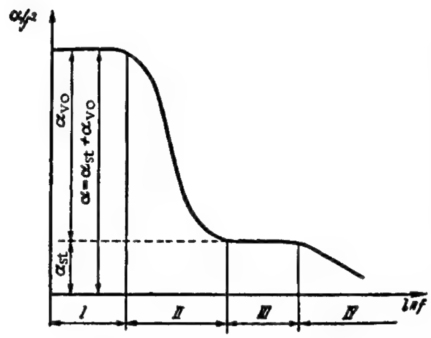


Figure 180. Relaxation curve

Figure 180 shows schematically the typical curve of the absorption coefficient $\alpha = \alpha_{st} + \alpha_{vo}$ (through the ratio α/f^2), as a function of frequency for certain liquids (the frequency logarithm is used in order to make the abscissa scale more convenient). Region I corresponds to frequencies for which both volume and shear viscosities have an effect upon the attenuation, and where the quadratic relation between absorption and frequency is satisfied; here α_{st} is the part of the absorption due to shear viscosity, and α_{vo} is the part due to volume viscosity. Region II is characterized by a decreasing volume viscosity. In region III only the shear viscosity is effective, and Stokes' formula for α_{st} holds true. Finally, in region IV α_{st}/f^2 decreases with increasing frequency. Regions II and IV are called the relaxation regions for the volume and the shear viscosities respectively.

These were the experimental facts requiring explanation when the relaxation theory of absorption was developed by L.I. Mandel'shtam and A.M. Leontovich (1937). This theory does not deal with the microstructure of the liquid, nor does it employ molecular models; but it is essentially a phenomenological theory, describing processes which are not at equilibrium. The main feature of this theory is that the equation of state of a liquid or gas contains, beside the pressure p, density ρ, and temperature T, still another parameter ξ*.

The parameter ξ may represent (as is the case for polyatomic gases) the concentration of excited molecules; in a chemical compound, it may represent the concentration of one component at chemical equilibrium. It may involve long-range-order effects, as in the case of the so-called structural relaxation**. Many processes may be pointed out which have a duration comparable to the period of an ultrasonic wave: the association

* There may be more than one such parameter in cases when there is more than one relaxation process.

** For instance, it is well known that ice is less dense than water. At the same time, ice is a crystalline solid, while water is a quasi-crystalline substance. In water, the molelcues tend to occupy quite definite positions with respect to each other. The fact that ice is less dense than water indicates that the "packing" of ice is looser than the "packing" of water. During the propagation of ultrasound in water, a transition to a structure more similar to that of ice may occur at points of rarefaction, i.e., where the density is less than the normal density of water. The transition from normal water structure to a looser "packing" is not instantaneous (as shown by theoretical and experimental data, the relaxation time of this transition is about 10^{-12} sec).

and dissociation of molecules under the action of ultrasound, polymerization and depolymerization, etc. None of these processes follow instantaneously the changes in pressure, temperature, or density in the wave. The parameter ξ accounts for the lag, i.e., it takes into account the nature of the deviation of the process from the equilibrium state. This deviation of the process from equilibrium leads, as indicated by the relaxation theory, to additional losses of acoustic energy (as compared to the ordinary losses due to shear viscosity), i.e., to increased absorption of ultrasonic waves; in addition to this, dispersion occurs. The absorption of ultrasonic waves in polyatomic gases is also described by the same theory.

The additional absorption coefficient α_r, pertaining to the relaxation process, is expressed, according to this theory, by the formula

$$\alpha_r = \frac{1}{2c_0} \frac{\omega^2\tau\left(c_\infty^2/c_0^2 - 1\right)}{1+\omega^2\tau^2};$$

c_0 is the velocity of sound for a very slow change of state of the medium; c_∞ is the velocity of sound for a very rapid change of state of the medium; τ is the relaxation time, as defined previously (see Chapter V, § 4), and ω is the angular velocity.

The total absorption coefficient is now written as

$$\alpha = \alpha_{st} + \alpha_r.$$

This formula will coincide with the formula previously given for α (and then $\alpha_r = \alpha_{VO}$), if the volume viscosity is given by the expression

$$\eta' = \frac{\rho_0\tau\left(c_\infty^2 - c_0^2\right)}{1+\omega^2\tau^2}.$$

As seen from this formula, the volume viscosity depends upon the frequency. For instance, if the temperature change in an acoustic wave takes place slowly at audible or low ultrasonic frequencies, then the processes of equipartition of energy among the degrees of freedom follow the temperature changes; and the energy losses by the sound wave (and also the volume viscosity) in this case are small. Thus, when the period of a sound wave $T = \frac{2\pi}{\omega}$ is much greater than the relaxation time τ, the volume viscosity is small. This is a general principle in relaxation theory, regardless of the specific type of relaxation process. It should also be noted that, for $T \gg \tau$, η' does not depend upon frequency, and the absorption coefficient is proportional to the square of the frequency. For $T \ll \tau$, the volume viscosity $\eta \sim \frac{1}{\omega^2}$, and α_{VO} is independent of the frequency. By measuring the absorption of acoustic waves from low to very high frequencies, it is possible, in principle, to determine the relaxation time τ. At present, however, it is only possible to produce in liquids vibrations up to 10^8 cycles. At the same time, a relatively small number of liquids are known for which these frequencies fall within, or exceed, the region of relaxation frequencies $\omega_r = \frac{1}{\tau}$. By using artificial methods of producing ultrasound, it is in most cases only possible to study the prerelaxation frequency region. Nevertheless, even the study of the prerelaxation region provides interesting material for investigations of rapid molecular processes in gases and liquids, not to speak of cases when the whole relaxation region can be investigated.

As mentioned above, it is not only the volume viscosity that is capable of relaxation (where the volume viscosity is determined by the various rapid molecular processes taking place with changes in volume). The shear viscosity, depending upon the transfer of momentum from one liquid layer to another, is naturally also related to the molecular transfer of momentum. At any rate, it is possible in principle to consider a liquid or gaseous medium, subject to shear vibrations of such a high frequency that the transfer of momentum will "lag" behind the changes in momentum. In this case, the medium will at first behave as a "jelly-like" substance, then as a solid. At very high hypersonic frequencies the shear viscosity, at least for a number of liquids (e.g., castor oil), decreases; in such cases, it is said to have "undergone relaxation".

These conclusions of the relaxation theory of absorption completely explain the nature of the relaxation curve in Figure 180, and provide an explanation for the experimentally observed values of the absorption coefficient for different liquids.

Let us now consider the velocity of sound in liquids and gases in which relaxation effects are observed. It follows from the theory of Mandel'shtam and Leontovich that the velocity of sound propagation must also be frequency dependent*; the formula is:

$$\frac{c_\omega - c_0}{c_0} = -\frac{1}{2}\frac{\omega^2\tau^2(c_\infty^2/c_0^2 - 1)}{1+\omega^2\tau^2},$$

where c_ω is the velocity of sound propagation at the frequency ω.

The relative change in sound velocity $\frac{c_\infty - c_0}{c_0}$ for liquids known to undergo relaxation effects is generally quite small, with an approximate value between 0.01% and 1-2%. It is only with the transition to hypersonic frequencies that the dispersion for some liquids attains a considerable value (see below, the last paragraph of the present section). By comparing the velocity at hypersonic frequencies with that at ultrasonic frequencies, it is possible to determine the relaxation time τ; just as it was done for absorption data.

Before completing this discussion of relaxation processes, let us look briefly at the potentialities of "relaxation spectrometry", a term which has begun to occur recently in scientific literature. The possibility of determining the relaxation time has already been indicated. However, even though this time is quite accurately known, it is not always possible to establish beyond doubt exactly which molecular mechanism is responsible for the relaxation effect. In this respect, acoustical data must be compared with some other kind of data. The "relaxation spectral line" is rather wide in comparison with what we have in optical spectroscopy. However, in cases when the relaxation times of two processes do not differ by more than approximately a factor of ten, the present accuracy of absorption measurements (10%) does not permit a differentiation between those two processes, to say nothing of a description of the "fine structure of the relaxation spectrum". It may be remarked that at present the resolving power associated with "relaxation spectrometry" is very low.

Hypersound. Scattering of Light by Elastic Thermal Waves. The highest ultrasonic frequency achieved in recent studies of the propagation of elastic waves in single crystals of quartz has been $2 \cdot 10^9$ cycles (see below, Chapter VIII, § 5). It is as yet impossible to attain artificially such high ultrasonic frequencies in liquids, because of the extremely high absorption taking place. Nevertheless, the frequency spectrum of artificially produced elastic vibrations is not limited to frequencies of 10^8 or 10^9 cycles, but stretches far beyond this. Elastic waves with frequencies above 10^9 cycles are known as hypersound, and they arise in all bodies due to their constant thermal agitation; it may be said that this thermal motion generates the hypersound. It is known that gas molecules, as well as the molecules in liquids and solids, are in a state of incessant motion. In gases this motion consists chiefly in the translational motion of molecules and in their continual collisions with one another. A crystalline solid may be represented by a three-dimensional lattice, at the sites of which atoms or ions are situated. Thermal motion in solids consists of continual random oscillations of these atoms; they do not move through large distances, as in a gas, but are only subject to small displacements about their equilibrium position. It is more complicated to provide an elementary description of the thermal

* There is a general theorem (by V.L. Ginzburg) stating that in a system with sound absorption there must also exist dispersion.

motion in liquids. Liquids are considered to be somewhat similar to solids with respect to their properties, and so their thermal motion is considered to resemble that of solids. The molecules of a liquid undergo small random oscillations about their instantaneous equilibrium points; as a result exchanging energy and momentum. The German physicist Debye, in his formulation of the theory of specific heats of solids, suggested that the thermal motion of solids may be considered as a superposition of elastic waves spreading in all possible directions. For this reason the elastic waves due to thermal motion are sometimes also called Debye waves. The maximum frequencies of Debye waves are 10^{13}-10^{14} cycles and are determined by the distances between the atoms or molecules of a given substance; the maximum wave lengths are determined by the dimensions of the body. This concept may naturally be extended to include gases and liquids. From this point of view, the fluctuations in density due to thermal motion can be represented as the result of the interference of Debye waves. Since Debye introduced the idea of thermal elastic waves, this theory has been checked and completely confirmed by various physical experiments.

Studies of hypersonic waves and their propagation in different media, particularly in solids and liquids (in gases hypersound is too rapidly attenuated; if the term "sound" is to have any meaning, the wave length must be longer than the mean free path of the molecules), are of very great interest. Such studies may provide much useful information for molecular theory, and so may be interesting with regard to the theory of the state of the medium involved. They have also proved useful for explaining a whole series of optical phenomena arising during the passage of light through transparent bodies. Finally, such studies are of interest from the acoustical point of view; they may serve to explain the fundamental properties of the propagation of elastic waves possessing the highest frequencies in the sound spectrum.

At present, there exists only a single method of studying the propagation of hypersound in liquids — the optical method, based on the study of scattered light. In order to arrive at even a very general idea of this method, and of the physical interaction between light and elastic waves, it is first necessary to discuss briefly the phenomenon of light scattering.

The scattering of sound waves by inhomogeneities in the refractive index, due to atmospheric turbulence, has already been discussed (see Chapter VI, § 3). Now let us consider light scattering by inhomogeneities caused by the propagation of hypersonic waves in liquids. Light scattering is a common, often observed, phenomenon. A searchlight beam may be observed in the dark because of light scattering by the atmosphere; we see sunbeams penetrate into a room through cracks for the same reason. Both the searchlight beam and the sun beams can be seen from the side. These are instances of large-scale scattering, i. e., scattering by small solid particles in the air, the sizes of which are greater than, or comparable to, the wave length of the light. In this case, scattering of the waves takes place at obstacles in a way similar to the scattering of water waves due to the presence of a small ball (see Figure 30). In addition to the main incident wave, there arise, by the laws of wave motion, additional scattered waves, spreading in all directions from the scattering body. These scattered waves will naturally have the same frequency as the incident waves; hence, this type of scattering is often called coherent.

In addition to coherent, large-scale scattering, physical optics also includes small-scale, or molecular, scattering, which is of great importance for investigation of matter. Such scattering is due to the change in motion of a molecule or a group of molecules. Such a change in motion leads to the formation of secondary sources of light, the radiation of which is in general insignificant and which may have directions and frequencies which do not coincide with that of the primary wave.

Thus, for instance, the sky above is blue due to light scattering by density fluctuations in the atmosphere. According to Rayleigh's well-known formula, the intensity of scattered light is proportional to the 4th power of the frequency of the incident light. Hence, the higher frequencies present in the spectrum of the sunlight are scattered more strongly, giving the sky its blue color. When the light traverses a greater thickness of atmosphere, for example at sunset, red and purple colors are observed. This is explained by the fact that for oblique beams higher frequencies (blues) are absorbed in the thickness of the lower layer of the atmosphere; as a result, lower light frequencies predominate in the transmitted light.

Rayleigh scattering does not involve any essential change in frequency and is not connected with any change in the internal state of the molecules. It is this kind of scattering which will be discussed in the following pages.

Another type of scattering, known as combination scattering (the Raman-Mandel'shtam-Landsberg effect), was discovered simultaneously and independently in 1928 by Mandel'shtam and Landsberg in the U.S.S.R. and by Raman in India. This type of scattering is connected with changes in the inner state of the molecules, caused by the incident light; in this case spectral lines appear which have a frequency different from that of the incident light*.

As already mentioned, the thermal motion of a medium can be envisaged as an ensemble of Debye elastic waves of all possible lengths and directions of propagation. Let the medium be a transparent liquid (e.g., benzene, carbon disulfide, etc.). When it is illuminated by a parallel beam of light, the light will be scattered by these Debye elastic waves. This scattering by inhomogeneities in density can be regarded as a reflection (the Bragg condition being satisfied, see Chapter VI, § 3) by a three-dimensional grating composed of the Debye waves. For standing Debye waves, light scattering observed at an angle θ will be caused by Debye waves with wave lengths Λ which satisfy the Bragg condition (see Figure 152)

$$2n\Lambda \sin\frac{\theta}{2}=\lambda,$$

where n is the refractive index of the medium, and λ is the wave length of the light. Thus, since the three-dimensional gratings formed by the elastic waves vibrating at the Debye-wave frequency Ω appear and disappear by turns (in the present case, they are formed of standing waves), evidently in addition to scattering, modulation also occurs. The latter effect is seen

* The action of the incident light changes the character of the vibrations of the molecules; these vibrations in turn modulate the basic wave. As Mandel'shtam graphically states: "In the same way as the spectrum of an ordinary radiotelephone transmitter carries in itself a complete conversation, everything which it is desired to tell, so does the spectrum of scattered light carry in it everything that a molecule tells about itself. In studying it, you learn about the state of the molecule". Investigations of the spectra of combination scattering make it possible to study molecular structure and the nature of intramolecular vibrations. Combination scattering is one of the most significant discoveries in physics.

as a splitting of the spectral line of the incident light. In fact, a change in the light frequency $\nu = \frac{c}{\lambda}$, where c is the velocity of light, occurs at an elastic-wave frequency $\Omega = \frac{v}{\Lambda}$, where v is the velocity of the Debye waves. The resulting change in frequency is

$$\pm \frac{\Delta\nu}{\nu} = \pm \frac{\Omega}{\nu} = \pm \frac{v}{\Lambda} \frac{\lambda}{c},$$

or, taking into consideration the Bragg condition,

$$\pm \frac{\Delta\nu}{\nu} = \pm 2n \frac{v}{c} \sin \frac{\theta}{2}.$$

This formula for the frequency change, derived by Mandel'shtam, determines two spectral lines (known as the Mandel'shtam-Brillouin doublet). The lines are situated to the left and to the right of the undeviated, central spectral line*, and differ from it in frequency by the quantity $\Delta\nu$; the frequency of the undeviated line is equal to that of the incident light. The three lines together are called a triplet, and they form the fine structure of the Rayleigh scattering lines**. The splitting of the Rayleigh scattering line to form a doublet, during light scattering by Debye waves, was predicted by Mandel'shtam. The splitting was then observed in the experiments of Landsberg and Mandel'shtam, as well as in experiments with quartz crystals by the Leningrad physicist E. F. Gross. Subsequently, Gross also discovered the fine structure of the Rayleigh scattering lines in liquids. In reality, the fine structure of the Rayleigh scattering lines proves to be more complex. The triplets themselves are somewhat diffuse, due to the damping of the Debye waves; in addition, a background of light exists, filling in the intervals between the lines and due in most cases to scattering caused by fluctuations in the orientation of the molecules (this is especially true for a number of gases). This background forms the so-called "wings" of the Rayleigh scattering.

The complex structure of the Rayleigh scattering spectrum is thus due to the behavior in time of the thermal fluctuations (their appearance and disappearance). Hence, by studying this spectrum it becomes possible to evaluate the kinetics of the fluctuation processes.

Let us illuminate the liquid under study with light corresponding to the blue line of the mercury spectrum (wave length $\lambda = 4358$ Å). If the refractive index n of the liquid is 1.5, and if the velocity of sound v in the liquid is $1.5 \cdot 10^5$ cm/sec (coinciding with the velocity of Debye elastic waves), then if light scattering is observed at an angle $\theta = 90°$ to the incident light, the previously given formula gives $\Lambda = 2.3 \cdot 10^{-5}$, corresponding, for the liquid, to a frequency $\Omega_{90°} = 0.75 \cdot 10^{10}$ cycles. Thus, we are dealing with hypersonic frequencies.

It should be noted that we cannot make use of light scattering in order to get at much higher frequencies Ω. In fact, even if the scattered light is observed from the direction opposite to the direction of the incident light (which is practically impossible because of the difficulty connected with isolating the scattered light against the background of the transmitted beam), then $\theta = 180°$ and $\Omega_{180°} = 1.05 \cdot 10^{10}$ cycles. By observing the scattering of shorter light waves, the frequency Ω can be increased, but only

* In light-scattering theory it is demonstrated that the undeviated line appears due to temperature fluctuations in the medium. For some liquids this line may be absent (e.g., water).

** It should be noted that in Rayleigh scattering by Debye waves in solids, due to the presence in the latter of two types of waves—longitudinal and transverse—there appear not one, but two, Mandel'shtam-Brillouin doublets. They are due to scattering by longitudinal and transverse Debye waves.

very slightly. Hence, the light scattering is determined by elastic-wave frequencies of $\Omega \sim 10^{10}$ cycles; all other frequencies of Debye waves which may be present in the medium under study (liquid) play no part in the light scattering*.

The value of the frequency splitting $\Delta\nu$ is very small ($\frac{\Delta\nu}{\nu} \sim 10^{-5}$ and the change $\Delta\lambda$ is from 0.05 to 0.1 Å); nevertheless, it may be observed quite clearly in experiment. For this purpose, however, optical instruments of very high resolving power are required; the experimental technique, too, must be well-developed. The difficulty of the experiments is also related to the very low intensity of the scattered light. Figure 181 shows, as an example of the fine structure of Rayleigh scattering lines, a toluene spectrum obtained by I. A. Fabelinskii and his co-workers. By measuring $\Delta\nu$ from these photographs of the scattering spectrum, the velocity v of a 10^{10} cycle thermal elastic wave (the velocity of hypersound) can be found using the Mandel'shtam formula.

Figure 181. Fine structure of the spectral line of Rayleigh scattering in toluene (t = 20°C)

The scattering spectrum is excited by mercury light (wave length $\lambda = 4358$ Å); the scattering angle is 90°. The distance between the central and the deviated lines is $\Delta\lambda = 0.04$ Å. The presence of a series of triplets is due to the properties of the optical interferometer.

The discovery by Gross of the fine structure of the Rayleigh scattering in liquids was extremely surprising. In effect, it meant that the attenuation of hypersound in liquids was not as great as had been previously assumed. Sharp doublet lines can actually be observed only when the sound attenuation over one wave length is small (only in such a case will there be many "rulings" in the diffraction grating formed by the hypersound), i. e., $\alpha\Lambda \ll 1$. Yet the calculation of the absorption α, using the corrected Stokes' formula and taking into consideration both the shear and the volume viscosities, leads, for all liquids, to a value $\alpha\Lambda \gg 1$ (for example, for benzene $\alpha\Lambda \approx 12$). At the same time, for benzene there is a quite sharp fine structure of the Rayleigh scattering lines, which requires that $\alpha\Lambda \ll 1$. This evident contradiction was one incentive to development, by Mandel'shtam and Leontovich, of the relaxation theory of ultrasound absorption in liquids. The basic principles of the theory have been discussed earlier in this section. As indicated by this theory, the absorption coefficient α_{vo}, determined by the bulk viscosity, at frequencies where $\omega\tau \gg 1$ (τ is the relaxation time), is independent of frequency and becomes constant. In liquids such as benzene,

* In reality, the frequency Ω is somewhat "spread out", and there is a certain interval of wave lengths $\Delta\Lambda$. But since $\Delta\Lambda$ is very small, it is possible to speak of elastic waves of a definite frequency, which are responsible for the scattering.

carbon disulfide, and carbon tetrachloride, the main part of the absorption at ultrasonic frequencies is due to the bulk viscosity η'. In the frequency interval beyond the relaxation region the absorption must be considerably less than that calculated from the law giving the absorption as proportional to the square of frequency. In this way, the possibility of observing the fine structure of the Rayleigh scattering of light in liquids is easily explained. Simply, at a frequency $\Omega \sim 10^{10}$ cycles, absorption is considerably less (in agreement with the relaxation theory) than that which follows from the corrected Stokes' formula.

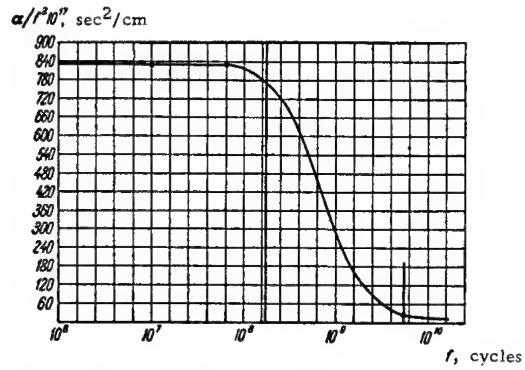


Figure 182. Relaxation curve, plotted according to data for the absorption of ultrasound and hypersound in benzene

Figure 182 shows the absorption coefficient (divided by f^2) as a function of frequency for benzene. Up to about $8 \cdot 10^7$ cycles, α/f^2 is constant (indicating that for this frequency range, $\alpha \sim f^2$ and Stokes' law holds true). Beyond this, there is a sharp drop in the absorption, until at 10^{10} cycles a further drop becomes unnoticeable — Stokes' law holds true once again; however, here the absorption is determined by the shear viscosity alone. The frequency band between the two vertical lines has not yet been experimentally investigated.

Investigations of the fine structure of the Rayleigh scattering lines for many liquids shows that for such liquids as benzene, carbon tetrachloride, and carbon disulfide, there takes place a dispersion of hypersound. Thus at 20°C the velocity of ultrasound in benzene is 1324 m/sec, while that of hypersound is 1470 ± 20 m/sec, i.e., the relative change in velocity is $\frac{\Delta v}{v} \cdot 100 = 10\,\%$. For carbon tetrachloride, it is 920 m/sec and 1040 ± 27 m/sec respectively, i.e., $\frac{\Delta v}{v} \cdot 100 = 12\,\%$*. From dispersion data and from relaxation-theory formulas it is possible to calculate separately the coefficients of absorption α_{vo} and α_{st} at a frequency of 10^{10} cycles.

* Experiments on the measurement of the velocity of hypersound in many liquids by means of studying the scattering of light in them were carried out by Fabelinskii and his co-workers at the Physics Institute of the U.S.S.R. Academy of Sciences.

§4. "Sound optics" (acoustic focusing systems)

"Sound Optics". In the preceding sections, it has been necessary more than once to employ systems for focusing ultrasound — lenses, mirrors, reflectors, concave or convex radiators, etc; now such systems will be considered in somewhat greater detail. The laws which govern sound-focusing systems are in many respects similar to the optical laws which form the basis for the calculation and manufacture of lenses, objectives, prisms, mirrors, and other optical instruments. Such similarity makes it possible to call this branch of acoustics "sound optics", or the acoustics of focusing systems*.

Of course, this analogy should be used carefully for many mistakes are possible, since the physical nature of light and sound are quite different, as has more than once been pointed out in this text. Another essential difference between acoustical and optical focusing systems is related to the fact that, due to the small wave length of light, the ratio of wave length to size of optical system is much less than it is in acoustics, including the ordinary range of ultrasonic wave lengths. For this reason, in acoustics diffraction phenomena are much more evident than in optics; and the concept of a ray can be used only with definite limitations.

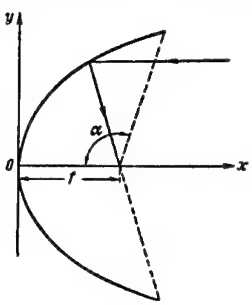


Figure 183. A parabolic mirror (reflector)

Mirrors, lenses, and zone plates will now be discussed, but not in great detail.

Concave Mirrors (Reflectors). Just as in the optics of light rays, the best form of concave mirror (reflector) for focusing sound waves or for obtaining a parallel beam of sound rays is a paraboloid of revolution. Spherical mirrors are capable of good focusing of incident sound waves only for small angles of aperture, in which case they differ little from parabolic mirrors. For large mirror angles, a part of the rays reflected from a spherical mirror miss the focal point, and the focusing is disturbed. Reflectors have many important uses, e.g., in hydroacoustic systems. Figure 183 shows a parabolic reflector with an aperture angle α. If the vertex of the parabola is at the origin O, and its axis coincides with the x-axis, then the equation of the parabola is:

$$y^2 = 2px,$$

where $p = 2f$ is the so-called semi latus-rectum of the parabola, and f is its focal length. In the case of a spherical mirror it can be easily demonstrated that, for a small angle of aperture, the equation of the circle is equivalent to the equation for a parabola

$$y^2 = 2Rx$$

* Rozenberg, L.D. Zvukovye fokusiruyushchie sistemy (Sound Focusing Systems). –Izdatel'stvo Akademii Nauk SSSR, 1949. It should be remarked that this terminology cannot be considered wholly suitable, since in electrical acoustics "sound optics" is the name given to the optical systems related to the reproduction of sound recorded on movie film.

with a focal length $f = \frac{R}{2}$, where R is the radius of the spherical mirror*.

For total reflection from a mirror, it is necessary that the specific acoustic impedance $(\rho c)_{\text{mirror}}$ of the mirror material be considerably greater than the specific acoustic impedance $(\rho c)_{\text{medium}}$ of the medium, since in this case the coefficient of reflection approaches unity (the difference in ρc for the mirror and the medium is the essential quantity, the density of the mirror material and the velocity of sound in it are in themselves unimportant). In air this condition is easily satisfied, due to the low value of $(\rho c)_{\text{medium}}$ for air and the high values of $(\rho c)_{\text{mirror}}$ for ordinary materials (solids). If water is the medium, mirrors are often constructed of two layers of solid material with an air space between them, in order to increase the coefficient of reflection of the mirror.

Parabolic or spherical mirrors can be used to concentrate sound energy, i.e., to serve as sound projectors. It is essential to note a certain difference in the projector. When the mirror is used as a reflector (i.e., as a focusing system), plane waves are transmitted, with fronts which are large. In the second case (projector), the radiated plane wave has a front approximately equal in breadth to the exit aperture of the mirror (the exit "pupil"). As we know, such a wave begins to spread out on reaching the critical distance $L_{\text{cr}} = \frac{R^2}{\lambda}$ (see Chapter III, § 3).

One more comment should be made in connection with the differences between optical and acoustical projectors. In an optical projector or spotlight, in order to obtain a narrow beam, the diameter of the exit pupil of the projector is made as small as possible, since the critical distance $L_{\text{cr}} = \frac{R^2}{\lambda}$ after which the beam begins to spread out will be very great (due to the small wave length of light), even for comparatively small values of R; thus, for $R = 25$ cm, the critical distance is 100 km. For a sound projector, because of the greater wave length of sound, in order to obtain a narrow beam a large exit aperture must be used, in order to make the critical distance greater. Thus, at a frequency of 5 mc, in order to obtain a plane wave front it is necessary, in the case of water, to make the exit pupil of the projector greater than 5 cm; and the critical distance thus achieved is still only 10 m. It should be noted that the satisfaction of this condition is necessary for accurate measurements of sound velocity in water (as well as in air) by interference methods.

From a piezoelectric crystal (e.g., quartz) let us cut out an X-cut plate, giving it the shape of a concave mirror; such a plate, when vibrating, will possess focusing properties. The ultrasonic waves will be concentrated at the focal point, which is situated on the acoustic axis. Such plates are used in order to obtain high acoustic power concentration at a point. Figure 184 shows photographs, obtained by the schlieren method, of an ultrasonic beam in water from a concave mirror made from a quartz plate. In these photographs the focusing action is clearly seen. The focusing is not sharp; one reason for this, in addition to those already mentioned, lies in the fact that the vibrations of a concave plate are not strictly radial.

* The equation of a circle with its center on the x-axis and which is tangent to the y-axis has the form

$$(x-R)^2 + y^2 = R^2,$$

so that for $x \ll R$,

$$y^2 = 2Rx.$$

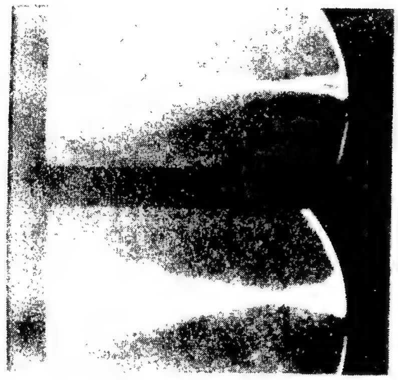


Figure 184. Focusing ultrasound in water at 8 mc, at different intensities

(The ultrasonic intensity in the lower photograph is greater than in the upper)

The velocity of propagation of longitudinal waves in quartz is different in different directions with respect to the crystal axes. For this reason, the resonance properties of a curved plate are not as sharply pronounced as in the case of a perfect X-cut plate. Using a concave radiator of barium titanate ceramic, this difficulty may be circumvented by performing the preliminary polarization in such a manner that sections of the plate vibrate strictly radially (i. e., in the direction of the radius of curvature of the plate).

Acoustic Lenses. Unlike optics, where the refractive indexes of all lens materials are always greater than unity (the velocity of light in air being greater than in any transparent body, solid or liquid), in acoustics the refractive index of a material may be either greater or less than unity. For $n>1$ (sound velocity in the lens material less than in the medium), converging-type lenses are always convex, and diverging lenses are concave (Figure 185), just as in optics. For $n<1$, the situation is reversed; convex lenses are diverging, while concave lenses are converging.

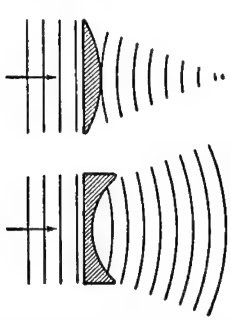


Figure 185. Incidence of a plane wave on (a) plano-convex and (b) plano-concave lenses; $n>1$.

The focal length F of a plano-convex or a plano-concave lens, with its plane side facing the source, is determined by the formula*

$$F = \pm \frac{\rho}{n-1},$$

where ρ is the radius of curvature of the lens; the minus sign corresponds to a virtual focus, i. e., the emergent beam diverges.

Lenses began to be used in acoustics long ago. For instance, carbon dioxide lenses were used

* For a doubly concave or a doubly convex lens, $F = \frac{n}{n-1} \frac{\rho_1 \rho_2}{d(n-1) + n\rho_1 + n\rho_2}$, where ρ_1 and ρ_2 are the radii of curvature of the lens, while d is its thickness along the principal axis.

to focus sound in air, the sound velocity in CO_2 being less than that in air. The development of ultrasonics has brought about a more extensive use of acoustic lenses. For a lens to be acoustically transparent (i.e., the loss of sound energy as the sound passes through the lens is kept at a minimum), the acoustic impedance of the lens material must be approximately equal to the acoustic impedance of the medium. This condition is much more difficult to satisfy than the condition for good operation of a reflector: $(\rho c)_{\text{mirror}} \gg (\rho c)_{\text{medium}}$.

Still another difficulty in the use and design of acoustic lenses should be noted. When lenses are made of solid material they are subject to the propagation of both longitudinal and transverse (or shear) waves (see Chapter IX). Even in the case of plane longitudinal waves incident normally upon a lens, the waves already reach the lens boundaries at oblique angles, due to the lens curvature; and, consequently, transverse waves arise, the velocity of propagation of which is less than that of the longitudinal waves. These transverse waves are refracted at angles other than those of the longitudinal waves, and this leads to a spreading of the image at the focal point of the lens. Hence, acoustic lenses made of solid material cannot ensure such sharp focusing as that of an optical system. The development of a theory of acoustic lenses, taking into account the presence of both longitudinal and transverse waves, encounters very great difficulties. Attempts to compensate the influence of the transverse waves experimentally have not yet been successful. This situation creates certain difficulties with regard to the functioning of such acoustic devices as S. Ya. Sokolov's ultrasonic "microscope" (see Figure 191).

Figure 186 shows the focusing of an ultrasonic beam by a plano-concave lens of plexiglas, placed in the path of ultrasonic waves propagating in

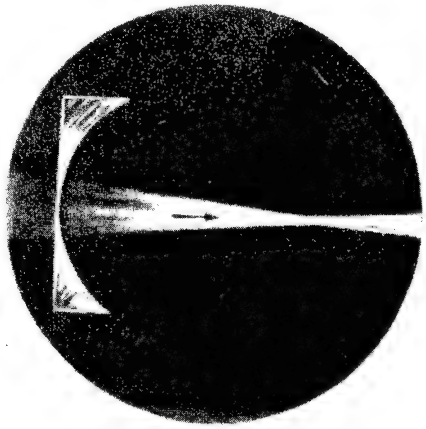


Figure 186. Focusing of an ultrasonic beam in water by a plano-concave plexiglas lens

Frequency 8 mc; photograph obtained by the schlieren method

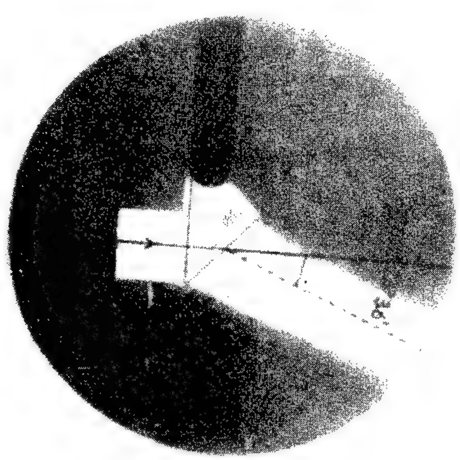


Figure 187. Deflection of an ultrasonic beam in water by a plexiglas prism (deflection angle 23°36')

distilled water (frequency 8 mc). The lens thickness along the acoustic axis is 1 mm; and the radius of curvature is 25 mm. The velocity of 8 mc longitudinal waves in plexiglas is $c_{pl} = 2880$ m/sec; the density of plexiglas $\rho = 1.89$ g/cm^3; therefore, the characteristic acoustic impedance $(\rho c)_{pl} = 3.3 \cdot 10^5$ g/cm$^2 \cdot$ sec. Since the sound velocity in water is 1500 m/sec, the refractive index $n = \frac{c_w}{c_{pl}} = 0.52$ (i.e., $n < 1$) and so the concave lens is of the converging type (in accordance with the previous discussion).

Figure 187 illustrates the action of a plexiglas prism upon the propagation of ultrasound of the same frequency in water.

Zone Plates. As focusing systems in acoustics, zone plates are also used. Consider spherical waves spreading out from a point source of ultrasound (Figure 188a), and their intersection with the plane PP. The lines of intersection of wave surfaces which are separated from one another by a half wave length will be concentric circles (Figure 188b). From Figure 188a it is not difficult to see that

$$R_n^2 = \left(f + n\frac{\lambda}{2}\right)^2 - f^2,$$

where n is the number of the wave surface, f is the distance from the source to the plane PP, and R_n is the radius of the nth circle in the plane PP. The concentric circles divide the plane PP into "Fresnel zones"; in passing from one zone to the next, the vibration phase changes sign. Consider the plane PP to be a screen of a special shape. Let the screen be open in all the Fresnel zones corresponding to the same phase of vibration (e. g., the odd zones, denoted by a plus sign) by means of suitable slits; at the same time the screen blocks off the zones of opposite sign (the even zones, denoted by a minus sign). It is evident that behind such a screen a plane wave will

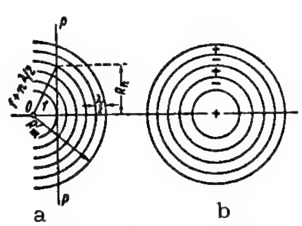


Figure 188. Fresnel zones

(a) Cross section in the plane PP of spherical waves from a point source; (b) concentric circles in the plane PP

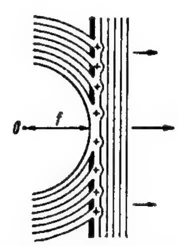


Figure 189. The effect of a zone plate

be propagated (Figure 189). It should be noted that only half of the energy of the incident wave is transmitted beyond the screen; the other half is reflected back, or else absorbed by the screen (if the latter is made of an absorbing material). It is evident, too, that if a plane wave impinges on such a screen, the waves will converge at some point behind the screen (the focal point). A screen such as this is called a zone plate. From the formula for R_n^2, the principal focal length of a zone plate is found to be

$$f = \frac{R_n^2}{n\lambda} - \frac{n\lambda}{4}.$$

For a system with a long focal length, i.e., if

$$f \gg \frac{n\lambda}{4},$$

then

$$f = \frac{R_n^2}{n\lambda}$$

and

$$R_n = \sqrt{fn\lambda}.$$

According to this formula, the radius of the first Fresnel zone is

$$R_1 = \sqrt{f\lambda}.$$

The wave nature of light is attested to by the fact that in optical instruments perfect focusing of a light beam into an ideal point cannot be achieved. As a consequence of diffraction, every optical device possesses a finite resolving power. Instead of a point, a diffraction pattern is obtained in the focal plane, consisting of a central spot with diffraction rings surrounding it. The same occurs in sound-focusing systems. The quality of a focusing system is determined by the size of the central diffraction spot at the focal point. This size depends upon the relation between the wave length, the diameter of the pupil of the system, and the focal length. In an ideal focusing system, all the energy entering the system should be concentrated within the first diffraction ring*. In actual systems only a part of the energy is concentrated within this ring, and the remainder is contained within the other diffraction rings.

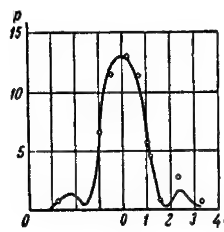


Figure 190. Distribution of acoustic pressure p in the focal plane of a zone plate

Figure 190 gives the theoretical distribution curve for the acoustic pressure in the focal plane of a zone plate with the following characteristics: $f = 23.1$ cm, $R_{\text{theo.}} = 36$ cm, $\lambda = 2.43$ cm, $n = 16$; the experimentally verified points are marked with circles (measurements made in air)**.

In conclusion, let us compare the described focusing systems in terms of their magnifying powers, where the magnification q_p is defined as the ratio of the acoustic pressure at the center of the focal spot to that in the incident wave. For a parabolic mirror, with an aperture angle $\alpha = 123°$;

$$q_p = 5.15\frac{R}{\lambda}.$$

For a zone plate with $\alpha = 63.5°$,

$$q_p = 0.81\frac{R}{\lambda}.$$

For a lens with a small refractive index and $\alpha = 90°$ $\left(\frac{(\rho c)_{\text{med.}}}{(\rho c)_{\text{lens}}} = 1.3\right)$,

$$q_p = 3.90\frac{R}{\lambda}.$$

* For a round aperture of a radius R, provided $\frac{R}{F} \ll 1$, where F is the focal length (long-focus system), the radius of the first ring is

$$r_0 = 0.61\frac{\lambda F}{R}.$$

** Rosenberg, L.D. Zvukovye fokusiruyushchie sistemy (Sound Focusing Systems). — Izdatel'stvo Akademii Nauk SSSR. 1949.

Thus, the highest magnification is that of the mirror; and the lowest is that of the zone plate.

Ultrasonic "Microscope". It is known that audible and ultrasonic vibrations can be transformed into electrical oscillations by means of a microphone or by piezoelectrical and magnetostrictive receivers. It is also known that electrical oscillations can be transformed into a visible image (for example, in television). Hence, the idea arose of the possibility of seeing objects by means of sound or ultrasound. Should it not be possible to realize audiovision by means of sound or ultrasound waves?

This is indeed possible. By employing sufficiently high ultrasonic frequencies, it is possible not only to observe objects placed, for example, in a liquid which is opaque to light, but also to magnify these objects many times (i.e., it is possible to construct an ultrasonic "microscope"— similar to an ordinary optical microscope)*.

The functional principle of the ultrasonic "microscope" is as follows. Consider an object, of which we want to obtain an ultrasonically magnified image, immersed in a liquid which fills a vessel (Figure 191). A narrow beam of ultrasonic rays, produced by the quartz plate 1, "illuminates" the object 2; the ultrasonic rays reflected by it are collected by an acoustic lens 3 upon the quartz plate 4. When the reflected ultrasonic

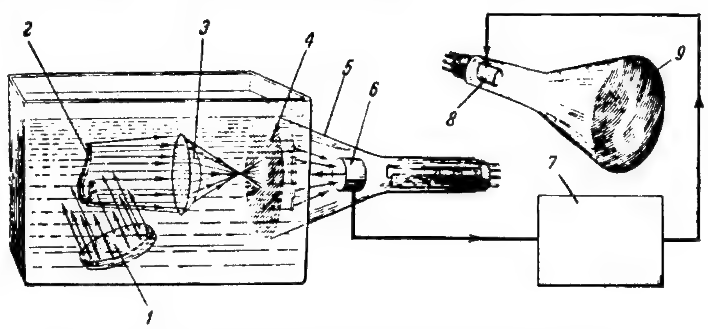


Figure 191. Operation diagram of Sokolov's ultrasonic "microscope"

rays strike the plate, electrical charges will appear on its surface due to the piezoelectric effect— at those points where deformations of the plate occur. The higher the intensity of the ultrasound at a given point on the receiving plate, the greater is its deformation at this point and the greater is the corresponding charge appearing on its surface. In this way, a latent image is formed on the receiving plate in the form of a certain distribution of electrical charges on its surface, which corresponds to the contour of the object.

Now the problem is to make the this image visible. This is achieved in the following way. The piezoelectric receiving plate 4 serves as the screen of a cathode-ray tube 5; a thin beam of electrons strikes the screen from the side opposite to the ultrasound, similar to the action of the cathode-ray tube of an electronic oscilloscope. The electron beam, striking the plate, causes secondary electrons to be ejected from it; the latter are collected by a special electrode 6 within the tube. The number of secondary electrons

* The ultrasonic "microscope" principle was first suggested by S.Ya. Sokolov. He was also the first to construct such an instrument.

leaving the piezoelectric plate when the electron beam strikes any point on it is proportional to the charge on the plate at the given point. As the electron beam sweeps over the surface of the piezoelectric receiving plate, which has different charges at different points, the current of secondary electrons flowing to the electrode 6 will vary. These variations are amplified by the amplifier 7 and fed into the grid (or modulator) 8 of the cathode-ray tube 9 of an electronic oscilloscope. The luminosity of the screen of the cathode-ray tube will vary corresponding to the number of secondary electrons emitted.

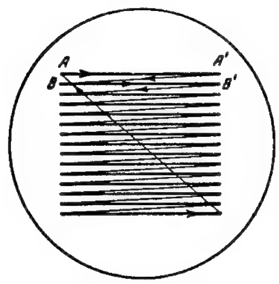


Figure 192. Scanning path of the electron beam over the screen

Distances between the horizontal lines have been exaggerated for the sake of clearness; in reality, the horizontal lines are closely adjacent.

The movement of the electron beam over the cathode-ray tube 5 is brought about by means of special electronic scanning circuits, similar to those used in television. The beam moves at constant speed (e.g., from left to right), beginning its motion at point A (Figure 192). On reaching point A', it jumps back almost to point A, but to a point which is a width of the spot below it, requiring for this an interval of time which is insignificant compared to the time for its movement from A to A'. Then, a similar movement takes place from point B to point B'. In this manner, the beam moves in lines from left to right, passing ultimately through every spot on the surface of the cathode-ray tube.

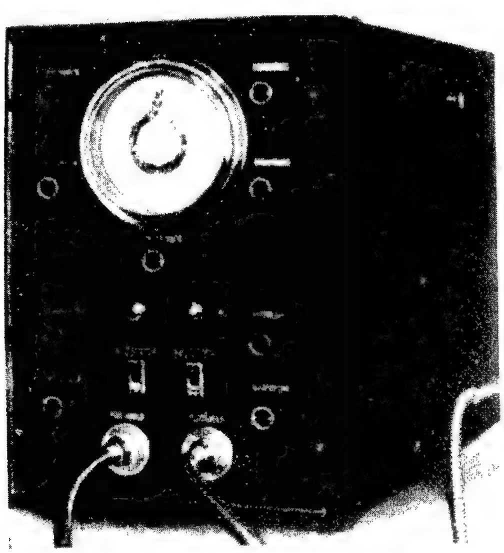


Figure 193. Sokolov's ultrasonic "microscope"

The beam scans the entire surface of the plate 20 to 30 times in one second. If the starting times of the sweeps of both tubes 5 and 9 coincide (i.e., if they are synchronized), then on the screen of tube 9 there will appear a visible image of the distribution of electrical charges on the receiving surface of the piezoelectric plate 4 — the image of the object. If the object moves, its image on the screen of tube 9 will move also.

Image magnification by means of such a system depends upon the ratio of the linear dimensions of the frames of tubes 5 and 9. In principle, quite

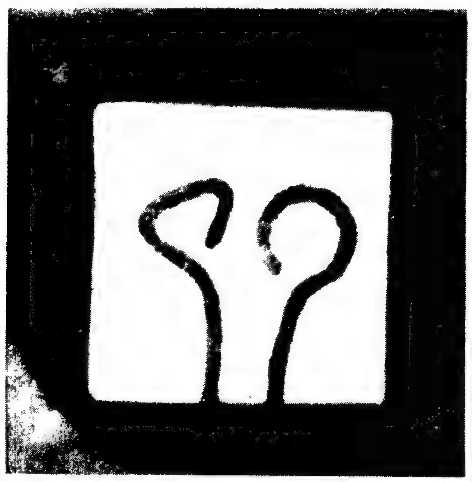


Figure 194. Photograph of an object image taken from the screen of an ultrasonic "microscope"

good magnification is possible. Figure 193 shows a photograph of the ultrasonic microscope, while Figure 194 is a photograph of two metal hooks in an opaque liquid. Since nearly all bodies are to some extent "transparent" to ultrasonic waves, the ultrasonic "microscope" may find quite extensive practical application.

§ 5. Second sound in liquid helium

All discussions of sound and sound waves in gases and liquids made so far have referred to pressure waves, in which compressions alternate with rarefactions, and the generation of which is always more or less due to the compressibility of the medium. It is also known that the propagation of sound waves is connected with alternating changes in temperature, since during compression the temperature rises somewhat, while during rarefaction it falls. In addition, it should be noted that every part of the medium which during passage of the sound waves is compressed and expanded (and which simultaneously becomes heated and cooled at the wave frequency) becomes a source of thermal waves. It is not difficult to understand how such waves originate. For example, they will be radiated by a wall, the temperature of which varies sinusoidally and which is situated

in a thermally conducting medium. The changes in the temperature of the wall have two effects. On the one hand, there is an alternating thermal expansion of the medium, due to the temperature changes; consequently the pressure changes, causing sound waves to be generated. On the other hand, due to the thermal conductivity of the medium, the changes in wall temperature are transmitted further and further into the medium. After a rise in wall temperature, during the next half-cycle a temperature drop takes place; due to heat conduction this, too, is conveyed to increasingly distant parts of the medium. As a result, thermal waves spread out from a wall of alternating temperature. These waves are characterized by very rapid attenuation with distance; at a distance of one thermal-wave length from the wall, their amplitude decreases by a factor of 535, i.e., they will have almost disappeared. Another characteristic feature is that their velocity of propagation is a function of frequency; the length λ of a thermal wave is given by the formula*

$$\lambda = \sqrt{\frac{2\pi\varkappa T}{\rho c_p}};$$

here, T is the wave period, $\varkappa$ is the thermal conductivity of the medium, ρ is its density, and c_p is the specific heat at constant pressure. The statement that during its propagation sound generates thermal waves is another way of stating that the sound loses a part of its energy due to heat conduction in the medium. A part of the sound energy is carried away from the wave in the form of rapidly damped thermal waves. In the same way, it may be stated that the viscosity losses occur because part of the energy of the sound wave is dissipated by the damped viscosity waves.

Relative changes in pressure amplitude in a sound wave are essentially greater than the relative changes in temperature amplitude; hence, thermal waves generated by the propagating sound do not play any important part.

However, there exists one remarkable instance in which, on the contrary, the thermal waves are only slightly attenuated and are accompanied to only a very slight degree by changes in pressure (i.e., by ordinary sound). The phenomenon in question is the propagation of "second sound" in liquid helium II. It is an established fact that the only liquid remaining unsolidified down to temperatures very near to absolute zero** is liquid helium.

The temperature of 2.19° K proves to be critical for liquid helium. Above this temperature, liquid helium (known as helium I) is a normal liquid; but below this temperature it turns into "liquid helium II" which has unique properties. For the vast majority of ordinary liquids, lowering of the temperature leads to an increase in viscosity. Helium II, however, possesses the remarkable property of s u p e r f l u i d i t y, discovered by P. L. Kapitsa in 1938. At temperatures from 2.19°K down to absolute zero, liquid helium II

* It should be noted that, when the wall vibrates in its own plane in a viscous liquid, so-called viscosity waves are produced, which have properties similar to thermal waves—i.e., they are also attenuated very rapidly and their velocity depends upon their frequency.

** Absolute zero is a temperature 273.16° lower than the temperature of melting ice on the centigrade scale. On the absolute (or Kelvin) temperature scale, absolute zero is taken as the initial point (0°K). For example, the boiling point of water on the absolute scale is

$$T = 100°C + 273.16°C = 373.16°K$$

At absolute zero, thermal motion ceases in all bodies.

flows through thin capillaries and cracks as though possessing no viscosity at all.

The theory of superfluidity and the hydrodynamics of superfluid liquids have been developed by L. D. Landau and his pupils. It is not possible to discuss this theory in any detail here, and therefore we will give only an explanation of some concepts related to the propagation of second sound in liquid helium II. At temperatures close to absolute zero, helium is a liquid which is almost completely devoid of thermal motion. With a temperature increase, thermal disturbances begin to appear; these do not appear continuously, but in quanta. If temperature disturbances are produced by any means in helium II, they will be propagated by mutual "collisions" of the thermal quanta. Just as in a normal gas the molecules transfer momentum to one another by mutual collisions, resulting in sound, so too, in the "gas" of thermal quanta pervading liquid helium II, thermal waves appear which are only slightly attenuated. These waves, however, have no relation to the highly damped thermal waves in a thermally conducting medium, mentioned previously, which are made possible only by the thermal conductivity of the medium. In these thermal waves, as distinguished from ordinary sound, the relative changes in pressure are slight, while the relative temperature oscillation amplitude is large. The name "second sound" for these waves was suggested by Landau.

The theoretical predictions of the possibility of propagation in helium II not only of ordinary sound, but also of second sound, were later confirmed in the experiments of V. P. Peshkov. Since, as already stated, second sound is characterized by large relative temperature amplitudes, Peshkov employed thermal methods to measure the intensity of emitted and received second sound. As a source of second sound, thin constantan wires (a few tens of microns in thickness) were used, through which an audio-frequency current was passed. A thin phosphor bronze wire served as the receiver. For measurement of the velocity of second sound, a phase-interference method was employed (similar to that described in Chapter IV, § 2).

The experiments demonstrated that in liquid helium II, in addition to ordinary sound with a velocity of 240 m/sec, second sound is actually propagated, the velocity of which, in full agreement with Landau's theory, varies from 20 m/sec at 1.36°K to 3 – 4 m/sec at 2.18°K. With a further rise in temperature, causing helium II to be transformed into helium I, second sound disappears.

§ 6. Propagation of sound in the sea

Properties of Sound Propagation in the Sea. During the propagation of sound, and especially of ultrasound in air, it is strongly absorbed, even in still air. Atmospheric turbulence and nonuniformities of temperature result in an even greater attenuation of sound, so that it is impossible to transmit sound of 5000 cycles and above over a distance of several kilometers.

In water, conditions for the propagation of sound and ultrasound waves are far more favorable. We are well aware of the fact that signaling and communication over large distances through the air are realized by means of radio waves. The idea naturally presents itself of also using radio waves for communication and the location of objects in water. It turns out,

however, that the absorption of radio waves in water is extremely great. Even for electromagnetic waves 10,000 m in length, the wave amplitude is diminished by a factor of ten every 3 m. Shorter waves — a few tens of centimeters or a few meters long, i.e., waves which could be used for radio location, are so much absorbed by the sea water that it is practically impossible to use them.

Still shorter electromagnetic waves — X-rays and radioactive gamma-rays — are diminished in amplitude by a factor of ten over every 15 cm. It is therefore hardly possible to use this type of radiation for underwater signaling and communication. With regard to light, the range of visibility in water is at best not over a few tens of meters. It is true that bright light under water can be seen rather far, but the contours of the source become diffused in the dark, resembling the weak light of electric lamps in a thick fog.

The use of sound and ultrasound waves remains the only means of signaling and communication under water. For these reasons, underwater acoustics (now called hydroacoustics) has recently begun to play an extremely important part in navigation.

Before considering the main practical tasks and achievements of this wide and important branch of modern acoustics, aspects of the propagation of sound and ultrasound waves in the sea will be discussed.

In relation to the properties of sound propagation in the free atmosphere, several phenomena caused by the nonuniform structure and turbulence of the atmosphere were explained. Just as the gaseous envelope of the earth is not a uniform and firm medium, neither is its liquid envelope — the sea. The water temperature and hydrostatic pressure vary with depth. Within the first hundred meters below the surface, the temperature distribution depends greatly on meteorological conditions — the season, time of day, wind velocity, cloudiness, etc. Ocean currents and convection lead to turbulence*. Due to the waves on the surface of the sea, and to undersea physicochemical processes, as well as to living organisms, bubbles of air are formed in the water, and these play an essential part in the propagation of ultrasound in the sea, as will presently be seen. Besides, during the propagation of sound in water, as already noted, the absorption is not as great as in air; and therefore an important part is played by the presence of boundaries which reflect sound waves — the surface of the sea and the sea bottom — especially in shallow parts of the sea.

All these factors lead to several important and interesting phenomena connected with the propagation of sound waves in the sea. A knowledge of the nature of the propagation of sound in the sea is extremely important for practical naval applications of sound and ultrasound waves.

Refraction of Sound Beams. The velocity of sound in the sea depends upon the temperature, pressure, and salinity of the water. This relation for the temperature range between 6° and 17°C is given by the following empirical formula:

$$c = 1410 + 4.21t - 0.037t^2 + 0.0175d + 1.14s \text{ m/sec.}$$

Here, t is the water temperature in °C; d is the depth in meters; and s is the salinity in grams of salt per liter of water. From the formula, it is

* The influence of marine turbulence upon the propagation of sound and ultrasound waves has so far scarcely been studied.

seen that the velocity of sound increases with a rise in temperature or salinity and with an increase in depth, i.e., with an increase in hydrostatic pressure. The velocity of sound increases by approximately 4 m/sec for a temperature rise of 1°C. For an increase in depth of 10 m (i.e., an increase of 1 atm in hydrostatic pressure), the velocity of sound increases by about 20 cm/sec. Fluctuations in the salinity play an essential part only in the vicinity of land, where the sea receives a large amount of fresh water from rivers.

On the average it may be considered that, under marine conditions, the velocity of sound fluctuates between 1450 and 1500 m/sec (i.e., changes by only 3.5 %). Nevertheless, even these small fluctuations in the velocity of sound cause considerable curvature of the path of sound beams.

In the same way as waves on a water surface, as they approach a shallow shore, move with decreasing velocity and are curved toward the coast line, so also are sound waves bent toward regions where their velocity of propagation is less.

In summer, when the upper layers of water are more heated and the temperature decreases with depth, sound beams from a source situated near the surface are bent downward. In winter, when the surface water is colder than that further below, the velocity of sound is less in the surface layer of water than it is in deep water, and sound beams are bent upward.

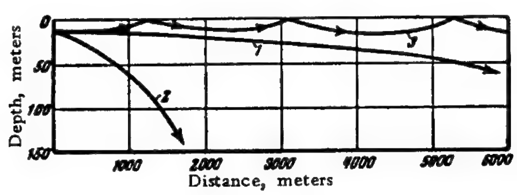


Figure 195. Curvature of the path of sound waves in the sea

1) temperature drops slightly with depth (normal case, summer); 2) excessive curvature of the sound beam (surface water much warmer than deeper water); 3) surface water colder than deeper water (winter).

Figure 195 shows typical cases of curvature of the path of sound beams in the sea in summer and in winter. Curve 1 shows the normal curvature of a sound beam, when the water temperature decreases slightly with depth (summer). In the surface layer, the water temperature usually changes by a few tenths of a degree per meter. The beams are curved downward in arcs with a radius of several kilometers. If the temperature changes in a different manner in different places, then the curvature of the beam also changes. In the presence of temperature fluctuations, which are in reality always present to some extent, the beams bend and fluctuate, resembling a picture viewed through the air over a hot surface. Curve 2 shows very strong curvature of the sound beam, when the temperature decreases rapidly with depth, as is often observed in southern seas. Curve 3 corresponds to winter conditions.

Owing to refraction, the distance over which a sound can be detected decreases in summer and increases in winter. Because the velocity of sound increases with an increase in salinity, sound beams curve toward fresher water. The following mnemonic will help in remembering the direction of bending of sound beams: the behavior of sound beams is like that of a traveler who is very thirsty on a sultry summer day; just as the traveler is attracted to cold and fresh water, so are sound beams attracted to places where the water is colder and less salty.

Since sound waves propagated in water are only absorbed a little, the maximum distance at which sound can be detected is mainly determined by refraction (especially for sound of low frequencies).

Jump Layer. At low underwater depths, due to the mixing of the surface layer by waves and the comparatively rapid heating of this layer, a "jump layer" is often formed in the sea. With increasing depth, the water temperature drops, at first rather slowly, and then more rapidly. Corresponding to these temperature changes, the water density increases with depth, first slowly, and then rather sharply. The layer with a rapidly changing temperature and density — the jump layer — has a different thickness in different seas — varying from a few meters to tens of meters or more. Below the jump layer, the decrease of temperature with depth (and the increase in water density) again becomes more gradual (Figure 196). Submarine sailors often call this layer the "liquid bottom", because a submarine in this layer easily achieves equilibrium, and can lie on it as if it

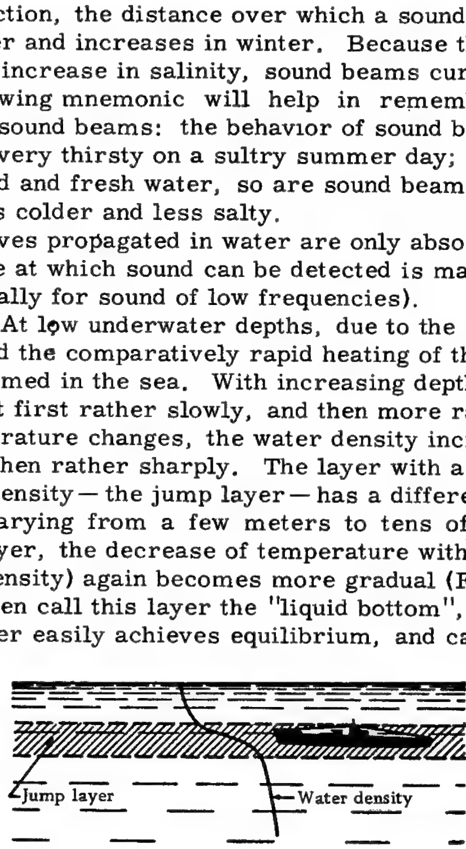


Figure 196. Jump layer

were the sea bottom. This is explained by the fact that, within the jump layer, a change in the weight of the submarine causes it to rise or to sink to a less extent than outside the jump layer. For a given change in weight, the vertical movement of the submarine outside the jump layer is many times greater than its corresponding movement within the layer, since the water density within the jump layer changes more rapidly with depth than outside the layer. For this reason, seaweed, air bubbles, plankton, minute sea organisms, etc. tend to accumulate in the jump layer.

The jump layer plays an important role in the propagation of sound in the sea. It bends sound beams to such an extent that for some angles of incidence this layer may become totally impervious to sound.

Effect of Air Bubbles. Near the surface of the sea there are usually many small bubbles of air. These bubbles may appear because of the breaking of waves or the passage of a ship (wake); an important part in the formation of air bubbles is also played by the plankton.

Air bubbles have a great influence on the propagation of sound — an effect which stands in no proportion to the small volume they occupy. The density of water is little changed by the presence of minute air bubbles.

However, since the air in the bubbles is quite compressible, the compressibility of the water containing the bubbles is greatly increased. For example, for an average of one air bubble 1 mm in diameter per cm^3 of water, the compressibility of water is increased by a factor of twenty (calculated at a depth of 10 m). As a result, the velocity of elastic waves is decreased by a factor of 4.5 !

Of course, such a great quantity of bubbles is never observed under actual conditions, but the example demonstrates the considerable change in the compressibility of water when air bubbles are present in it.

The amount of air bubbles contained in sea water depends upon the season, the time of the day, and also the weather. In warmer water, gases are liberated; in colder water, the number of air bubbles is less, since some of them are dissolved. The presence of air bubbles in water increases the absorption of sound very much. This is very easily verified by tapping a teaspoon against a glass of boiled water and a glass of tap water. Boiled water contains no air bubbles, while tap water contains a large amount of them. Therefore, in the first case a quite clear sound is heard, while in the second case the sound is dull; the sound would become even more dull if the glass were filled with carbonated water.

The mechanism of sound absorption by air bubbles is rather complicated. This absorption has many causes, two of which may be regarded as fundamental: first, heat transfer from the bubble to the liquid during the periodical changes in the bubble volume, which occur due to the action of the passing sound wave; second, partial scattering of the energy of the sound wave, caused by the air bubble itself becoming a source of sound. Because of the small size of the bubble, this radiation, or scattering, takes place in all directions. The mechanism of sound absorption by air bubbles is in many respects similar to the relaxation absorption of sound in polyatomic gases, discussed briefly in Chapter V.

When the size of a bubble is small compared to the wave length of the sound, i.e., at very low sound frequencies, the temperature difference between the bubble and the water is completely equalized. When the size of a bubble is larger than the wave length of the sound, i.e., at very high sound frequencies, this equalization cannot take place at all. However, in both cases there are no thermal losses. For intermediate bubble sizes and sound frequencies, however, energy losses due to the transfer of heat from the bubble (heated during compression) to the water play an essential part. In Figure 197a the change in temperature and volume of a bubble are shown schematically.

Consider that at the moment t_1 the acoustic pressure causes compression of the bubble. In the (p, v) diagram point 1 corresponds to the moment t_1 (Figure 197b). During compression the bubble is heated, but so rapidly that the heat has no time to be transferred to the surrounding liquid. At the moment t_2, the temperature of the gas in the bubble becomes greater than that of the liquid; point 2 corresponds to this moment on the (p, v) diagram. Further, during the time interval from t_2 to t_3, the pressure on the bubble due to the sound wave may be considered approximately constant (at any rate, changing very gradually); in other words, during this interval of time the volume of the bubble may be considered constant. At the same time, the heat accumulated by the bubble during its compression will be transferred to the surrounding liquid and will heat it. The temperature of the

bubble will become lower, and consequently the pressure of the gas in the bubble will decrease. Point 3 on the (p, v) diagram corresponds to moment t_3. Beginning at moment t_3, compression of the bubble is replaced by expansion, due to the decrease of the acoustic pressure upon the bubble. The volume of the bubble increases sharply, while its temperature decreases sharply; at moment t_4 the temperature of the bubble is lower than that of the surrounding liquid (point 4 on the (p, v) diagram).

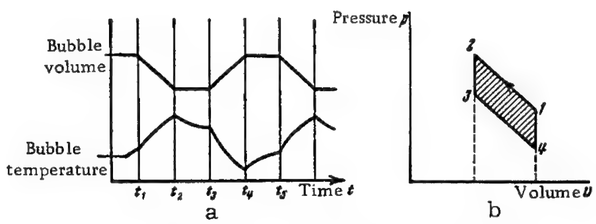


Figure 197. The change in volume and temperature of a bubble during the passage of a sound wave

The wave is considered to be trapezoidal to make the picture clearer.

During the time interval from t_4 to t_5 the volume of the bubble does not change, but its temperature rises, due to heating by the surrounding liquid; the point representing the state of the bubble — its volume and pressure — changes from position 4 to the original position 1. Thus, a closed cycle is obtained in the diagram, just as for the equilibrium process in the elementary volume of a polyatomic gas during the passage of a sound wave through it. As already noted, this means that as a result of the sound wave work has been performed, which was spent in heating the liquid. The work expended to compress the bubble is greater than the work obtained from its expansion, because the compression encounters resistance from the increased elasticity of the gas in the bubble.

Hence, while contracting and expanding due to the acoustic pressure, the bubble transfers heat to the liquid during its vibrations. The heating occurs at the expense of the energy of the sound waves, and the latter are gradually attenuated. The absorption is especially noticeable when the sound frequency coincides with the natural frequency of the bubble, i. e., in the case of resonance, when the vibrations of the bubble are at a maximum. This frequency can be obtained from the approximate expression

$$f \approx \frac{0.66}{2r},$$

where r is the bubble radius in centimeters, and f is in kc.

Thus for a bubble with a radius of 1/10 mm, the natural frequency is approximately 33 kc. Air bubbles of large size, present in water, rapidly float to the surface, owing to the considerable buoyant force; and the remaining bubbles are mostly minute ones with diameters of a fraction of one mm. For this reason, the presence of air bubbles largely affects ultrasonic frequencies of just the range employed in hydroacoustics, i. e., about 15,000-100,000 cycles.

Effect of Temperature Inhomogeneities. Because of currents and the warming of the sea surface, and the resulting processes of convection and mixing, there are always present in the sea, to some extent, temperature inhomogeneities, which have a substantial effect upon the propagation of sound, and especially of ultrasound. Figure 198 shows a typical temperature curve, obtained by trailing a sensitive low-inertia thermometer through the ocean along a horizontal plane at a depth of 50 m.

The working principle of such a thermometer is similar to that of the microthermometer, described in Chapter VI, § 3. During the recording, a platinum wire 2 cm long and 0.25 mm in diameter, connected in one arm of an electrical bridge, was in direct contact with the sea water, thus making it possible to record temperature changes. In order to eliminate rolling effects, the apparatus was installed in a submarine, moving in a straight line at 2 to 4 knots.

The horizontal temperature changes represented in Figure 198 show that the deviation of temperature from the average value is about 0.04°C; near the ocean surface, however, the deviations may be considerably greater.

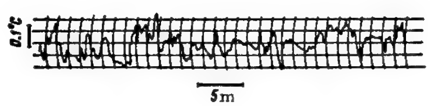


Figure 198. Temperature curve obtained by trailing a microthermometer in a horizontal plane 50 m under the ocean's surface

It should be mentioned that fluctuations in the flow velocity itself, caused by turbulence, do not have such a great effect on the propagation of ultrasound as they do in the atmosphere. The reason is that the sound velocity in water is considerably greater than in air, while the flow velocity, and with it the eddy velocity, is much less in the sea than in the atmosphere.

Temperature fluctuations, the effect of which in the air was discussed in Chapter VI, do not take place as rapidly in the sea as in the atmosphere, since sea-currents are relatively slow. Hence, to a first approximation, temperature inhomogeneities in the sea may be regarded as quasi-stationary, and in any case, even if they change, the change is gradual. It may be considered that temperature inhomogeneities of various sizes are distributed throughout the sea water, acting like concave and convex lenses, which spread out or focus sound beams.

Temperature inhomogeneities in the sea result in a series of important effects in the propagation of ultrasound. The effects are in the main the same as those mentioned in Chapter VI in the discussion of the effect of turbulence upon the propagation of sound in the atmosphere; in just the same way, in the sea fluctuations in sound velocity occur, as well as disorientation of the phase-surfaces, fluctuations in signal level, and scattering by temperature inhomogeneities.

Reverberation in the Sea. The phenomenon of reverberation, or residual sounding, which was discussed in the section dealing with the acoustics of buildings, also plays a very large part in the propagation of sound in the sea. Reverberation in a closed building is determined by sound reflection from the boundaries of the room and from objects in it; while reverberation in the sea is caused mainly by the reflection of sound from various undersea inhomogeneities and by scattering at the air bubbles present in the water. In shallow seas, an important part is also played by the waviness of the sea surface and by the uneven profile of the sea bottom. Reverberation effects at ultrasonic frequencies, due to the inhomogeneities in the sea water and due to air bubbles, were discovered by V. Anastasevich and studied in detail by Yu. Sukharevskii.

Inhomogeneities in water density, as well as air bubbles, are mainly concentrated in the surface layer and in the jump layer; consequently, the reverberation phenomenon is most significant in cases when sound waves are propagated in the horizontal direction (horizontal sonic depth finders and sonar, see below). If, after a horizontal sonic or ultrasonic pulse has been sent, the sender is switched over to reception, then immediately after

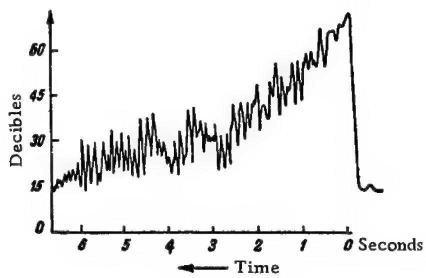


Figure 199. Recording of undersea reverberation

Duration of the rectangular signal is 0.2 sec; the ultrasound frequency is 20 kc; the instrument recorded only the changes in the signal level (the envelope)

the end of the pulse, reflected signals will reach the receiver; thus, residual sounding, or reverberation, is detected. Reflections from nearby bubbles and inhomogeneities will arrive first, then those from more and more distant ones, and the reverberation level gradually drops with the passage of time. The character of the reverberation damping depends on the sound frequency, the duration of the pulse, the quantity of bubbles and inhomogeneities and their distribution with depth, the contours of the sea bottom, the absorption, and other factors; the damping is very irregular (Figure 199). The reverberation, as it gradually dies away, oscillates and undergoes fluctuations. Figure 200 represents a recording of undersea reverberation on an oscillograph (the signal itself was also recorded). Reverberation plays a detrimental part in underwater acoustics. After an ultrasonic pulse has been sent, its echo-signal, reflected from an object, may arrive against a background of reverberation effects, which strongly mask the echo. In many cases strong reverberation considerably diminishes the range of action of the pulse method.

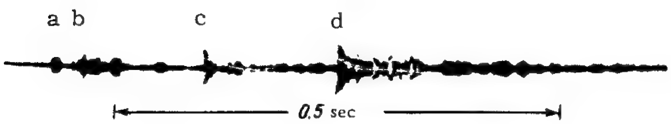


Figure 200. Oscillogram of undersea reverberation

At moment a an ultrasonic pulse (frequency, 23.5 kc) is sent horizontally; at b the emitter [now as receiver] is connected to the amplifier; at moments c and d, the amplifier sensitivity is increased.

Deep-Water Sound Channel. Superdistant Sound Propagation. A very interesting case is that of sound propagation at great depth, where the temperature decreases slowly, approaching the value corresponding to maximum water density, and where conditions are very stable and constant throughout the whole year. Beginning at a depth of about 1200 m, where the water temperature remains practically constant, changes in sound velocity occur only because of changes in pressure. Figure 201 shows the changes in temperature, pressure, and sound velocity with the depth of the sea. At a depth of approximately 1200 m, there is a minimum value for the velocity of sound*; at greater depths, due to the increased pressure, the sound velocity again increases. Since sound beams are always bent toward regions of the medium where their velocity is at a minimum, they are concentrated in the layer with a minimum velocity of sound. A sound beam emanating from a point A (Figure 202) within such a layer, and which has only a

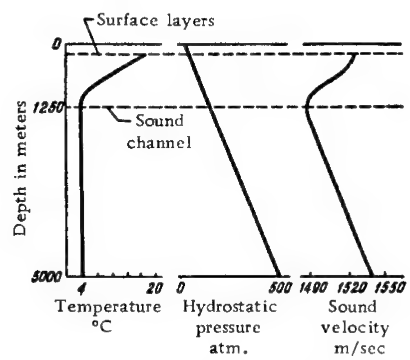


Figure 201. The changes in temperature, pressure, and sound velocity with the depth of the sea

slight inclination to the horizontal, will always remain within the layer. Such a layer is called a deep-water (or underwater) sound channel.

Here the same phenomenon of the concentration of waves in a layer, as in the passage of light through an ultrasonic "grating" (see Figure 178), exists.

* In different oceans and seas, the layers of minimum sound velocity may occur at different depths; the value of 1200 m refers to the Atlantic Ocean. In some seas, the axis of the sound channel may be situated at comparatively small depths. [This channel is also known as the SOFAR channel.]

Similar waveguide-like properties are shown, for instance, in optical mirages. It is known that a mirage is a very rare phenomenon, possible only in southern latitudes and in the presence of a layer of sharp temperature nonuniformity in the atmosphere. Light beams are able to travel along such a layer far beyond the horizon, carrying images of very distant objects — mountains, oases in the desert, and cities.

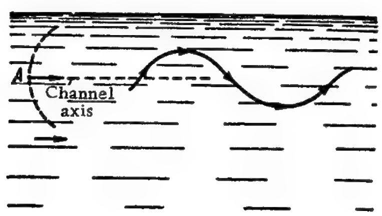
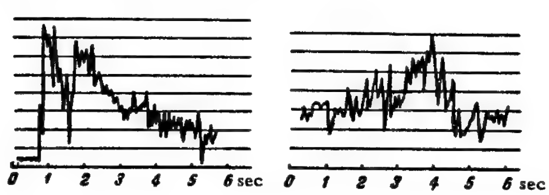


Figure 202. Path of sound beam in the deep-water sound channel

The initial direction of the beam is inclined slightly to the channel axis.

For a beam of light, the refractive index depends mainly upon the air temperature, i.e., the air density; for radio waves, in addition to temperature, the refractive index depends upon the water-vapor content in the atmosphere. For this reason, a "mirage" is more often observed for radio waves in the radar band — from a few dozen centimeters up to a few meters in wave length. In southern latitudes, especially over the sea, radiolocators often give a distinct reflection of radio waves from shores which are hundreds of kilometers away from the station, while the radar range under ordinary atmospheric conditions extends as far as the line of sight, or the horizon (for a radar station on the surface of the earth, a few tens of kilometers).

Due to the waveguide-like properties of the deep-water sound channel in the sea, superdistant propagation of sound is possible. The explosion of a small bomb, only a few kilograms in weight, which took place within such a channel, could be detected over a distance of 5000 km, with a deeply immersed receiver! This colossal distance was traversed by the sound in an hour! The tape of the recording apparatus showed a recording of the signal which lasted more than 30 sec.



a b

Figure 203. Recording of the explosion of a small charge: a) close to the explosion center and b) at a distance of 560 km; depth of explosion — 100 m

Figure 203a shows a recording of the explosion of a small charge, made at a point close to the explosion center; while Figure 203b shows the recording of a similar explosion made at a distance of 560 km. In these experiments (by Rozenberg and others), the sound was received at the surface of the sea, while the explosion itself took place at a depth of 100 m. The curves differ greatly from each other. In the first case the sound pulse arriving at the receiver increases sharply, then slowly falls off. This falling off is the result of undersea reverberation, discussed above. In the second case, the arriving signal gradually increases, with fluctuations in intensity, and having attained a maximum value 4 sec after the arrival of the beginning of the signal, it stops rather abruptly.

Figure 204 shows schematically the path of the beams, in order to explain the trend of the signal recordings at great distances from the explosion center. At first glance, it would seem that the sound pulse following path 1 must arrive later than the one traveling along path 2. However, calculations show that beam 1 arrives at the observation point O' first, and beam 2 arrives second. The more times a beam is reflected from the water surface, the later the corresponding sound pulse arrives. This is explained by the fact that beams 1 and 2, for instance, travel along part of their paths through layers of water in which the sound velocity is greater. In reality, not one or two, but a great number of beams, arrive at the reception point, every one with its own particular trajectory. As a result, a prolonged sounding of the signal is obtained at the observation point. The comparatively sharp fall in the signal level is caused by the fact that, beginning at a certain period of time after the arrival of the horizontal beam, pulses begin to arrive along path 3, which have undergone large absorption during reflection from the sea bottom. The sea bottom is mostly layers of silt, which often reach dozens of meters in thickness. Acoustic waves falling on the silt bottom are strongly absorbed; this absorption is especially high at ultrasonic frequencies.

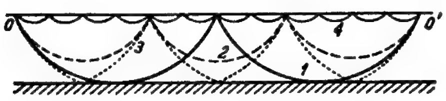


Figure 204. Path of sound beams, explaining the recordings in Figure 203

The phenomenon of superdistant sound propagation in the sea, in addition to its scientific interest, may have several important practical applications. For instance, it has been suggested to use the deep-water sound channel for signaling by airmen in case of emergency (steps have already been taken toward practical implementation of these suggestions). A few sound detectors placed at large distances can, by the difference in arrival time of the sound of an explosion, locate the spot where a depth-charge has been dropped. Also, the superdistant propagation of sound will, apparently, make it possible to obtain information about events taking place in distant regions of the ocean.

Independently of the research of American acousticians, the superdistant propagation of sound in the sea was discovered by Soviet scientists (Rozenberg and others). The phenomenon was given a strict theoretical basis in the work of L. M. Brekhovskii.

Sonic Depth Measurement. Sonic Depth Finders. One of the first important uses of sound waves in water was their application in the measurement of the depth of the sea. In principle, such measurements are very simple; they were mentioned in Chapter II, where an example was given of the measurement of the sound velocity in air by the echo method. If a sound signal is sent from a ship on the surface, e. g., due to an explosion (Figure 205), then the pulse will be reflected from the bottom and will

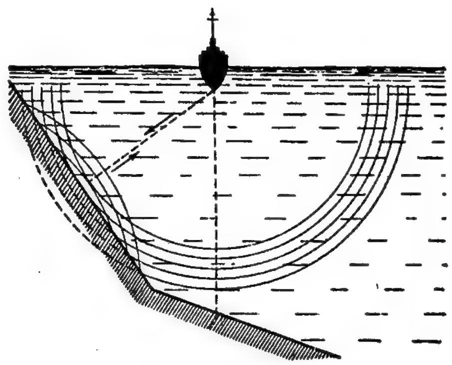


Figure 205. For undirected transmission of a sound pulse, the depth of the sea under the ship may be measured erroneously

return after a time $t = \frac{2L}{c}$ (where L is the depth of the sea). The problem consists in measuring the period of time between the moment of explosion and the reception of the reflected sound signal — the echo. If the velocity of sound in water is known, the depth of the sea can be found at once using the above formula.

However, for a long time it was found impossible to use this principle to measure the sea depth with sufficient accuracy and reliability. The reason is that spherical sound waves are propagated by an explosive source. If the bottom has an uneven profile, as in Figure 205, the first reflected signal may arrive not from the part of the bottom which is directly under the ship, but from parts nearer the ship, but situated to the side. For this reason, large errors are possible in depth measurements with an undirected source of sound waves.

An even greater difficulty lay in the necessity of measuring the small intervals of time between two individual sound pulses. Due to the high velocity of sound propagation in water, the time between the outgoing pulse and the returning echo is extremely small at depths which are not very great. For instance, if the sea depth is 25 m, then the time interval between outgoing and returning pulses is

$$t = \frac{2 \cdot 25}{1500} = \frac{1}{30} \text{ sec.}$$

If it is desired to perform measurements with an error of not more than 1 m (4 %), then the time interval must be determined within $\frac{1}{30} \cdot 0.04 = \frac{1}{750}$ sec.

For a depth of 2.5 m (an accurate knowledge of such depths is extremely important in river navigation), in order to obtain a result accurate to 20 cm, it is necessary to measure the time interval between the outgoing signal and the returning echo to 1/3750 sec. The interval being measured in this case is only 1/300 sec. Such small time intervals between two individual sound pulses have become simply and accurately measurable only by means of electronics.

The employment of an explosion or the sound of a bell is quite useless for continuous depth recordings, i. e., for the recording of the bottom profile during the motion of a ship. If, instead of an explosion, sound waves of low frequencies are used, then again depth measurement becomes very inaccurate, since waves produced at low frequencies are practically spherical, while for accurate determination of the depth immediately under the ship it is desirable to have a sharp directivity pattern for the radiation and reception of the sound vibrations.

P. LANGEVIN
(1872-1946)

As already mentioned, to obtain directed soundings it is necessary for the dimensions of the sound source to be greater than the wave length of the sound radiated by it. For example, for a frequency of 1500 cycles, the wave length in water is 1 m. It is practically impossible to construct a radiating surface several meters in size; hence at such frequencies the sound energy is not concentrated.

The employment of low frequencies leads, however, to other difficulties: the radiated sound pulse cannot be of long duration, since for moderate depths the reflected pulse will arrive before the radiated pulse has been completed, whereby measurement of the time interval t between sending of the sound and reception of the echo becomes impossible. At low frequencies (large wave lengths) this condition causes too few sound vibrations to be radiated during the pulse. A reflected pulse consisting of a small number of vibrations will not have an adequate effect on the receiver, and the latter will not record an echo from the sea bottom. It is true that other methods may be employed for measurement of the depth of the sea, in addition to the pulse method; e.g., the method of the acoustic inteferometer described above. However, using low sound frequencies, the same difficulties are encountered in obtaining a sharp directivity pattern of the source and receiver.

It is only as a result of the immense achievements of acoustics and radio engineering during the last 25-30 years that accurate measurement of the sea depth has been accomplished, as well as various other applications of the pulse method to underwater acoustics. It has become possible to radiate powerful ultrasonic waves into the water and thereby to detect weak ultrasonic signals propagating through the water; it has also become possible to achieve sharp ultrasonic beams and to radiate and detect ultrasonic pulses of short duration, containing, as a result of their high frequency, a great number of ultrasonic waves. The development of electronics has made possible high amplification of weak voltages and accurate measurement of time intervals.

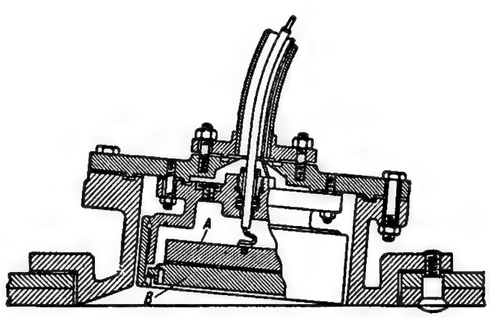


Figure 206. Langevin's piezoelectric quartz source (detector)

Quartz plates (mosaic) are placed between two steel electrodes A and B

The first accurate depth measurements were carried out in 1918 by the famous French scientist Langevin, who used a piezoelectric quartz source and detector and employed ultrasound instead of sound waves. Figure 206 represents the source (also serving as the detector) of ultrasonic vibrations used by Langevin for sea-depth measurements, while Figure 207 is a photograph of Langevin's piezoelectric quartz source (detector) for sending ultrasonic waves in the horizontal direction. In order to obtain a large surface area, not one but a numerous set of quartz plates were used (a mosaic),

with the same thickness and the same cut (X—cut), since it would be difficult to produce a single quartz plate of such large dimensions. The quartz-plate mosaic was placed between two steel electrodes. One of the steel plates, in contact with water, produced ultrasonic vibrations. The system as a whole had a natural frequency of 18-20 kc. Voltage pulses were supplied to the steel electrodes at regular time intervals, by means of a small motor running at a strictly constant speed, which interrupted the connection to the voltage source. The decaying ultrasonic pulses, after being reflected from the bottom, returned as echoes and were detected by the same ultrasonic source, which then served as a detector. The reflected pulses received were amplified by a special device resembling a loop oscillograph, and then recorded on photographic paper; the profile of the sea bottom was thus directly obtained.

Figure 207. Langevin's source, designed to send ultrasonic pulses in the horizontal direction

At present, sonic depth finders have come into wide-spread use not only in ocean navigation, but also in river navigation and hydrography. Many sonic depth finders exist, of various designs. Instead of quartz sources and detectors, magnetostrictive and Rochelle salt types are generally used in modern equipment. Figures 208 and 209 are photographs of a magnetostrictive source (detector) which forms part of a modern sonic depth finder.

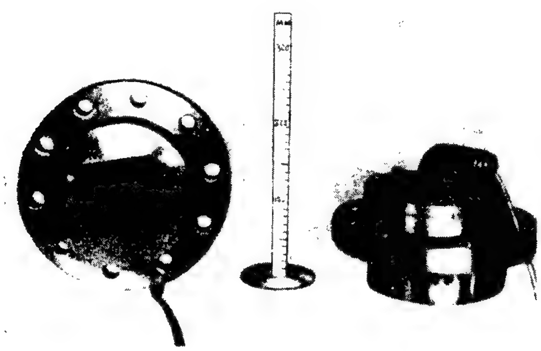


Figure 208. Magnetostriction source (detector) in a case

Usually, two separate transducers are used for the source and the detector. They are attached to the bottom of the ship, so that the radiation is directed straight downward. Also, an outboard design may be used (especially in river depth finders), in which the source and detector are placed in a streamlined chassis shaped like a fish.

The principle of operation of a common type of depth finder is evident from Figure 210. Via the reduction gear P the motor M turns a drum, to which an arm with a pen S is attached. At every revolution of the drum, the contacts K are broken, activating the relay in the contact box 3, which supplies a short electric pulse of about 1000 volts to the magnetostrictive generator 4 (represented in greater detail in Figure 211).

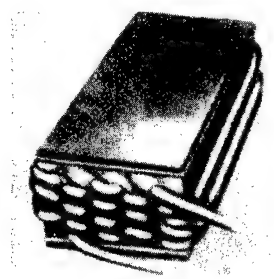


Figure 209. Stack of magnetostrictive plates with winding, removed from its case

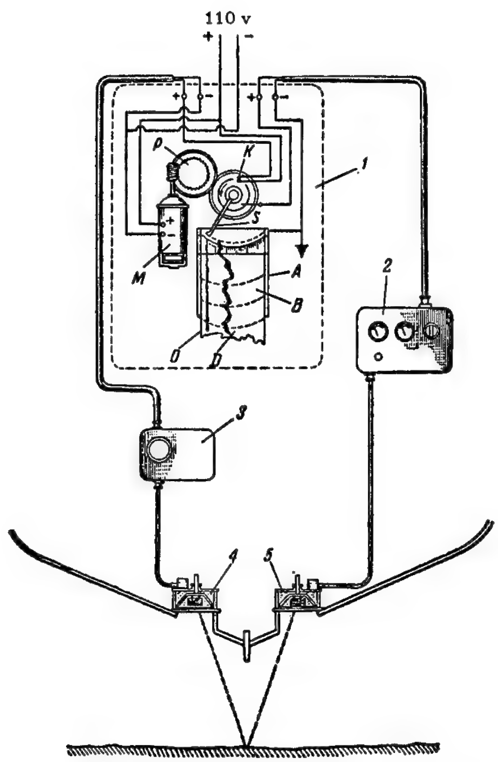


Figure 210. Diagram of an undersea sonic depth finder

1) recording instrument; M) d.c. motor; P) reduction gear; S) pen mounted on a drum; K) contacts; A) grounded metal plate; B) graphite paper; O) zero line, corresponding to submersion of the equipment; D) recording of the bottom profile; 2) echo-signal amplifier; 3) contact box with starting relay, 4) magnetostrictive source, 5) magnetostrictive detector

For every revolution of the drum, the pen S traverses a definite path over a special paper (impregnated with iodine salts or covered with a thin layer of graphite), touching it lightly. At a constant speed the motor draws the paper over the base-plate A. When the pen crosses the zero line O, contacts K are opened and the source sends an ultrasonic pulse downward from the ship; at this moment the pen S receives a portion of the voltage of the electrical pulse, and the paper under the pen is blackened (with iodine paper the salts are decomposed and iodine liberated, while with graphite paper the thin gray layer is cut through by the discharge, uncovering the black underlayer), causing a short black mark to appear. After being reflected from the bottom, the ultrasonic pulse is picked up by the magnetostrictive detector 5; after passing the amplifier 2, the electric pulse passes to the pen S, causing the appearance of another black mark. During the passage of the ultrasonic waves to the bottom and back, the pen will describe an arc which is proportional to the depth of the sea under the ship. Line O on the paper strip corresponds to the zero line, while line D represents the profile of the sea bottom under the ship. The record thus

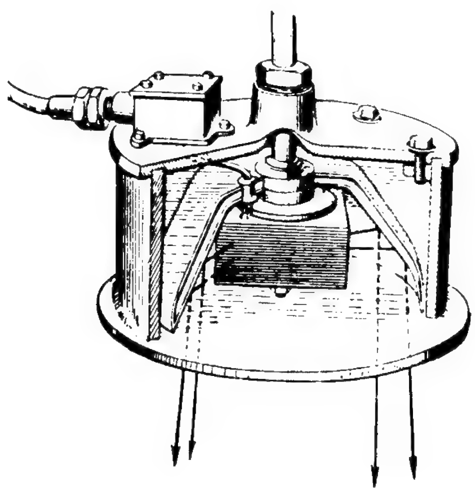


Figure 211. Design of a magnetostrictive source (detector) for a sonic depth finder

The hollow conical reflector is shown; the stack of magnetostrictive plates is immersed in the water.

obtained of the depth beneath the moving ship is known as a bathygram. The depths are read directly from the paper strip in meters, according to a previously determined scale. By observing the readings of the sonic depth finder, a navigator always knows the depth of the sea beneath the ship (usually, in addition to the recording, the instrument readings are observed directly on a dial with neon bulbs which light up at the spot on the dial corresponding to a given depth).

Sonic depth finders play a very important part in sea and river navigation. In rough weather, when the ship's coordinates cannot be accurately determined by astronomical (stellar) methods, depth-finder data may contribute very substantially to determination of the position of the ship. At present, for nearly all the seas depth charts are available which have been compiled by means of sonic depth finders, and these charts make it possible to determine a ship's position. It is also interesting to note that the greatest depths (e.g., the Pacific Chasm*, with a depth of $11{,}034 \pm 50$ m) have been discovered by means of sonic depth finders.

Ultrasonic depth finders can operate while the ship is under way. During its motion the ship becomes a source of numerous sounds, and the noise of the engines and various auxiliary machines is transferred through the ship's hull into the water. During the ship's motion eddies are formed in its wake, and also serve as a source of noise. However, the main source of noise are the propellers, whose rotation produces noise of high intensity. Similar to airplane propellers, the rotation of a ship's propeller produces rotation sound. This sound is easily heard while diving near a passing motor boat. All these sounds arising during the motion of the ship make it impossible to operate at sonic frequencies, because they create a high level of background interference. Hence, the use of ultrasonic waves is also profitable from this point of view. Though the spectrum of ship noise contains ultrasonic components, they are not as sharply defined as low frequencies; and hence the noise level is lower at ultrasonic frequencies.

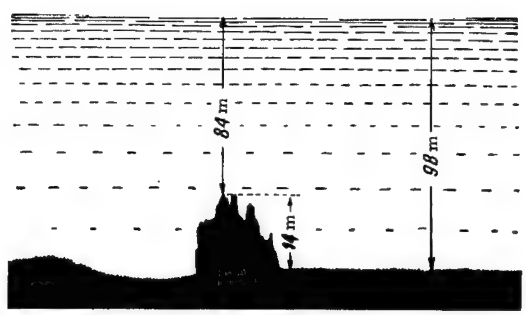


Figure 212. Outline of the sunken Lusitania, recorded by a sonic depth finder

By means of depth finders it is not only possible to measure the depths of seas and rivers, but also to find sunken ships and detect submarines hiding at the bottom. Figure 212 represents the outline of the sunken ocean-going vessel Lusitania, obtained by means of a sonic depth finder. The depth finder has also found important applications for the fishing industry. By means of ultrasonic waves it is possible to locate schools of food fish. The bladders of the fish, which contain air, reflect (scatter) ultrasonic waves to a sufficient degree that an echo returning from a school of fish may be recorded by a depth finder. Figure 213 represents

* The Mariannas deep-sea chasm, 11°20.9' N. lat., 142°11.5' E. long.

a depth-finder recording of a school of horse mackerel; from such a recording, an idea may be formed of the size of the school. On large trawlers, the depth finder may be quite useful in searching for food fish.

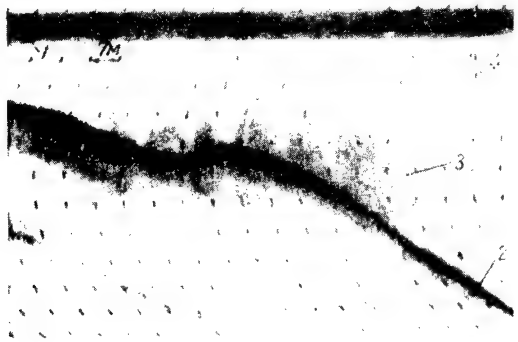


Figure 213. A recording of a school of horse mackerel 480 m long

Obtained by A.S. Shein over a sloping bottom in the vicinity of Cape Aiya (Black Sea). 1) sea surface, 2) sea bottom, 3) school of mackerel

Horizontal Echo Sounder. Undersea Sound Ranger [Sonar]. By sending out ultrasonic pulses in the horizontal direction, it is possible by means of an echo sounder to locate enemy submarines, mines, and obstacles such as icebergs and underwater reefs. Such horizontal echo sounders are called undersea sound rangers, or sonar.

Modern sonar is a powerful weapon both for antisubmarine warfare and for directing submerged submarines toward a target. While radar makes it possible to detect enemy surface craft and airplanes, and to direct fire against them, sonar enables detection of a submarine and, once it is located, its destruction by depth charges. A sonar unit mounted on a submarine can give the direction and distance of a ship, i.e., the data necessary for a torpedo attack.

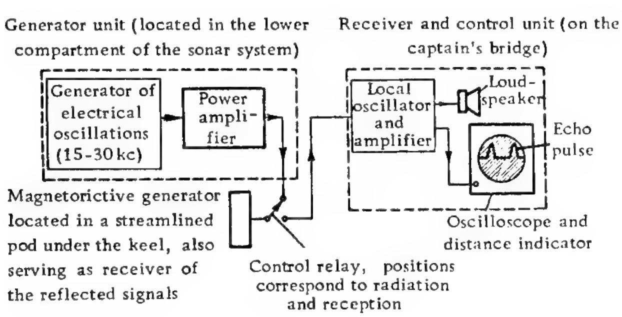


Figure 214. Basic design of a sonar system

The basic scheme of sonar is shown in Figure 214. Electrical oscillations with a frequency of 15 to 30 kc are applied to an amplifier, which amplifies the power to one kilowatt. In the form of short pulses the alternating voltage from the anplifier is applied, via a special gating relay, to the magnetostrictive generator. The duration of the pulses is about 0.1 sec or less, with intervals of several seconds between pulses; the length of the intervals may be changed at will, depending upon the distance to the object.

The magentostrcitive generator, which also serves as the receiver (Hydrophone) for the reflected signals, is placed in a collapsible streamlined pod attached to the underside of the ship (Figure 215). The source-receiver can be rotated through a circular arc, as it searches for a target.

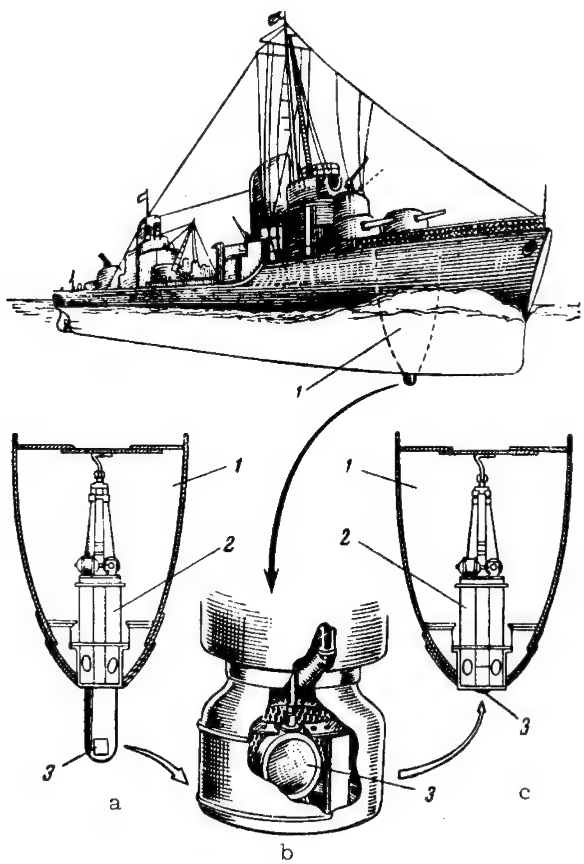


Figure 215. Mounting of a sound-ranging source (receiver) on a ship

1) Compartment for acoustic system; 2) device for retracting the sound-source (receiver) system into the ship; 3) sound source (receiver) within the streamlined pod; a) operating position; b) streamlined unit retracted into the ship; c) head of the streamlined unit, sound source (receiver) shown in cross section.

At the moment a signal is sent, the source-receiver is connected, via the contacts of the gating relay, to the output of the power amplifier; immediately after sending of the signal, it is switched to the reflected-signal amplifier; in this way, after the pulse has been sent, the whole system is made ready for reception.

The amplifier of the echo signal is usually a heterodyne amplifier, the output of which is a low-audio-frequency signal (about 600-1000 cycles), instead of an ultrasonic signal. The amplified oscillations of the reflected signal are mixed with (added to) the signal from the local oscillator (heterodyne). If the signal frequency is f_c and the heterodyne frequency is f_r, then as a result of mixing, frequencies $2f_c$, $2f_r$, $(f_c + f_r)$, $(f_c - f_r)$ etc., appear. A filter placed after the mixer selects the difference frequency $f_c - f_r$, and does not pass any other frequencies. For a signal frequency of 25 kc and a heterodyne frequency of 24 kc, a filter tuned to the difference frequency $f_c - f_r$ provides a 1 kc output signal.

Thus, the ultrasonic echo signal is transformed into an audio-frequency signal. From the output of the heterodyne amplifier, the voltage is sent to the loudspeaker and to the vertical-deflection plates of the electronic oscilloscope. From the image on the oscilloscope screen, it is possible to measure the distance to the reflecting object, provided a suitable sweep is chosen, the beginning of which coincides with the time of sending of the pulse. The difference, as compared with the use of the pulse method discussed above for measurement of the ultrasound velocity in air (or of the distance, if the velocity is known), consists in that in the previous method the repetition frequency was a few dozen cycles per second, while here only a single pulse may be sent during several seconds. Hence, in order to obtain a clear image on the oscilloscope, long-persistence tubes are used.

But the oscilloscope alone is not sufficient to describe the complicated phenomena taking place during propagation of sound in the sea. Often it is difficult to determine the cause of an incoming signal — whether it is produced by a submarine, by a mine, by a large fish in motion, by reflection from water layers containing many air bubbles, or even by reverberation effects.

An experienced undersea-acoustic-system operator can draw many important conclusions from the properties of the signal which he receives through the loudspeaker; the mode of its increase and fading out, as well as its "tone color" and its changes in tone, indicate the nature of the reflecting object — whether it is approaching, receding, or stationary. A change in the pitch of the arriving echo-signal indicates that the observer and the object are moving in relation to one another. This phenomena is known as the Doppler effect, and is often encountered in everyday life. Everyone has certainly noticed the change in pitch of the whistle of an approaching locomotive, or similar effects for automobile horns or the sounds of passing airplanes. The explanation of this phenomenon is quite simple.

Imagine that we are standing on a stationary underground escalator, with another escalator moving upward next to our own. Let us assume that two rows of passengers pass us every second. If we compare them to compression waves, then we may say that radiation occurs with a frequency of 2 cycles per second; in other words, two waves meet us every second.

Now let our own escalator begin to descend with the same speed with which the other one moves upward. It is clear that now four rows of passengers rather than two will move past us every second. In this way, the frequency of the "source" has been doubled. It is obvious that, when an observer moves not toward the source but away from it, the frequency becomes lower.

In a way similar to the above example, the pitch of sound becomes higher when a source moves toward a stationary observer or when an observer moves toward a stationary source; but it is lowered when the source moves away from the observer. The frequency of the sound heard by the observer is

$$f = f_s \pm f_s \frac{v}{c} = f_s \left(1 \pm \frac{v}{c}\right),$$

where f_s is the source frequency, v is the source velocity, and c is the velocity of sound. The positive sign is taken if the source moves toward the observer, while the negative sign corresponds to a source moving away from the observer. When an undersea-sound-system operator receives a reflected signal which corresponds in its characteristics to a reflection from a submarine, he can, by the change in tone of the loudspeaker signal, decide whether the submarine is approaching or moving away from the ship.

Several seconds may pass between two pulse transmissions in search of submarines, so that it would be very difficult to notice a change in the pitch of the signals, if the tone of the transmitted signal were totally absent during the interval between pulses. Here the operator is aided by the reverberation in the sea. Actually, the operator hears, after the signal has been sent, the tone of this signal owing to reflections at air bubbles and inhomogeneities in the sea water; the reverberation fades gradually.

Since the air bubbles and the inhomogeneities in temperature and salinity of the sea water are at rest (or move only very slowly), the Doppler effect is in this case absent and the frequency of the reverberation tone does not change. It is true that, if the ship containing the sonar system is in motion, then of course the frequency of the reverberation tone is not the same as the frequency when the ship is anchored; but, provided the ship moves at a constant speed, the reverberation tone remains unchanged.

When an undersea-sound-ranger detects reflections from an object, the reflected signal returns against a background of reverberation; and if the object itself is in motion, then there is a change in the pitch of the echo, as compared with the pitch of the reverberation. In this way, reverberation, although an undesirable phenomenon which interferes with detection, is at the same time not devoid of usefulness.

The range of a sonar system depends, to a very great degree, upon the conditions of sound propagation in water. The average range of a system for detecting a submarine is several kilometers, but it can be lowered considerably by strong refraction effects and a high reverberation level (Figure 216).

It may seem possible to increase the range by increasing the intensity of the ultrasonic pulse radiated. However, it is impossible to increase considerably the acoustic power of the pulse; as explained below (Chapter VIII), at high ultrasonic intensities the phenomenon of cavitation arises. But even if a sizeable increase in power were feasible, the range would still be increased only very little. In the presence of strong refraction, when the

beams are bent downward away from the surface of the water, an increase in power (in order to increase the range) is even less effective.

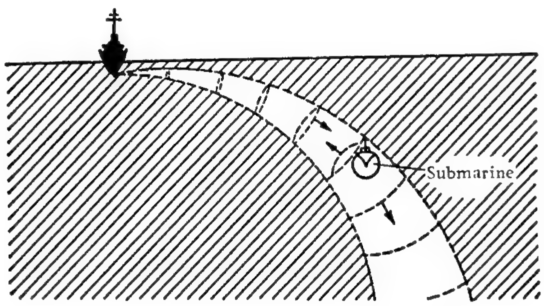


Figure 216. Influence of refraction on the operation of a sonar system

Consequently, though sonar is an effective means of detecting submarines and surface ships, the complexity of marine conditions often makes the sound ranging difficult. Acoustic methods of undersea detection can be effectively utilized only by an experienced operator, with a good knowledge of undersea sound propagation and of the sound-ranging apparatus of the ship or submarine.

Noise-Type Ranging Systems. As already mentioned, every moving ship is a powerful sound source. A ship may be located by its noise in the same way as an airplane is located by the sounds of its propeller and engine. A sonar system works by sending out ultrasonic pulses and receiving echo signals reflected from the object; thus it is an active means of sound ranging. Noise-type ranging is passive sound ranging. By detecting the noise made by an enemy ship or submarine, it is possible to find the direction of the source of the noise. Then, taking readings from two points situated at some distance from each other (along a baseline), it is also possible to find the distance to the ship being located.

Different types of ships have different noise spectra, but for nearly all ships the most common noise frequencies are below a few thousand cycles. For this reason noise ranging usually employs audible frequencies. This involves numerous difficulties. For example, it is necessary to stop the engines of the ship doing the detecting, since they create noise which forms a high interference level*. But even if the detecting ship or submarine is stopped, the sea itself is a source of various types of sound interference. An undersea-sound-system operator hears the noise of waves breaking and of waves beating against his own ship. In the vicinity of the shore the noise produced by the surf beating against the rocks is heard. Considerable experience is necessary in order to isolate the sounds belonging to a faraway ship from the sum total of sounds arriving at the hydrophone**.

* Recently noise ranging has also been performed at ultrasonic frequencies (a few tens of kc), without stopping the ship, since at ultrasonic frequencies hydrodynamic interference and propeller noise are less pronounced than at audible frequencies.

** There also exist special methods for selecting a useful signal from a random background noise, the level of which exceeds that of the signal itself.

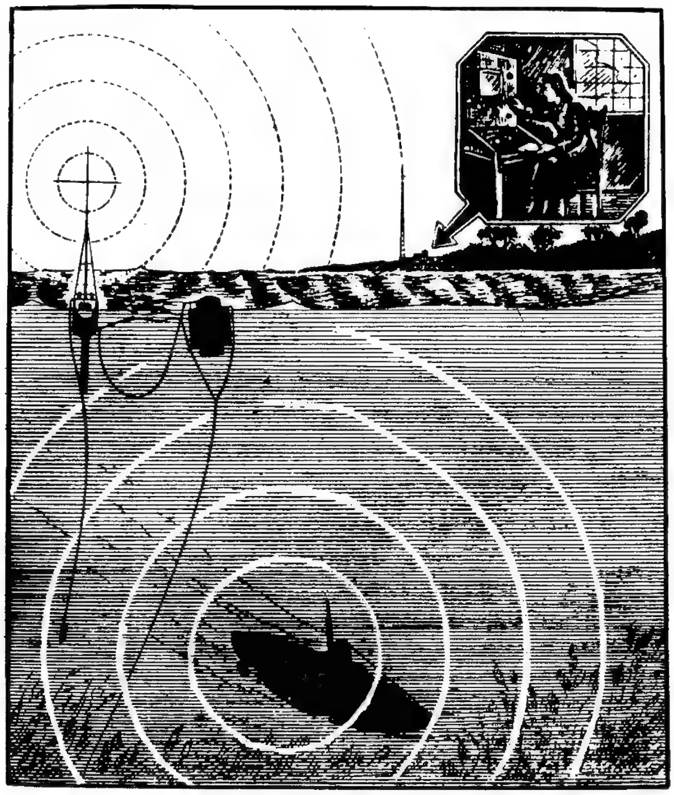


Figure 217. A radio buoy with supply batteries

Many sea animals, including many species of fish, have turned out to be the source of various noises. The saying "dumb as a fish" proves to be wrong, and some fish produce noises with an intensity 100 db above the threshold of hearing ($2 \cdot 10^{-4}$ bar).

The frequency spectrum of the noise produced by fish depends on many factors and has a range up to a few thousand cycles. Besides fish, there are various other species of sound-producing marine animals. For instance, 50 meters deep in tropical seas some shrimp have been found which produce clicking noises. In the spectrum of the noise produced by these shrimps, high audible frequencies predominate, extending from a few thousand cycles to 15-20 kc. The level of the noise produced by these shrimps is 20-40 db higher than that of other sea noises. Measurements show that the maximum sound pressure due to the clicking of a single shrimp is 200 bars at a distance of 1 m.

The detection of noise can not only be performed from a ship, but also a large distance away from the part of the sea where the noise source (enemy submarine or surface craft) is situated. This is achieved by means of a radio buoy. From the buoy a hydrophone is lowered to the desired depth, where it detects sounds in the sea; the hydrophone is connected through an

amplifier to a small radio transmitter (Figure 217). This transmitter transmits over long distances the sounds coming in from the sea in its vicinity. If many radio buoys are placed throughout the sea, the radio receivers of a coastal station will give information about the "acoustics" of a vast and distant region of the sea.

In contemporary naval warfare noise ranging has played an immense part. Besides being one of the basic methods of antisubmarine warfare, noise ranging serves as a powerful means of detecting surface ships by the submarines themselves, enabling them to find their targets without surfacing. Sonar is the eyes of a submarine; and the noise ranger is its ears.

Chapter VIII

ULTRASONIC WAVES OF HIGH INTENSITY. SHOCK WAVES

§ 1. Powerful ultrasonic waves

Ultrasonic Waves of High Intensity. In recent years, there has been an ever-increasing interest in high-intensity ultrasonic waves in liquids. This interest is the result of the large number of practical ultrasonic applications connected with the effects of ultrasound on materials — made possible through the use of ultrasonic vibrations of medium and high intensity. At the same time, during the propagation of intense ultrasound, both in liquids and in gases, numerous new physical phenomena appear which are of great scientific interest.

Acoustic quantities for a plane ultrasonic wave propagating through water

I, wt/cm^2	p, atm*	v, cm/sec*	x, cm* $f=10^6$ cps	w, cm/sec^2* $f=10^6$ cps	w in g-units $f=10^6$ cps
1	1.7	11	$2\cdot10^{-6}$	$7\cdot10^{7}$	$7\cdot10^{4}$
10	5.1	33	$6\cdot10^{-6}$	$2\cdot10^{8}$	$2\cdot10^{5}$
100	17	110	$2\cdot10^{-5}$	$7\cdot10^{8}$	$7\cdot10^{5}$
1000	51	330	$6\cdot10^{-5}$	$2\cdot10^{9}$	$2\cdot10^{6}$
5000	120	800	$1\cdot10^{-4}$	$5\cdot10^{9}$	$5\cdot10^{6}$
10000	170	1100	$2\cdot10^{-4}$	$7\cdot10^{9}$	$7\cdot10^{6}$

* The table shows sound-pressure amplitude p, velocity v, displacement x, and acceleration w.

Let us agree to consider intensities from a fraction of a watt/cm^2 to a few watts/cm^2 as medium ultrasonic intensities in a liquid. Intensities over 10 watt/cm^2 will be regarded as high. In order to have a better idea of the quantities associated with intense ultrasonic waves, a table is given in which (for various intensities, I, of a plane ultrasonic wave propagating in water) have been computed, according to the formulas for a plane sound wave (Chapter II, § 4) the following quantities: the sound-pressure amplitude p in atmospheres, the acoustic particle velocity v, the displacement x, and the acceleration w in cm/sec^2 as well as in gravity units (x and w were computed for a frequency of 1 mc).

Figure 218. Ultrasonic fountain in water
Frequency 1.5 mc, intensity 15 watt/cm^2.

From the above table it is seen that, even for the comparatively low intensity of 1 watt/cm^2, the sound pressure is 1.7 atm (for a standing wave the sound pressure is twice as great at the pressure maxima); for a very high intensity — 1000 watt/cm^2 — it is 51 atm. At a frequency of 1 mc maximum accelerations are produced (which change in direction twice during each period) of respectively $7 \cdot 10^4$ and $2 \cdot 10^6$ times the acceleration of gravity!

It should be noted that it is possible at present to use quartz plates to produce in water continuous plane waves (sinusoidal vibrations) with frequencies of about 1 mc (without focusing), which have intensities of 50-60 watts/cm^2.

A. K. Burov and his co-workers obtained, for 1.5 mc waves radiated into water during a short interval (a few seconds), a broad beam with an intensity of 200-300 watt/cm^2. Using pulse techniques, an intensity up to 500 watt/cm^2 was achieved.

If an ultrasonic beam is directed upward, toward the surface of the liquid, the liquid expands; and with a further increase in intensity, a fountainlike effect appears. Thus, for an ultrasonic intensity in water of 150--200 watt/cm^2 and a frequency of 1.5 mc, the fountain reaches a height of 40-50 cm, giving a quite effective demonstration.

Figures 218 and 219 are photographs of fountains produced with intensities of 10-15 watt/cm^2 at a frequency of 1.5 mc.

Figure 219. Ultrasonic fountain in transformer oil

Frequency 1.5 mc, intensity 15 watt/cm^2.

The first photograph shows a fountain in water; the second photograph shows one in oil.

In the first case, in addition to the drops of water thrown out, an extremely fine mist is seen to be formed; it is also interesting to note that in both photographs chains of separate droplets are visible, with equal separations between the droplets.

Figure 220 shows the effect of intense ultrasound (about 100 watt/cm^2) upon a piece of plexiglas. During a second's irradiation, the piece is so heated as to melt at the edges; the pattern of the melting follows, to some extent, the configuration of the ultrasonic field. In this way, the extent of nonuniformity of the ultrasonic field may be estimated from the photograph. The melting of plexiglas thus represents one method of visualizing an intense ultrasonic field.

An ultrasonic beam of high intensity (50 watt/cm^2 or above), in liquid, which even for a few seconds strikes living tissue (e.g., the human body) causes very severe and painful burns and destruction of the tissue.

The intensity of ultrasound may be increased considerably by the use of focusing systems — lenses or large concave mirrors, with surfaces consisting of a mosaic of quartz plates or of barium titanate ceramic. Thus, if a lens increases the intensity by a factor of 100, a beam of

50 watt/cm^2 will reach an average intensity at the focal point of 5 kw/cm^2*. To obtain high intensities in the megacycle range, it is necessary, as a rule, to use piezoelectric quartz plates. It is also possible to use barium titanate plates**; however, the highest intensities of continuous radiation so far achieved have been obtained with piezoelectric quartz plates.

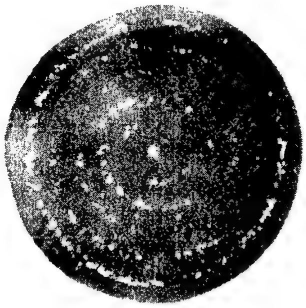


Figure 220. Piece of plexiglas which has been subjected to ultrasonic radiation from a circular quartz plate (negative print)

A vibrating X-cut quartz plate, radiating in both directions, provides the following sound power per unit area:

$$J \cong \frac{7.2 f^2 U_{\mathrm{amp}}^2}{\rho_0 c_0} 10^{-14} \text{ watt/cm}^2,$$

where $\rho_0 c_0$ is the specific acoustic impedance of the medium; f is the natural frequency of the plate; and U_{amp} is the amplitude of the voltage applied to the electrodes of the plate.

With the plate adjusted so that it radiates only from one side ("single source", see below), the displacement amplitude of the plate is doubled, and the sound intensity increases by a factor of four. The following formula is easy to remember for the intensity of the unidirectional radiation of a quartz plate vibrating in water:

$$J = 2 f^2 U_{\mathrm{amp}}^2 \text{ watt/cm}^2,$$

where f is in megacycles and U_{amp} is in kilovolts.

Producing Powerful Ultrasound. Powerful radio oscillators are required in order to produce high ultrasonic intensities. Suppose it is required to obtain an intensity of 500 watt/cm^2 from a quartz plate 5 cm in diameter. The area of the plate is about 20 cm^2; hence, in order to generate the required power it is necessary to have a high-frequency oscillator with an output of 10 kw, assuming that all the power supplied by the oscillator is transformed into ultrasound (100% efficiency). Actually, the efficiency is 50-70%, so that the oscillator output must be taken as 15-20 kw. An oscillator of such capacity for the radiation of ultrasound in continuous waves becomes a rather impressive piece of equipment.

With a quartz plate as the ultrasound source, the voltage applied to the electrodes of the plate must be high, of the order of several kilovolts. Thus, according to the above formulas, in order to obtain in water an intensity of 10 watt/cm^2 at a frequency of 1 mc it is necessary to supply to the plate a high-frequency voltage of about 4 kv; at a frequency of 400 kc, about 8 kv is required. It is therefore necessary to take safety precautions against possible electrical breakdown in various parts of the equipment. (The most troublesome place is the mounting of the quartz plate.) Usually, the plate

* Neglecting losses in the lens, in reflection from the lens, and in the transformation of longitudinal into transverse waves. This elementary calculation does not take account of one very important phenomenon —a large increase in ultrasound absorption at high intensities (see below).

** For high intensities and continuous radiation of ultrasound barium titanate plates are strongly heated, and require good cooling. But they are more convenient than quartz plates, since they do not require such high voltages as the latter (in order to obtain a given ultrasonic intensity).

is placed in a special case filled with transformer oil. The electrodes are deposited on the plate in such a manner as to leave a strip open along the edge. The transformer oil must be dried under vacuum for several hours before pouring it into the case, to remove water and gas. This results in a considerable increase in the dielectric strength, so that the breakdown voltage is increased substantially. The depositing of electrodes on a plate which is to generate considerable acoustic power requires a special technique. If the plate is coated with silver or chromium by the usual method of vaporizing the metal under vacuum* (or by galvanic methods of depositing the electrodes), then during intense vibration the metal layer peels off, and metal dust gets into the transformer oil, immediately leading to electrical breakdown and to destruction of the plate. A method has been developed of applying the electrode coating as a combination of metal layers (the metal is vaporized under vacuum) and layers of BF-type glue; this system of layers has a "tight grip" on the plate, permitting operation at high ultrasonic intensities.

Mounting the Quartz Plates. Connection of the Quartz Source to the Oscillator. There exist very many different systems of mounting the quartz plates. It may be said that everyone designing ultrasonic apparatus to be used with liquids has his own particular method of mounting the quartz ultrasonic source, and that there are still no generally accepted design rules. We shall mention just a few types of mountings for quartz plates suitable for producing ultrasound of medium and high intensity.

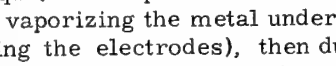


Figure 221. Mounting for quartz plates

1) Plastic cup, 2) quartz plate; 3) brass high-voltage connection; 4) screw for tightening the high-voltage connection; 5) air "cushion"; 6) brass cover, serving at the same time as a ground contact; 7) lead seal; 8) rubber gasket; 9) wall of liquid container; 10) movable ring attaching the radiator to the container wall.

Figure 221 is the diagram of a simple mounting for a quartz plate, while Figure 222 is a photograph of it. With such a holder an ultrasonic intensity up to 10-15 watt/cm^2 may be obtained in water, at a frequency of 1.5 mc.

The air "cushion" operates as a reflector from the reverse side of the plate, so that the plate acts as a unidirectional radiator (into the liquid). The electrodes are deposited on the plate by the method described above, using BF glue applied up to 2-4 mm from its edge (the plate diameter being 30 mm). A plate with this type of mounting can be subjected to a high-frequency field of 1.5-2 kv/mm.

Figure 223 represents diagrammatically another type of mounting, in which the plate is placed in a case of transparent plastic (e.g., plexiglas), filled with purified transformer oil. The outlet through which ultrasound leaves the case is separated from the surrounding liquid by a thin nylon film, which is "transparent" to ultrasound. Instead of the nylon film a

* The quartz plate, together with a piece of the metal to be vaporized, is placed in a vacuum chamber. The metal is vaporized by heating it with electric current, and particles of it are deposited on the surface of the quartz plate (carefully cleaned of all dirt), forming a thin layer.

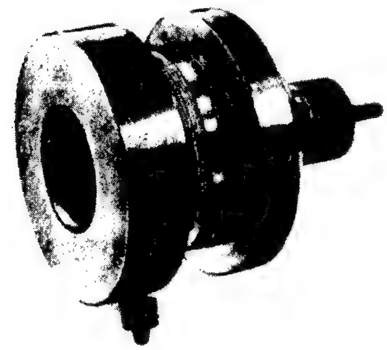


Figure 222. Photograph of the mounting in Figure 221

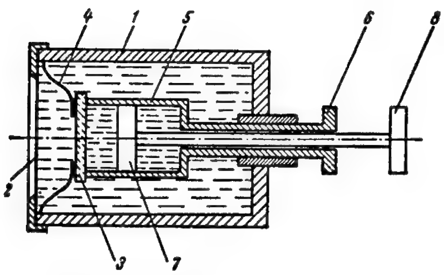


Figure 223. Quartz-plate mounting for high ultrasonic intensities

1) Case; 2) nylon film; 3) quartz plate; 4) curved spring; 5) cup; 6) lever and piston rod, regulating the pressure on the plate; 7) reflector with air cushion; 8) lever and piston rod, regulating the position of the reflector

glass plate may also be used. If the thickness of such a plate is $\frac{\lambda}{2}$, where λ is the ultrasonic wave length in the plate material, then the plate is "transparent" to ultrasound. The quartz plate is pressed against a movable piston — a cylinder serving as the ground electrode — by means of curved spring of special shape, which serves as the second electrode. Optimum pressure on the plate is achieved by moving the piston rod. The second piston rod has at its end the reflector with the air layer; maximum radiation from the plate occurs when the distance between the plate and the reflector is equal to an odd number of quarter wave lengths of ultrasound in oil. In this case, the waves from the reflector will be in phase with the vibrations of the radiating side of the plate.

In producing ultrasound in general, and intense ultrasound in particular, an essential problem is the impedance matching between the output stages of the oscillator and the ultrasonic source. Figure 224 shows the most

common circuits for connecting the quartz ultrasonic source to the oscillator, in order to produce powerful ultrasound. In diagram a, the oscillating circuit LC in the output stage of the oscillator is inductively coupled to the quartz circuit $L_1 C_1 K$; in diagram b, this connection is made by means of an autotransformer LL_1. The latter diagram is less convenient because the leads to the quartz source are at all times under high potential with respect to ground. For this reason, blocking capacitors C' and C'' are often introduced into this circuit, and one of the quartz electrodes is grounded.

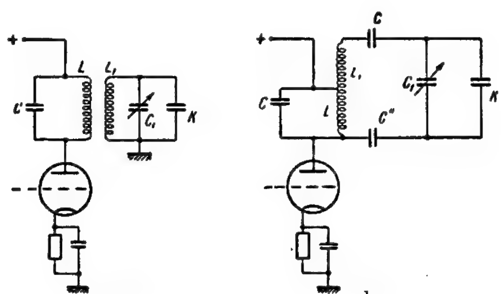


Figure 224. Diagrams for connecting a high-frequency voltage source to a quartz plate

When working with high voltages, 10 kv and above, electrical breakdown may occur in the quartz mounting, due to a whole series of causes, partly discussed above. Most often this occurs at the edges of the plate, because of the oil becoming dirty; heating of the oil during continuous radiation by the plate also plays a part. As a rule, at electrical breakdown the plate cracks and becomes useless. An automatic safety device is used to prevent this, whereby at the beginning of breakdown high voltage is removed from the electrodes of the plate.

During the propagation of ultrasonic waves in liquids, at medium and especially at high intensities, several important and interesting phenomena occur which are discussed below.

§ 2. Acoustic "wind"

Acoustic Current ("Acoustic Wind"). Faraday (1831) was the first to note the air currents formed during the vibration of a membrane. Savart discovered that, as opposed to the case with heavier particles (e.g., grains of sand) which form Chladni patterns*, an extremely fine powder (such as talcum powder) does not collect along the node lines. The fine powder collects above the vibrating membrane in a cloud, which after a short

* Different portions of the membrane surface vibrate with different amplitudes; the mode of vibration of a membrane may be different, depending upon its geometrical shape, its dimensions, the manner in which it is clamped along its edges, and the excitation frequency. If grains of sand are poured onto the membrane surface, then during vibration they collect at points with a minimum displacement amplitude. Beautiful symmetrical patterns of various kinds are formed; these are known as Chladni patterns, since the physicist Chladni was the first to observe them.

while distributes itself over points on the membrane where the vibration amplitude is maximum. Faraday explained this phenomenon as due to the action of air currents, arising, in his opinion, at these maximum-vibration points. In a vacuum no such phenomenon occurs. Furthermore, it was discovered (by Dvořák and, independently, by Meyer) that any air resonator acted upon by a sufficiently powerful sound is subject to a force directed from inside its mouth. Four such resonators, mounted together on steel rods, similarly to a cup anemometer, can be made to rotate by the action of a powerful sound. Air movement of medium strength is observed during intense vibrations in a Kundt tube and in many other phenomena.

At frequencies in the megacycle range, this "wind" or current is easily observed (e.g., in water), beginning at intensities of a fraction of watt per cm^2, provided that the diameter of the quartz plate is less than that of the vessel.

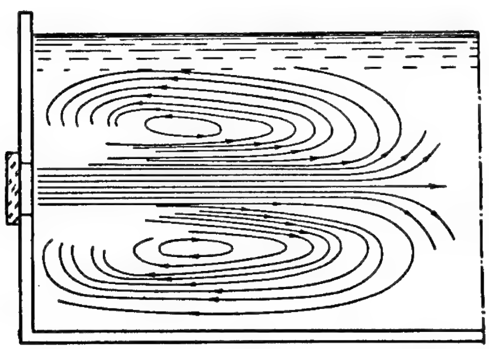


Figure 225. Diagram of the streamlines in an acoustic current

For observation of the current, some drops of ink may be added to the liquid, or some aluminum powder; with a lateral illumination of the vessel, it is easy to measure the velocity of the current, by means of a photograph with known exposure time, since individual particles in the liquid show up as dashes in the photograph, and the length of these dashes indicates the velocity. Figure 225 shows a diagram of the streamlines; this pattern may be observed by photographing illuminated, moving aluminum-dust particles. It can be seen that, along with the current of liquid away from the plate, a countercurrent also exists. Obviously, if a quartz plate radiates sound into a tube with a diameter less than that of the plate, no current can arise, since in this case there is no possibility of a countercurrent. The velocity of the acoustic current is not great; it is usually from a fraction of one cm/sec to a few cm/sec for medium intensities, and reaches a few tens of cm/sec for high ultrasonic intensities.

A very interesting method for the observation of the streamlines in an acoustic current can be demonstrated at the boundary of two immiscible liquids, glycerine and vaseline oil*. When a drop of colored water (ink

* This was done by L.K. Zarembo and V.V. Shklovskaya-Kordi at the Laboratory of Anisotropic Structures of the U.S.S.R. Academy of Sciences.

may be used) is lowered into the vaseline oil, then, since the density of the oil is less than that of water, the drop slowly sinks down in the shape of a sphere, and stops at the surface of the glycerine layer, because the latter is heavier than water. After a short while, the drop bursts under the action of surface-tension forces, forming a comparatively thin, colored disk on the glycerine surface. If the center of the quartz plate coincides with the oil-glycerine boundary and the disk is close to the plate, then during vibration of the plate, the disk begins to move under the action of the acoustic current. With time, the disk assumes more and more a form showing the transverse distribution of velocities in the current. Thus, at the beginning the disk resembles a horseshoe (Figure 226a); the pattern

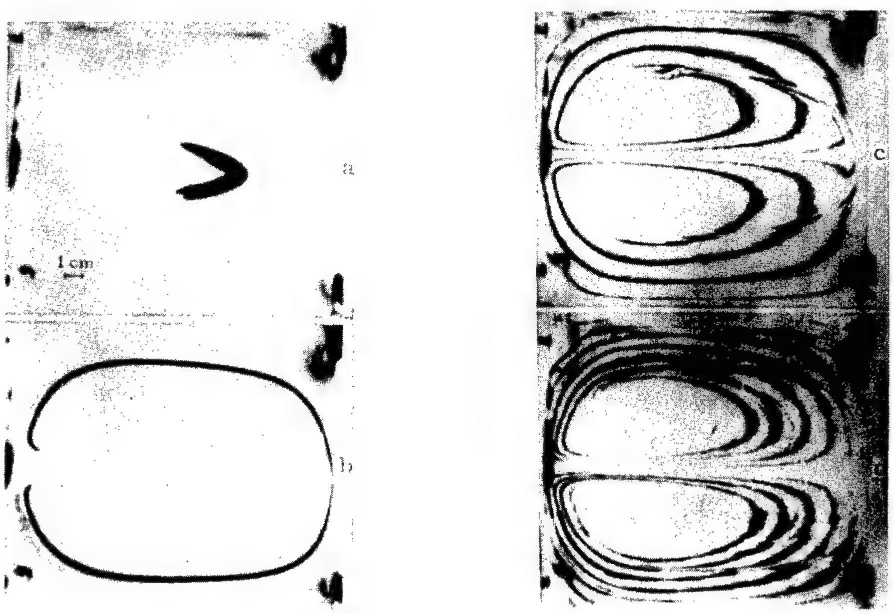


Figure 226. Acoustic current at glycerine–vaseline oil boundary; frequency 1.5 mc, voltage at the quartz plate 600 v; the following periods of time have elapsed:

a) Zero time; b) 3 min; c) 9 min; d) 24.5 min (quartz plate at center of left side of photographs).

in Figure 226b occurs 3 min after that in Figure 226a. The photographs were taken from above, through the vaseline-oil layer, with the quartz plate operating at 1.5 mc. The intensity was 4 to 5 watt/cm^2.

The theory of acoustic wind is quite complicated and is far from being completely developed. This theory, formulated by the American physicist Eckart, states that, if an acoustic current is caused in a tube of radius R by the vibrations of a quartz plate with a radius $r \ll R$, while at the same time $r \gg \lambda$, where λ is the ultrasonic wave length (the wave, to a first approximation, is considered plane), then the current velocity v along the axis of the ultrasonic beam is given by the formula

$$v = \frac{\omega^2 r^3 I}{\rho c^4}\left(2+\frac{\eta'}{\eta}\right)G_0.$$

Here $\omega = 2\pi f$ is the angular velocity; ρ is the density of the liquid (gas); c is the ultrasound velocity; I is the ultrasonic intensity; η is the coefficient of shear viscosity; and η' is the coefficient of volume [bulk] viscosity. The quantity G_0 is a number which depends on the ratio $\frac{r}{R}$, i.e., it represents the geometrical characteristics of the system*. The formula for the velocity of the current pertains to the case of a plane wave and holds true only for comparatively low intensities. It is derived on the assumption of laminar flow; but in reality the flow becomes turbulent (in water, for instance) beginning with intensities of a few watt/cm^2 in the megacycle range, so that the formula no longer applies. In this case, the current velocity v is no longer proportional to the intensity but is proportional to its square root. It is of interest to note that the acoustic current becomes turbulent at comparatively small Reynolds numbers $R = \frac{rv}{\nu}$ (where ν is the kinematic viscosity), about one order of magnitude smaller than for the flow of a liquid in a pipe. Possibly this is related to the fact that the field in the vicinity of the radiating quartz plate is extremely nonuniform, with sharp gradients of acoustic pressure both along and across the beam.

For low intensities, the velocity of the acoustic current is proportional to the ultrasonic intensity and to the square of the frequency. Also, the lower the density of the medium and the sound velocity in the medium, the greater is v. For this reason, in gases, where ρ and c are small in comparison with liquids, the acoustic current must be taken into account even at low audible frequencies, while in liquids a noticeable current arises only at ultrasonic frequencies. In order to form some idea of the physical mechanism of the formation of the acoustic current, it may be considered (and this is the viewpoint held by most writers) to arise, in the case of plane waves, as a result of the radiation-pressure gradient in the liquid, caused by the absorption of ultrasonic waves. The drop (gradient) in radiation pressure, caused by absorption, sets the liquid into motion. From this point of view, the velocity of the acoustic wind should be proportional to the absorption coefficient of the ultrasound, and this also follows from theory.

Measurement of the Absorption of Ultrasound by Means of the Velocity of the Acoustic "Wind". In this connection, it is interesting to note the following fact. It should be possible to determine the ratio of volume viscosity to shear viscosity by measuring, under given experimental conditions, the velocity of the acoustic current (with known values of ω, r, G, ρ and c), and by determining independently, by another method, the sound intensity I. Some time ago this was recognized as a means of determining η', and relevant experiments have actually been performed. In experiments carried out in 1948 by the American acoustician Liebermann the velocity was determined by means of the already described method of photographing the motion of aluminum-dust particles, while the intensity I was determined using a radiometer (see below). In such experiments, it is important to take into account the fact that the radiometer is not only acted upon by the radiation pressure, but also by the hydrodynamic pressure of the stream, which must somehow be eliminated**. This was achieved in the following way. At the beginning of the quartz-plate vibration, the radiometer shows a rapid deflection, and then almost stops. After some time, another, slow deflection is observed. This is easily explained in the following way. The initial deflection is caused by the action of radiation pressure, propagated

* The formula for G_0 is

$$G_0 = \frac{1}{2}\left(\frac{r^2}{R^2} - 1\right) - \log\frac{r}{R}.$$

** During radiometric measurement of the intensity of ultrasound, errors introduced by the acoustic current must always be taken into account.

at the velocity of sound; then to this is added the effect of the hydrodynamic pressure caused by the acoustic wind. In this way, it is possible to separate the two effects. However, this should be performed with care, since acoustic wind cannot be regarded as arising in the immediate vicinity of the vibrating quartz plate and then spreading out. This wind arises at any point at which a wave is being propagated*; however, some time is required for it to form (development time). Unfortunately, this problem has been little studied, and it is not possible to say exactly what this time is or upon what it depends. Experiments with acoustic wind have resulted in the determination of η'/η for several liquids; however, it has since been demonstrated that in these measurements the quantity actually measured was the over-all absorption coefficient, produced by the combined effect of η and η'. The method in itself does not represent a new, independent method of measuring the volume viscosity.

§ 3. Waveform distortion in ultrasonic waves

Waveform Distortion in Fluids. The equation of state for an adiabatic process in a gas (the propagation of sound is such a process) is the equation of the adiabatic curve:

$$\frac{p}{p_0}=\left(\frac{\rho}{\rho_0}\right)^{\gamma},$$

where, for example, for air $\gamma=\frac{c_p}{c_v}=1.43$. This is a nonlinear equation, since no direct proportionality exists between the change in pressure and the change in density. The change in density during compression is somewhat greater than that during rarefaction.

The theory of the propagation of plane sound waves in gases, excluding damping, but including the nonlinearity of the equations of motion and the equation of state, was given as long ago as the time of Poisson, and in a more complete form by the famous German mathematician Riemann. In this theory, in contradiction to the usual acoustical treatment, where the pressure amplitude is regarded as small (actually, as infinitesimal) in comparison to the mean pressure in the medium and where the acoustic particle velocity is considered to be small in comparison with the velocity of sound, such limitations were not introduced. In other words, the finite nature of the sound-wave amplitude was recognized, and consequently the nonlinearity of sound propagation. For this reason, sound (or ultrasound) waves which are sufficiently intense, and for which nonlinear effects become evident, are called waves of finite amplitude. Yet, these waves of finite amplitude are not yet shock waves. The latter will be discussed later and it will be seen that, due to the presence of "discontinuities", the equations of hydrodynamics no longer apply, and that only the conservation laws may still be used. Waves of finite amplitude occupy an intermediate region between shock waves and waves of infinitely small amplitude. It will be

* For this reason, it cannot be assumed that the introduction of a screen in front of the radiometer (e.g., a screen of nylon film), which is "transparent" to ultrasound but "opaque" to the acoustic current, will eliminate the current beyond the screen. The current will still be there, although its nature will be different, depending on the geometry beyond the screen and on the ultrasonic intensity. The presence of a screen does, however, weaken to some extent the effect of the acoustic wind upon the radiometer.

shown below that ultrasonic waves of finite amplitude propagating in a liquid may have the properties of weak, periodic shock waves.

According to the Riemann theory, the velocity of propagation c of a point on the wave, with respect to a stationary medium (an ideal gas), is given by the formula

$$c=c_0+\frac{\gamma+1}{2}v.$$

The points along the profile of the [particle] velocity v which have a maximum amplitude v_0 (Figure 227, point 1) move with a velocity $c=c_0+\frac{\gamma+1}{2}v_0$; those with a minimum amplitude (point 3) move with a velocity $c=c_0-\frac{\gamma+1}{2}v_0$.

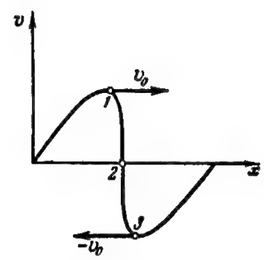


Figure 227. Profile of the [particle] velocity v in a wave of finite amplitude

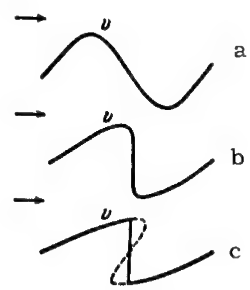


Figure 228. The change in the velocity profile as a wave of finite amplitude is propagated

Simultaneously, point 2 moves with a velocity c_0. As a consequence, there is a "twisting" of the sinusoidal wave and a discontinuity is formed (Figure 228). Of course, the case shown in Figure 228c (dashed line) cannot take place in reality, since the velocity (or any other quantity characterizing the wave: pressure, temperature, or density) cannot have three different values at one and the same moment. In this case, the wave does not correspond to the dashed segment in Figure 228c and, instead, an infinitely steep break is formed in the medium. The distance a wave will have covered when a break forms in a nonviscous, thermally nonconductive medium is

$$L_{\text{break}}=\frac{\lambda\rho_0c_0^2}{(\gamma+1)\pi p_{10}},$$

where λ is the wave length and p_{10} is the amplitude of the sound pressure at the sound source. In reality, this steep break cannot occur, since wave attenuation will cause a smoothing-out of the front. The nonsinusiodal (sawtooth) form of the wave represents a composite wave with a set of frequencies — harmonics; it has been shown that any nonsinusoidal wave can be considered to be the combination of a wave of fundamental frequency ω with its harmonics of frequencies 2ω, 3ω, ... According to the Riemann theory, the amplitude p_2 of the 2nd harmonic is

$$p_2=(\gamma+1)\frac{\omega x p_{10}^2}{4\rho_0c_0^3}.$$

Thus, the pressure amplitude of the 2nd harmonic increases with an increase in frequency, and over the distance traveled by the wave it is proportional to the square of the sound pressure p_{10} of the primary wave.

Such is the case for gases. If it is desired to apply the formulas for c, L_{break}, and the pressure amplitudes of the harmonics to liquids, it would appear sufficient simply to substitute unity for γ (see Chapter VII, §1). This has been done by many investigators; however, it proves to be erroneous.

Actually, if the acoustic excess pressure $p_e = p - p_0$ is not large, during propagation of the wave, then the equation of state (the equation of the adiabatic curve) can be written

$$p_e = p - p_0 \cong p_0\gamma \frac{\Delta\rho}{\rho_0} + p_0 \frac{\gamma(\gamma-1)}{2}\left(\frac{\Delta\rho}{\rho_0}\right)^2, \qquad \text{(a)}$$

where $\Delta\rho = \rho - \rho_0$. Thus, for small values of p_e (and therefore, small $\Delta\rho$) the following relation may be considered to hold for liquids in general:

$$p_e \cong A\frac{\Delta\rho}{\rho_0} + \frac{B}{2}\left(\frac{\Delta\rho}{\rho_0}\right)^2, \qquad \text{(b)}$$

where A and B are certain coefficients. This expression for p_e contains, in addition to the term representing the direct proportionality between p_e and $\Delta\rho$, a second, quadratic term, while higher powers of the ratio $\frac{\Delta\rho}{\rho_0}$ become insignificant, since $\Delta\rho$ is very small. By comparing formulas (a) and (b), we obtain

$$A = p_0\gamma,$$
$$B = p_0\gamma(\gamma - 1),$$

and so

$$\gamma = \frac{B}{A} + 1.$$

The ratio B/A can be shown to represent the bulk modulus of elasticity K_0 of the liquid** at atmospheric pressure, divided by the internal pressure of the liquid p_{int}, i. e.,

$$\frac{B}{A} = \frac{K_0}{p_{int}}.$$

Statistical measurements give values of γ for various liquids which lie within limits from 7.7 (water) to 11.45 (mercury). In order for a linear relationship to exist between pressure and density, γ must be 1 and B must be 0; therefore, the nonlinearity of the adiabatic curve for air (for which $\gamma = 1.43$) exerts less influence on waveform distortion than the nonlinearity of the "adiabatic curve" for water (for which $\gamma \cong 7$).

Such is the case for the propagation of waves of finite amplitude in liquids when attenuation effects are not taken into account. However, in reality attentuation is always present and, unless it is considered, the formulas for the amplitudes of the harmonics (2nd, 3rd, etc.) become totally inapplicable for large values of the distance x, since these amplitudes must become infinite — an impossible situation. The phenomenon of waveform distortion has been quite well investigated for gases; but for liquids the general opinion has been that such a phenomenon does not exist, even during the propagation of high-intensity ultrasound. This assumption was supported by the fact that the internal pressure in liquids (which amounts to thousands of atmospheres), caused by molecular forces, is considerably higher than the acoustic excess pressure in the waves which represent the propagation of ultrasound (a few atmospheres or, at most, a few tens of atmospheres).

Actually, however, due to the comparatively low attenuation of ultrasound in liquids (e. g., the attenuation in water is about 1000 times less than

* This result can be derived more rigorously using the equation of motion of the liquid.

** For small changes in pressure, the cubical dilatation $\frac{\Delta V}{V}$, where V is the volume of the liquid, is proportional to the change in pressure Δp. Therefore, the volume elasticity of a volume of liquid V subjected to a given pressure p can be represented by its bulk modulus:

$$K_0 = V\frac{\Delta p}{\Delta V}.$$

that in air, other conditions being equal), the nonlinearity of the equations of state and of motion results in an appreciable "piling-up" of waveform distortion and thus leads to the formation of harmonics.

Experimental Observation of Ultrasonic Waveform Distortion in Liquids. Figure 229 is a diagram of the equipment for observation and study of distortion in an ultrasonic wave propagating in a liquid. This equipment is

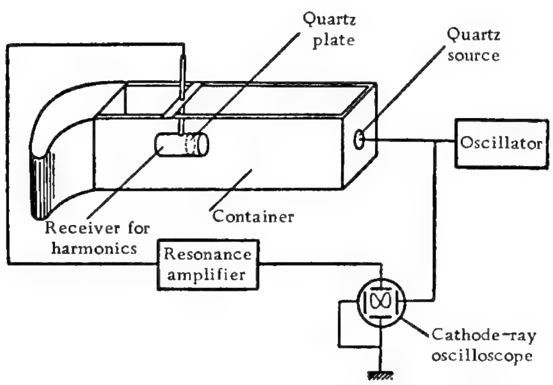


Figure 229. Block diagram of equipment for the observation of harmonics

comparatively easy to assemble for lecture demonstrations in physics. Sinusoidal voltage from a high-frequency oscillator (frequencies of 1-2 mc are convenient) is applied to a quartz source radiating ultrasonic waves into a chamber containing water (for this source, the mounting shown in Figure 221 may be used). For the detection and observation of the harmonics the application of a few volts to the quartz is in general sufficient. For this purpose, the voltage from a standard oscillator (e.g., type GSS-6) may be amplified in voltage and power. Still, it may be desirable to work with higher voltages (a few hundred volts) and to have an oscillator with a high-frequency output of 10 watts or more. The principal difficulty of these experiments is that all possible precautions must be taken to prevent the formation of standing waves, i. e., it is necessary to create conditions for obtaining traveling waves only. For this purpose, the bottom and walls (50 to 70 cm in length) of the chamber must be covered with a layer of rubber and at the end of the chamber a muffler must be constructed, i. e., the back part of the chamber must be made in an irregular shape with slanting walls. It is useful to place glass wool in the muffling wall, since it is a good absorber of ultrasound in water.

The receiver is very simple in design. In the front part of its duralumin or brass case a quartz plate is mounted, the natural frequency of which is close to the frequency of the 2nd harmonic which is to be selected and studied. The electrodes on both sides of the plate may come into immediate contact with water, but when liquids with a conductivity higher than that of water are used, it is necessary to insulate the plate from the liquid. For this purpose, the receiver plate may be set in a special mounting, the case of which is filled with transformer oil. In working with water, the tube may be used without a back wall. Care must be taken to provide a possibility of moving the receiver through the chamber and of adjusting the plane of the receiver parallel to the plane of the radiating plate.

The voltage appearing across the electrodes of the receiver plate is fed to a resonance amplifier with moderate gain — a few hundred or a thousand (if the voltage applied to the radiating plate is low, only a few tens of volts are delivered by the amplifier). The amplifier is tuned to receive the harmonic to be selected (2nd, 3rd, etc.). The resonance amplifier is used to filter out the fundamental frequency, the intensity of which in the spectrum of the propagating distorted wave is far greater than that of the desired harmonic.

This equipment forms an acoustic-spectrum analyzer, tuned to definite lines in the acoustic spectrum (the harmonics). From the output of the resonance amplifier, voltage of a frequency $2f$, $3f$ etc., is fed to an instrument which measures the amplitude of this voltage (electronic voltmeter). An electronic oscilloscope may also be used for this purpose; in order to check the isolation of the desired harmonic, a Lissajous figure may be observed on the screen. For this purpose, to one pair of deflection plates is applied the voltage from the amplifier output (the harmonic), while to the other deflection plates a part of the voltage from the oscillator is applied. When the receiver is moved with respect to the source, the figure will undergo the corresponding changes (compare Figure 78).

As already mentioned, the principal difficulty in the isolation of harmonics lies in the necessity of eliminating standing waves and filtering out the fundamental frequency. However, the receiver plate is "opaque" to the fundamental frequency, since its thickness is $\lambda/4$, where λ is the wave length of the fundamental (the natural frequency of the plate being $2f$). Therefore, the fundamental wave of frequency f is reflected from the receiver plate, and forms standing waves. To decrease this reflection, the surface of the receiver plate may be covered with a thin rubber disk (1 mm thick); the sensitivity of the receiver remains sufficiently high. For the same purpose it is possible to employ a filter plate, placed in front of the receiver. This filter (for a frequency of 1.5 mc) may be a brass plate 0.5 mm thick. If the thickness of a plate is $\lambda'/2$, where λ' is the wave length of an ultrasonic vibration in the material of the plate, then it is "transparent" to a propagating ultrasonic wave. Hence, the plate should be positioned in such a way that its thickness is equal to $\lambda'/2$ for the desired harmonic; the harmonic component of the wave will then pass through the plate. At the same time, the wave at the fundamental frequency will be reflected from the plate, so that only a small fraction passes through. To observe the harmonics, the receiver (with the rubber disk) is first placed near the source and (after tuning the amplifier to the desired harmonic) moved through the chamber, while the harmonic amplitude is measured. When a filter plate is used, the reciever is fixed at the far end of the chamber, while the filter plate is moved through the chamber.

Figures 230 and 231 show the experimental results of measurements of harmonic amplitudes, obtained with equipment of this kind*. The measurements were made at a frequency of 1.5 mc in tap water and in transfomer oil. Quartz plates with resonance frequencies of 1.5, 3.0, and 4.5 mc were used as ultrasonic receivers. The amplifier gains at these frequencies were 20, 50, and 100, respectively. As seen from the curves for the 2nd and 3rd harmonics, an increase in the distance between source and receiver

* The measurements were performed by L.K. Zarembo, V.V. Shklovskaya-Kordi, and the author.

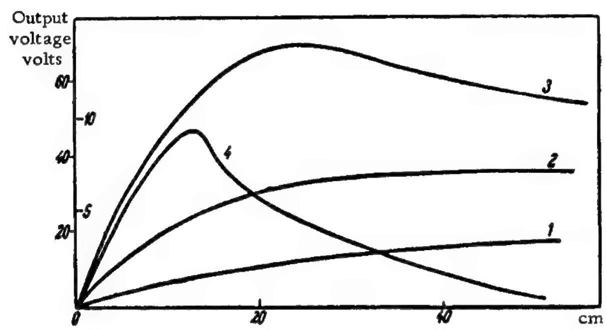


Figure 230. Pressure amplitude of 2nd harmonic as a function of distance from source

In water: 1) voltage applied to source V = 500 v, intensity at source I = 0.5 watt/cm^2; 2) V = 1 kv, I = 2 watt/cm^2; 3) V = 2 kv, I = 7.8 watt/cm^2. In transformer oil: 4) V = 2 kv, I = 7.8 watt/cm^2. The ordinate scale on the left is for water; the scale on the right is for transformer oil.

(or filter, in the second case) causes the harmonic amplitude to increase gradually to a maximum, and then to decrease. This behavior of the harmonics can be explained by absorption effects. In the initial stage, at small distances from the source, an increase in waveform distortion takes place, according to the Riemann theory*. However, beyond this the energy dissipated by the fundamental wave, e. g., to the 2nd harmonic, is fully absorbed (it should be recalled that the absorption of acoustic waves

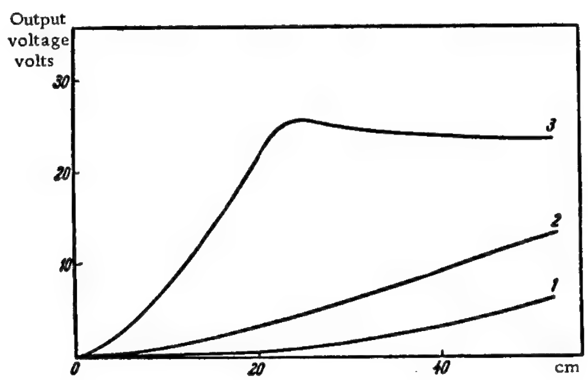


Figure 231. Pressure amplitude of 3rd harmonic in water as a function of distance from source

1) V = 500 v; 2) V = 1 kv; 3) V = 2 kv

* It may be noted that the experimental verification (using water) of the formulas for the amplitudes of harmonics p_2 and p_3, at the given value of $\gamma \cong 7$ and with low intensities and short distances, shows the Riemann formulas to hold true not only qualitatively, but also quantitatively. For longer distances the formulas no longer apply, because absorption effects become appreciable.

in a liquid, where there are no relaxation processes, is proportional to the square of the frequency). An energy balance occurs; and at the distance where such a balance takes place the amplitude of the harmonic remains constant. With a further increase in distance, the energy losses by the 2nd harmonic become greater than the supply of energy from the fundamental wave. Thus, beyond this point the harmonic amplitude will decrease. The same holds true for higher harmonics.

It may be remarked that for such a liquid as transformer oil, with greater viscosity (and thus greater acoustic absorptivity) than water, the distance at which the maximum is reached for the amplitude of the 2nd harmonic is quite a bit shorter than it is for water*.

Direct Observation of the Sawtooth Waveform. The above method for studying the distortion of an ultrasonic wave of finite amplitude in a liquid has certain advantages as well as disadvantages. Among its advantages is the high sensitivity of the method, making it possible to detect, at a frequency of 1 mc, high-order harmonics in water at intensities less then a few tenths of 1 watt/cm^2. A disadvantage is the comparatively complicated and laborious procedure of measuring the absolute values of the harmonic amplitudes.

At intensities beginning with a few watt/cm^2, it is possible to observe directly the form of the propagating ultrasonic wave and to follow its distortion with an increasing distance between radiator and receiver.

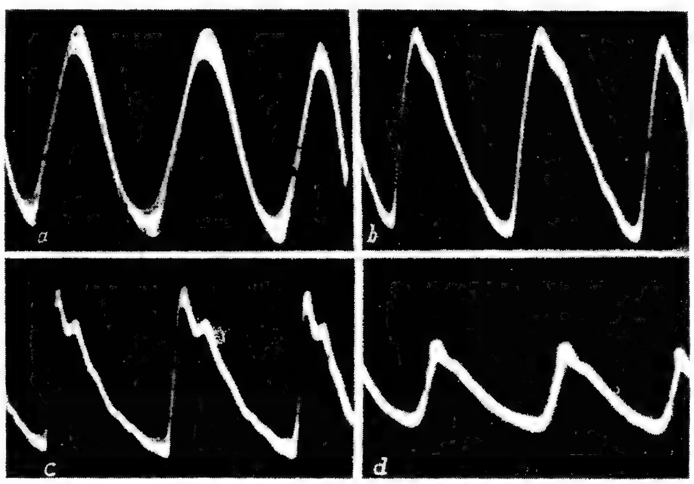


Figure 232. Change of the ultrasonic waveform in water (frequency, 1 mc; intensity, 40 watt/cm^2), as a function of distance from source: a) 2 cm; b) 10 cm; c) 20 cm; d) 48 cm

* Theoretical analysis shows this distance (known as the stabilization distance) for acoustical Reynolds numbers $\mathrm{Re} = \frac{\rho}{b\omega} \ll 1$ (see § 4 of this chapter) is given by the formula $x_0^* = \frac{\ln 2}{2\alpha_1^0}$ where α_1^0 is the low-amplitude absorption coefficient for the fundamental frequency.

It will be seen from this formula that, other conditions being equal, the stabilization distance is greater in liquids with a greater shear (or volume) viscosity.

For this purpose the equipment described above must be modified in such a way that the receiver and amplifier pick up the distorted waveform without altering it, i.e., that the receiving channel of the equipment has a sufficiently wide pass band*. Figure 232 shows photographs (taken from the screen of a cathode-ray oscilloscope) of the gradual distortion of the waveform of a sinusoidal (at the source) ultrasonic wave, frequency 1 mc, obtained in water with an ultrasonic intensity immediately at the source of 40 watt/cm^2. The radiation of ultrasound lasted for 1 or 2 sec. As a receiver a quartz plate was used with a resonance frequency of 11.5 mc (the high natural frequency of the plate ensured a wide band for the detected frequencies); the plate was 25 mm in diameter, and was placed in a case filled with transformer oil.

The photographs clearly show that the wave, which is nearly sinusoidal 2 cm from the source, begins to be distorted further away. A steep front appears, and the wave more and more assumes a sawtooth profile. With increasing distance from the source the amplitude gradually decreases; and the front, after attaining a maximum steepness, becomes somewhat smoother. The wave, however, continues to propagate as a sawtooth wave.

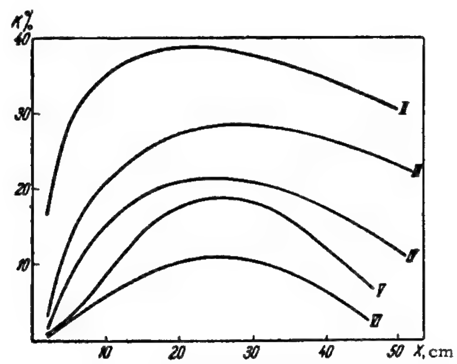


Figure 233. Result of a graphical analysis of the percent content K of harmonics in the curves in Figure 232

II) second harmonic; III) third harmonic; etc.

The sawtooth profile indicates that here we are dealing with waves which are similar to weak periodic shock waves in a liquid. A perfect sawtooth curve has a second-harmonic amplitude which is 50% that of the fundamental frequency. Figure 233 presents a graphical analysis of the spectral composition of the waveforms shown in Figure 232. At a distance of 20 cm, the 2nd harmonic is only 40% of the amplitude of the fundamental wave. From Figure 233, it will also be seen that from 20 to about 40 cm from the source the percent content of harmonics is little changed, which means that within these limits the waveform changes only very little. It may therefore be assumed that, over a certain part of its path, the distorted wave maintains a constant shape. At still greater distances, the wave amplitude becomes small, with a gradual transition back to a sinusoidal waveform. All this holds true for a plane wave. In actual experiments,

* This was done by V.A. Burov and the author.

beginning at a distance R^2/λ (where R is the radius of the source plate), the ultrasonic beam, as we know, begins to diverge; and the wave gradually becomes spherical. For a diverging wave distortion occurs to a substantially lower degree than for a plane wave, due to the relatively rapid energy loss (the wave fills an ever-increasing volume of liquid).

An interesting phenomenon may be observed during study of the diffraction of light by high-intensity ultrasonic waves*. As distinguished from light diffraction by ultrasonic waves of low amplitude (where a diffraction grating is formed by sinusoidal waves), for high-intensity waves a sawtooth diffraction grating is formed, i.e., the form of the curve is asymmetrical. Accordingly, in the first case a symmetrical pattern is observed for the intensities of spectral lines of different orders, with respect to the central maximum (the zero-order spectrum, see Figure 179). In the second case, however, sharp asymmetry is observed in the spectral pattern. Figure 234 shows a series of diffraction spectra, obtained for the diffraction of light by 573 kc ultrasonic waves of different intensities, in distilled water. The photographs show the spectral pattern to become more asymmetrical as the intensity increases (from top to bottom in Figure 234).

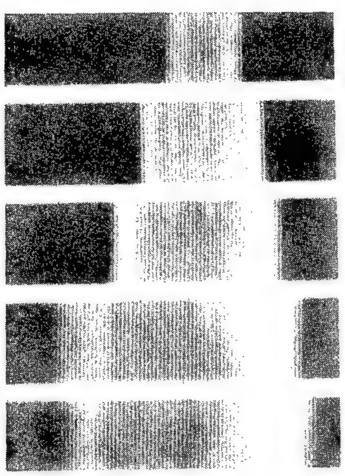


Figure 234. Spectra of the diffraction of light by ultrasonic waves, at different intensities (f = 573 kc)

The bottom photograph was obtained with an intensity of about 15 watt/cm^2; distance from sound source, 13 cm.

§ 4. Absorption of plane ultrasonic waves of finite amplitude

Effect of Waveform Distortion upon Absorption. Since the absorption in liquids and gases, in the absence of relaxation processes, is proportional to the square of the frequency, it is obvious that a wave of nonsinusoidal form must be damped to a greater extent than a sinusoidal wave. This must occur, because a nonsinusoidal wave (e.g., a sawtooth wave) contains high-frequency components — harmonics, the absorption of which is much higher. Consequently, the absorption coefficient of the 2nd harmonic is 4 times as great as that of the fundamental wave, that of the 3rd harmonic is 9 times as great, etc. The harmonics themselves derive their energy from the main wave. For this reason, the propagation of ultrasonic waves of finite amplitude must be accompanied by greater absorption in the case of transition to a sawtooth wave than for the propagation of a sinusoidal wave. The physical mechanism of the increased absorption in the case of a sawtooth wave can be explained without applying spectral concepts. In a sawtooth wave there is a steep forward frontal surface; therefore, over a short distance great changes in translational velocity and in temperature occur. The first leads

* This has been done at Leningrad State University by I.G. Mikhailov and V.A. Shutilov.

to increased viscosity effects, and the second leads to increased thermal-conduction effects in comparison with a sinusoidal wave. This was first realized when Fox (U.S.A.) discovered that the absorption coefficient in water at a frequency of 10 mc is 5 times greater at an intensity of 4 watt/cm^2 than at low intensities; he also demonstrated that this increase is not related to cavitation. In a more viscous liquid — carbon tetrachloride — where absorption is much greater than in water, an increase in the absorption coefficient at higher intensities also occurs, but to a smaller degree. Subsequently it was established* that for lower frequencies and for high ultrasonic intensities the rate of increase of the absorption coefficient in water is even greater, and this phenomenon is also not related to cavitation processes. For liquids with higher absorption, this change is less pronounced.

Methods for Measurement of the Absorption Coefficient. Before discussing further the absorption of intense ultrasonic waves, we shall dwell briefly on the measurement of this absorption, in comparison with measurements of the absorption of low-intensity ultrasound. In order to measure the absorption coefficient for low-amplitude ultrasonic waves, it is in principle necessary to measure, for a plane wave, the ultrasonic intensity at two points along the wave path, or else to compare the pressure amplitudes at these points. For the same purpose, it is possible to use a quartz-plate receiver which responds to the same frequency as that of the source plate. This is done, as mentioned previously, by means of the pulse method or using a standing-wave interferometer (see Chapter VII, § 1). However, in the case of high-intensity ultrasonic waves the absorption coefficient cannot be measured in this way. Indeed, if the wave is distorted, it is necessary for the receiver (if a quartz plate is used) to have a sufficiently wide reception range, so that all the harmonic components present in the distorted wave are picked up equally well**. Previously, the various investigators performing measurements of the absorption coefficient of ultrasound in various liquids paid no attention to the voltage on the quartz, i. e., to the ultrasonic intensities with which they were working. At the same time, the receiver used was a quartz plate responding to the same frequency as the source, provided the ultrasonic intensity was not too low. The obvious result was that such measurements were inaccurate in many cases. For these reasons, in dealing with intense ultrasonic vibrations, other methods are required for absorption-coefficient measurements.

There are several suitable methods, of which three will be described here. The most reliable one seems to be the calorimetric method. As ultrasound is absorbed, its energy is lost and eventually transformed into heat. Thus, by measuring the increase in temperature ΔT of a liquid during a definite period Δt, in a vessel of given volume V with thermally nonconductive walls, where the liquid has a known specific heat C, it is possible to determine the energy which has entered the vessel and there been completely transformed into heat.

The quantity of heat absorbed by the liquid is

$$Q=\rho VC\Delta T,$$

* By L. K. Zarembo, V. V. Shklovskaya-Kordi, and the author.

** However (see below), the form of the curve, due to possible cavitation effects, may differ from a sawtooth and may have some asymmetry, making it difficult to interpret data obtained with a wideband receiver.

where ρ is the density in g/cm^3; and C is the specific heat of the liquid in cal/g·degree. Since 1 cal = 4.19 joule (mechanical equivalent of heat), the energy absorbed by the liquid is $\Delta E = 4.19 \rho V C \Delta T$ joule. With an ultrasonic intensity of I watt/cm^2, and a source area of S cm^2, the energy absorbed by the liquid during the period of ultrasonic irradiation Δt sec is

$$\Delta E = JS\Delta t \text{ joule.}$$

Now, combining these two formulas, it is possible to determine I:

$$\mathrm{I} = 4.19 \frac{\rho V C \Delta T}{S \Delta t} \text{ watt/cm}^2.$$

For water

$$\rho = 1 \text{ watt/cm}^3 \text{ and } C = 1 \ \frac{\text{cal}}{\text{g·degree}}$$

so that

$$\mathrm{I} = \frac{4.19\, V \Delta T}{S \Delta t}.$$

In this way, if V, S and Δt are given (the latter may called the "exposure time"), measurements of ultrasonic intensity are reduced to the measurements of ΔT, the increase in temperature in the vessel. It is convenient to use a Dewar flask for this experiment, since its walls do not conduct heat, and minimum heat transfer from the liquid in the vessel to the surrounding medium is ensured. The mouth of the flask is covered with a thin nylon film, and the vessel is placed a certain distance from the source, after which the period of ultrasonic irradiation is measured with a stop-watch. Immediately after the irradiation is over, the lid with the nylon film is removed from the flask, and the temperature of the liquid is measured with a thermometer, while the liquid is stirred.

There is another thermal method for measuring the ultrasonic intensity of waves of finite amplitude, in which the intensity is measured by means of thermoelectrical transducers. In principle, it is similar to the first method; the difference is that temperature measurements are made with a thermocouple. The latter is a metal wire to both ends of which wires of some other metal are soldered, forming junctions. If one of the junctions [the measuring junction] is heated, while the other [the reference junction] is kept at a constant temperature, then across the open ends of the wire an electromotive force [emf] is generated. If the ends are connected through a galvanometer, a current will flow in the circuit. Different metals produce different thermal emf. For temperature measurements in the range from 0 to 100°C, copper-constantan thermocouples are used. This couple yields approximately $4 \cdot 10^{-7}$ amp when the temperature of the junction is raised by 1°C. When a liquid is heated in a calorimeter (Dewar flask) it should be possible simply to introduce a thermocouple into the liquid and to determine the amount of heating. For common liquids, however, which have low absorption coefficients, the sensitivity of such a measurement is so low that it is not useful in practice. The thermocouple must be placed in a medium with good ultrasound-absorption qualities, e.g., castor oil. By placing the couple in a hermetic vessel with windows which are "transparent" to ultrasound, it is possible to determine the acoustic intensity by the amount of heating. Thermoelectric receiver-transducers with solid ultrasonic absorbers are more convenient in practice and simpler in operation; rubber

or diacetate film* are good ultrasonic absorbers in the megacycle range. When one junction of a thermocouple is dipped into rubber or diacetate glue, a thin absorbing coating is produced after the glue has dried. The dimensions of such a receiver may be quite small ($\sim$ 0.5-1 mm). Very often not a point receiver but an extended receiver is necessary, e.g., in order to determine the average intensity over the cross section of an ultrasonic beam; the ultrasonic field in the neighborhood of the source may be nonuniform, so that averaging over the whole cross-sectional area of the beam is required to find the average intensity of the ultrasound. In this case the receiver shown in Figure 235 can be used. Here, in an ebonite capsule, are placed over 60 measuring junctions of copper-constantan, coated by a thin, ultrasound-absorbing layer of diacetate. The thermocouples are connected in series. The reference junctions are brought out behind the capsule and, of course, are not covered with the absorbing material. In this way, the heating of the absorber is measured relative to the temperature of the medium in which the receiver is immersed. This multijunction receiver has substantially higher sensitivity than a single-junction receiver.

Figure 235. Thermoelectric receiver with 61 measuring thermocouple junctions of copper-constantan under a thin, ultrasound-absorbing diacetate film

To use this receiver, the absorber with its thermocouples is placed in an ultrasonic field and voltage is applied to the radiating quartz plate. The acoustic energy absorbed by the layer of absorbing material heats this layer, and the thermocouple shows a rise in temperature. However, in conjunction with this process, another one takes place; since the absorber is heated more rapidly than the medium, the difference between the temperatures of the medium and the absorber is constantly increasing. This difference causes a reversing process to occur — heat conduction, which cools the absorber. If the ultrasonic source operates for a sufficiently long time, then thermal equilibrium is finally reached, when the quantity of heat supplied

* A diacetate film is often used as a base for photographic and movie film. Diacetate glue is obtained by dissolving in acetone the film, cleaned of the emulsion.

by the ultrasound equals the quantity of heat carried away from the absorber due to heat conduction. Nevertheless, in the initial period the heating rate of the absorber is proportional to the ultrasonic intensity; therefore, in measurements using thermoelectric receivers, the sound source is usually switched on for short intervals only.

Similar to thermoelectric receivers are ultrasonic receivers employing thermistors. A thermistor is a resistor with a very high temperature coefficient of resistance. By coating a spherical thermistor with an ultrasound absorbing material, a thermistor receiver is obtained. The resistance of a thermistor can be measured, for instance, by means of a bridge circuit (similar to Figure 142), together with automatic recording of the voltage or current.

Besides thermal methods (calorimetric, thermoelectric, and thermistor methods), mechanical methods are employed in intense ultrasonic fields; one of these is the radiometer method.

It has already been noted (Chapter II, § 4) that when a plane acoustic wave, with an intensity of I watt/cm^2, strikes an ideally reflecting surface, the latter is subjected to a pressure (the radiation pressure) of

$$P=\frac{2I}{c}.$$

If the liquid in which the measurements are being performed is water, then, substituting the sound velocity in water into the equation, we obtain

$$P=132I \text{ bar}.$$

Thus, by measuring the radiation pressure P, it is possible to determine the sound intensity at a given point in space.

A radiometer for this purpose is shown diagrammatically in Figure 236. If the receiving area (disk) of the radiometer is S, then the force acting on the radiometer disk is

$$F=2\cdot 10^7\frac{I}{c}\text{ dyne},$$

where I is in watt/cm^2.

By measuring the radiometer deflection in an ultrasonic beam, it is possible to determine F, and then I.

It has been noted previously that a large error is introduced into radiometer measurements by the acoustic wind. Its effect can be diminished by shielding the radiometer by means of a nylon diaphragm which is "opaque" to the acoustic current, but "transparent" to ultrasound. However, such a diaphragm, though it diminishes the error caused by the acoustic wind, does not eliminate it completely (see § 2 of this chapter).

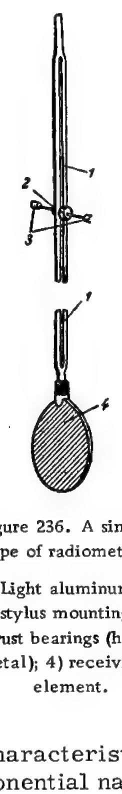


Figure 236. A simple type of radiometer

1) Light aluminum arm; 2) stylus mounting; 3) thrust bearings (hard metal); 4) receiving element.

Measurements of the Absorption of Waves of Finite Amplitude. It has been previously mentioned that the absorption of waves of finite amplitude may be substantially greater than that of waves of small amplitude. Another characteristic of the absorption of waves of finite amplitude is the nonexponential nature of this absorption. Plane waves of small amplitude are

absorbed in such a way that the decrease in amplitude (e.g., in sound-pressure amplitude) over a given small section of the path traversed by the wave is proportional to the length of this section. Such a simple law, however, is true only for waves of small amplitude. In a sinusoidal wave of small amplitude the absorption coefficient is constant, depending only upon the frequency and upon the properties of the medium (its viscosity and thermal conductivity). In a wave of finite amplitude, the absorption not only depends upon these factors, but upon the pressure amplitude and the distance from the source as well. Close to the source, when waveform distortion is still very small; the absorption coefficient of waves of finite amplitude does not differ from that for waves of small amplitude; but as the distance increases, waveform distortion builds up, and the coefficient increases. Finally, at maximum distortion, the absorption coefficient attains its maximum value. As we have indicated during the discussion of waveform distortion, the wave retains a relatively stable form over a certain portion of its path, and the relative content of harmonics remains approximately constant (i.e., the spectral composition of the wave does not change). Over this distance the (maximum) value of the absorption coefficient also remains unchanged. With a further increase in distance, the wave is gradually transformed into a wave of small amplitude, and, accordingly, the absorption coefficient decreases gradually, approaching the value for a wave of small amplitude. To illustrate this behavior of the absorption coefficient of a wave of finite amplitude, Figure 237 shows curves for the coefficient of

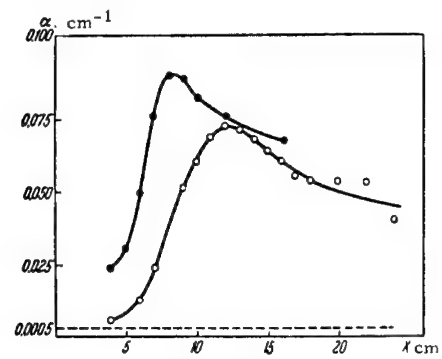


Figure 237. Absorption (in terms of energy) at 1 mc for different ultrasonic intensities, as a function of distance from source (in distilled water)

o–50 watt/cm^2; •–100 watt/cm^2. Dashed line corresponds to absorption for waves of small amplitude.

(energy) absorption for different ultrasonic intensities in distilled water, as a function of the distance from the source (a quartz plate)*. As seen from the figure, the absorption coefficient reaches a maximum after a certain distance, approximately 7 to 10 cm (at this point maximum waveform distortion is also observed); and then it decreases. The coefficient of absorption for small-amplitude waves is shown in the figure as a straight dashed

* The measurements, which were calorimetric ones, were conducted at the Laboratory of Anisotropic Structures of the U.S.S.R. Academy of Sciences by V. A. Burov

line. In the experiments represented in the figure, α is more than a hundred times greater than the absorption coefficient for small-amplitude waves.

In a liquid like water, there is strong waveform distortion for intense ultrasonic waves and considerable increase in absorption in comparison with the same conditions for small-amplitude waves. In liquids of higher viscosity, waveform distortion is less pronounced, and consequently the increase in absorption is not so great*. It may be desired to obtain the general absorption laws as functions of the properties of the liquid and of the ultrasonic intensity and frequency. A general consideration of this problem is made possible by utilizing the Reynolds number for wave processes.

We have already had occasion to employ the Reynolds number (see Chapter VI, § 3) as a criterion for the transition to turbulent flow. This number also characterizes the similarity of two flows. For instance, consider the flows of two liquids (with kinematic viscosities ν_1 and ν_2) through pipes of radiuses r_1 and r_2, with flow velocities v_1 and v_2; the flows are similar if their Reynolds numbers are equal, i.e., if

$$\frac{v_1 r_1}{\nu_1} = \frac{v_2 r_2}{\nu_2}.$$

The Reynolds number for sound propagation may be written

$$\mathrm{Re} = \frac{v\lambda}{\frac{b}{\rho}} = \frac{v\rho\lambda}{b}.$$

Here, the acoustic particle velocity corresponds to the flow velocity, the wave length λ corresponds to the characteristic linear dimension, and $\frac{b}{\rho}$ corresponds to the kinematic viscosity; this is actually the same kinematic viscosity, but corrected for the bulk viscosity ($b = \frac{4}{3}\eta + \eta'$, where, as previously, η is the shear viscosity and η' is the bulk viscosity).

By making use of the relation for a plane wave $v = p/\rho c$, where p is the acoustic excess pressure and c is the velocity of sound, this number may be written

$$\mathrm{Re} = \frac{p}{b\omega}.$$

It can be shown that if p_{10} is the pressure amplitude in the neighborhood of the ultrasonic source, then for $\mathrm{Re} = \frac{p_{10}}{b\omega} \gg 1$ viscosity forces cannot prevent the formation of a discontinuity, and the stable form of the wave is a sawtooth. For $\mathrm{Re} < 1$ waveform distortion may also occur, but the stable waveform is nearly sinusoidal.

The absorption coefficient α of the sawtooth waveform is then expressed by the following formula, which is found to be in good agreement with experimental results:

$$\alpha = \alpha_1^0 \frac{\gamma+1}{\pi} \mathrm{Re}_x = \alpha_1^0 \frac{\gamma+1}{\pi} \frac{p_x}{b\omega},$$

where $\mathrm{Re}_x = \frac{p_x}{b\omega}$, p_x is the acoustic excess pressure at the point x at which α is being determined, $\gamma = \frac{B}{A} + 1$ for liquids (see above) and $\gamma = \frac{c_p}{c_v}$ for gases, and α_1^0 is the absorption coefficient for a wave of small amplitude.

Thus, the absorption coefficient α proves to be proportional to the Reynolds number, or to the acoustic excess pressure**. From this formula let us determine for high intensities the ratio α/α_1^0 for water. For instance, let $\mathrm{Re}_x = 100$, which for water and a frequency $f = 1$ mc corresponds to an acoustic pressure $p_x \cong 22$ atm. Then

$$\frac{\alpha}{\alpha_1^0} \approx 0.44 \cdot 2\pi \mathrm{Re}_x$$

or

$$\frac{\alpha}{\alpha_1^0} \cong 280.$$

* In this connection it is important to note that, because the viscosity of transformer oil decreases with heating, the absorption coefficient does not decrease (as should have been the case for waves of small amplitude), but increases. This fact can be explained only by the essential part played by waveform distortion, i.e., by the appearance of harmonics.

** Contrary to the absorption of a wave of small amplitude, which does not depend on p.

If $p_x \cong 2.2$ atm at this frequency in the region of stable waveform, then $\alpha/\alpha_1^0 \cong 28$; thus, even for a comparatively low intensity the absorption coefficient for a sawtooth wave is 28 times greater than the absorption coefficient for a small-amplitude sine wave.

The sizeable increase in the absorption of ultrasonic waves of finite amplitude in low-viscosity liquids has, in addition to its scientific significance, considerable practical importance as well. This phenomenon must be taken into account in various measurements of the absorption coefficient for ultrasonic waves in liquids, in designing long-focal-length acoustic focusing systems, and in working with intermediate (and even more with high) ultrasonic intensities in low-viscosity liquids such as water. This increase in absorption (along with cavitation, see below) may be responsible for the fact that an increase in source power is in many cases not followed by an increase in the distance of propagation of acoustic waves.

Beside waveform distortion, there are other reasons for the increased ultrasonic absorption with increased intensity. Among these are the energy losses during the formation and maintenance of the acoustic current; and the rise in ultrasonic cavitation, which also consumes the energy of the propagating ultrasonic wave, and leads to additional absorption. These problems, however, are at present only subjects for future research.

§ 5. Cavitation

The phenomenon of cavitation is very commonly observed during the motion of liquids and consists in the formation in the liquid of breaks or cavities, filled with the vapor of the liquid and with gas which is dissolved in the liquid. When these cavities collapse, they produce strong local compression shocks, reaching thousands of atmospheres in intensity. For instance, cavitation occurs as a result of the rotation of a ship's propeller or the blades of a water turbine as well as during the rapid flow of liquid through the throat of a pipe, etc. Cavitation is important in industry, where it is sometimes a useful, and sometimes a harmful, factor. The compression shocks produced by the collapse of cavitation bubbles, with their high pressures, cause erosion of metals; in poorly designed ship propellers and turbine blades, where cavitation effects are prominent, the metal surface is worn away and the part becomes useless. For some useful aspects of cavitation, see below.

In the following experiment, performed by I. G. Polotskii, an effect appears which is similar to cavitation. If water vapor is passed through a thin glass tube into cold water, then the vapor in the bubbles, on entering the cold water, is condensed, and the bubbles collapse rapidly.

The propagation of intense sound and ultrasound waves in liquids is also accompanied by cavitation; the latter is known as ultrasonic cavitation, although it occurs at audible frequencies as well.

Actually, cavitation is merely a boiling of the liquid; the only difference is that during ordinary boiling the bubbles are filled with vapor, while in cavitation they may be filled either with vapor or with gas. The boiling point of water is 100°C at normal atmospheric pressure. At this temperature the pressure of the saturated vapor equals the external atmospheric pressure; and in general, it may be said that a liquid starts "boiling" when the latter condition is true for a given temperature. Thus, if at 20°C

the saturated-vapor pressure of water is ~ 0.023 atm, then when the pressure on the water is reduced by 0.977 atm (1 atm — 0.023 atm), it should start boiling. However, when very pure water is so expanded, the boiling does not begin, since the liquid is in an unstable, or metastable, state. The reason for this is that in perfectly pure water there are no gas or vapor bubbles; these bubbles decrease the cavitation resistance of a liquid and cause boiling to start in ordinary water. In order for a given bubble of radius R to exist in a liquid for a prolonged period, the internal pressure in the bubble must be equal to the external pressure. For a very small bubble, the internal pressure is determined by the surface tension and given by the formula

$$p_{\text{in}}=\frac{2\sigma}{R},$$

where σ is the surface tension of the liquid. If the pressure outside the bubble becomes greater than p_{in}, the bubble will collapse (due to the condensation of vapor, or to the dissolving of gas); if it is less, then the bubble will expand. Since, in order for an perfectly pure liquid to boil, a bubble must appear in the medium, let us consider its initial dimensions to be small, e.g., $R = 10^{-6}$ cm. Then, for water ($\sigma = 80$ dyne/cm), $p_{\text{in}} = 16 \cdot 10^7$ bar = = 160 atm; and for such a bubble to form, it is necessary to produce in the liquid a rarefaction of 160 atm. In order to make possible the formation of a bubble with a size comparable to the size of a molecule of the liquid, it is necessary to apply to the liquid a negative pressure [to expand the liquid] which is nearly equal to the internal pressure of the liquid, i.e., $\approx 10^4$ atm. So far the maximum negative pressure obtained experimentally at which bubbles begin to appear is 280 atm for very pure water (rarefaction was produced by the centrifugal method). This negative pressure is not as high as that indicated by the preceding discussion, since there exist in water certain points (often called weak spots in the liquid) where bubble formation is made easier; these points are also called cavitation centers. They may be solid inclusions, not well dissolved in the liquid, or small gas bubbles which do not dissolve because their internal pressure is lower than that given by the above formula (because of decreased surface tension). The presence of certain organic liquids causes a decrease in the surface tension of many liquids, e.g., water. Small amounts of these liquids may be the cause of the existence of gas cavitation centers. In liquids which have not been purified by special methods, the cavitation resistance corresponds to only a few atmospheres. It has already been mentioned that acoustic pressures in intense ultrasonic waves may be several atmospheres, in some cases even tens of atmospheres. Under such conditions, however, cavitation becomes impossible. A bubble appearing in the negative pressure phase collapses upon the arrival of the compressional half wave. During the collapse of the bubbles, immense pressures arise, of the order of several thousand atmospheres*, and a spherical compression wave spreads out from the collapsed cavity.

* The forces arising during the collapse of gas bubbles in water were computed by Rayleigh. If the initial radius of a bubble is R_0, and if it decreases to a value R under the action of a hydrostatic pressure p_0, then at a distance $x = 1.59R$ there is a pressure

$$p = 0.15 p_0 \frac{R_0^3}{R^3}.$$

Thus, if a bubble is compressed to one-twentieth of its initial volume, and if $p_0 = 1$ atm, then $p = 1260$ atm.

Ultrasonic Cavitation. In spite of the large number of works published on ultrasonic cavitation, this phenomenon has not yet been adequately studied. This is due to the fact that the appearance of cavitation depends upon many factors, which are often difficult to control. Existing data indicate that the cavitation threshold (the minimum acoustic pressure at which cavitation appears) depends upon frequency; it becomes higher as the frequency increases. This rise in the threshold becomes appreciable at frequencies of about a few megacycles. The cavitation threshold also depends upon the viscosity of the liquid, but this dependence is very weak; if the viscosity of the medium changes from 0.01 to 1 poise, the cavitation threshold is raised only by a factor of two.

The presence of cavitation can be observed visually as a foggy cloud of bubbles in the ultrasonic field. If measurements in the ultrasonic field are performed with a low-inertia piezoelectric receiver, then the onset of cavitation is accompanied by a sharp, irregular variation of the amplitude of the arriving signal. At ultrasonic frequencies and high intensities, the appearance of cavitation is accompanied by a hissing sound, resembling that of a tea kettle beginning to boil. This is related to the collapse of the cavitation bubbles, which causes a loud noise. In addition to the continuous (white) noise, which extends down to audible frequencies, there are in the cavitation-noise spectrum isolated spectral lines, corresponding to the fundamental frequency, and to its harmonics and subharmonics. The cavitation intensity may be evaluated from the level of cavitation noise.

Cavitation has various effects; and some instances will be given below in which cavitation appears to play the main role. Recently, intense ultrasonic vibration has found wide application for various biological purposes. Among other effects ultrasound produces partial, and in some cases even total, deactivation of various microorganisms. The compression shocks formed by the collapse of bubbles have a destructive effect on bacteria. This biological effect of ultrasound is utilized in medicine, e.g., for sterilizing fresh-water reservoirs, and for other purposes. The effect of ultrasound upon tuberculosis bacilli, streptococci, and staphylococci is to reduce their activity.

Cavitation may also cause the erosion of metal surfaces (separation of minute particles). This phenomenon, in some cases detrimental (e.g., the destruction of ships' propellers), can also be used to obtain a well-dispersed suspension of a metal in a liquid.

The emulsifying effect of ultrasound also seems to be connected with cavitation. An emulsion is a suspension of minute particles of one liquid in another liquid, the two being mutually insoluble; e.g., milk is an emulsion of fat in water (these dispersed systems are called hydrosols).

To prepare an emulsion (e.g., benzene in water) the method of agitation (shaking) the benzene together with the water is commonly used. However, by this method it is impossible to obtain benzene droplets smaller than 4-5 microns, or at any rate the amount of smaller drops is very small. Ultrasound of considerable intensity, however, is capable of producing a small-grained and more homogeneous emulsion, since the ultrasonic vibrations break up large drops of the emulsion. The main cause of this dispersive action of ultrasound is cavitation. By means of this "grinding" action of ultrasound it becomes possible to produce extremely fine-grained photosensitive layers, permitting high enlargement.

Cavitation is responsible for many chemical changes taking place in an ultrasonic field, for instance, the oxidation of potassium iodide. The depolymerization of certain high polymers also seems to be determined by the cavitation effects of ultrasound.

Many industrial uses of ultrasound depend upon specific cavitation effects. For instance, an ultrasonic field has been shown to be a better and quicker means of cleaning and washing certain machine parts. For example, optical lenses to be polished are cemented into a polishing machine; and, afterward, the cement is usually washed off with acetone. Ultrasonic washing in acetone is quicker than hand washing, and is widely used when it is necessary to clean thoroughly small objects of complicated shape, e.g., clockwork parts, electron tube elements, etc.

Beside their "grinding" effects, ultrasonic waves have in many instances quite an opposite effect on small particles. Small particles of smoke or fog, caused to vibrate under the action of sufficiently intense ultrasound, collide with each other and stick together, forming larger particles; this phenomenon is known as aerosol coagulation (aerosols are gases containing suspensions of minute liquid or solid particles).

One reason for the different effects of ultrasound on hydrosols (dispersion) and aerosols (coagulation) lies in the absence of cavitation in gases, since it is cavitation which breaks up the grains in an emulsion.

If, in a glass tube containing a little lycopodium powder or cork dust, sound waves of a sufficiently high frequency (10 kc or above) are produced, then the dust concentrates at the pressure nodes. The tube used for this demonstration, with a piston to change the length of the air column, is commonly called a Kundt tube. If the sound frequency is known, and the half wave length is measured as the distance between two adjacent clusters of dust particles, the velocity of sound is easily determined. Instead of dust, the tube may be filled with smoke; the smoke particles will also concentrate at nodes of the acoustic pressure.

Let us now recall the preceding sections, in which we discussed the propagation of ultrasonic waves of finite amplitude. The measurements of the waveform distortion and the absorption of waves of finite amplitude which were discussed there referred to cases when cavitation processes had no time to develop fully, since quite high frequencies were used, while the time of ultrasonic radiation (exposure time) amounted to only a few seconds. However, at high intensities, even for short exposures, cavitation does develop strongly, leading to some new effects in wave propagation. Among such effects is an additional waveform distortion in the propagating sawtooth vibration.

In observations of waveform distortion in a 1 mc ultrasonic vibration in water (see Figure 232), it was noticed that this form changes in time; with prolonged irradiation and sufficiently high intensity, the wave, especially in freshly drawn tap water, becomes altogether unstable. Thus, at intensities above 30-40 watt/cm^2, the waveform becomes so unstable that successful photographs cannot be taken from the screen of an oscilloscope. Figure 238 represents two oscillograms obtained in tap water at 22 cm from the source, and at an intensity of 35 watt/cm^2. Oscillogram (a) was obtained 0.5 sec after the beginning of radiation. A completely symmetrical sawtooth waveform is evident; the fluctuation near the peak (compare

Figure 232) seems to be explained by the insufficiently wide frequency response of the amplifiers of the oscilloscope. After a few (5-7) seconds, the amplitude decreases and the waveform becomes asymmetrical. The sharp peak of the sawtooth is cut off on the side of negative pressure. Then this effect is heightened, while the amplitude becomes unstable, remaining somewhat lower than the initial amplitude. Oscillogram (b) was obtained 35 sec after the beginning of radiation; the asymmetry of the waveform is obvious, as well as the somewhat lower amplitude in comparison with the first oscillogram. Some smoothing of the forward wave-front is also seen.

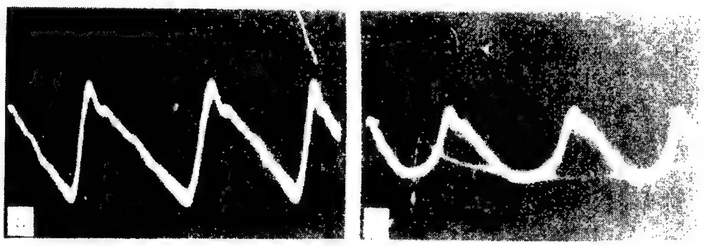


Figure 238. Oscillogram (a) taken 0.5 sec after the beginning of radiation (water; frequency 1 mc; intensity at source, 35 watt/cm^2). Oscillogram (b) taken 35 sec after the beginning of radiation

These phenomena seem to be caused by cavitation processes. Thus, the asymmetry in the form of the curve can be explained by the large losses in the wave during the rarefaction half-period, due to the appearance of cavitation, leading to a particular acoustical rectifying effect. The cutting off of the lower point of the sawtooth occurs at an acoustic excess pressure of about two atmospheres. This quantity corresponds approximately to the cavitation resistance of gas-rich water, such as tap water. The decrease in amplitude with time and its instability also seem to be explained by the gradual development of cavitation.

The phenomena taking place in distilled water are mainly the same as those just described, only they are weaker and require more time to develop.

Consequently, in addition to ultrasonic waveform distortion, caused by nonlinearity of the equation of motion and the equation of state, there is another kind of distortion, due to cavitation. This should lead to additional attenuation of the ultrasonic wave; however, this problem has not yet been investigated. In cases where it is necessary to prevent cavitation during the propagation of intense ultrasonic waves in a liquid (e.g., during certain measurements), it is possible to apply a compensating pressure to the liquid (provided design conditions allow this). Obviously, the compensating pressure must exceed the acoustic excess pressure in the ultrasonic wave.

§ 6. Shock waves

In the preceding sections, we have dealt with weak periodic shock waves in a liquid. During explosions (when pressures are considerably greater

than atmospheric), during detonation phenomena, during combustion of gaseous mixtures, during supersonic gas flow in the nozzles of jet airplanes, around moving turbine blades, projectiles, and airplanes moving with supersonic speeds, and in many other cases which are common in technology, shock waves are produced; and these are essentially different from ordinary acoustic waves. The ordinary theory for the propagation of sound waves of small amplitude is totally inapplicable, since for shock waves completely different, qualitatively new phenomena occur, which are unlike those accompanying the propagation of acoustic waves. We will now consider in somewhat greater detail this important field, which is closely related to fluid dynamics (particularly the dynamics of compressible gases, i.e., gas dynamics) and which has a multitude of practical applications.

The theory of shock waves is a vast subject, involving many branches of physics and chemistry: acoustics, the kinetic theory of gases, gas dynamics, optics, thermodynamics, the kinetics of chemical processes, etc. Shock waves have received especially great attention because modern aerodynamics deals with objects moving at velocities which approach and exceed the sound velocity. In addition, shock waves, as previously mentioned, are produced in explosions; and a clear idea of shock-wave formation and propagation is essential to investigations of the explosion mechanism itself.

Formation of Shock Waves. For a more distinct understanding of shock-wave formation, let us consider a long cylindrical tube of constant cross section. Let a piston move inside the tube at a uniform velocity u. The piston will produce a finite change in pressure (a compression) in its immediate vicinity. This compression can be regarded as the result of numerous small sudden changes in pressure, following one another; i.e., the smooth curve of the piston motion can be replaced by a broken line consisting of a large number of segments.

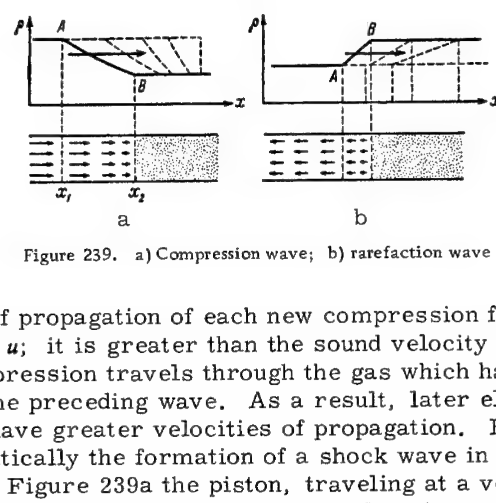


Figure 239. a) Compression wave; b) rarefaction wave

The velocity of propagation of each new compression following a preceding one is $c + u$; it is greater than the sound velocity c, since the wave of each new compression travels through the gas which has already been compressed by the preceding wave. As a result, later elementary compression waves have greater velocities of propagation. Figure 239 represents diagrammatically the formation of a shock wave in this manner. In the upper part of Figure 239a the piston, traveling at a velocity u, is in position x_1, with a compression wave $x_2 - x_1$ in length spreading ahead of it. The passage of successive elementary waves causes the steepness of the curve AB to increase more and more (dashed lines), so that a sudden jump in compression (compression shock) is formed. In the lower part of

Figure 239a the arrows indicate arbitrary values of the velocities of propagation of compression in various parts of the compression-wave front AB.

Now let the piston, which has been pushing the gas ahead of it from left to right, stop instantaneously. A rarefaction is now formed in front of it, which will also spread from left to right. However, in this case no shock wave is formed. Indeed, during the rarefaction each successive wave travels more slowly than the preceding one; and the curve AB has an increasingly greater slope. We therefore come to the conclusion that no rarefaction shock waves can exist*.

In an ideal gas, in which the viscosity and thermal conductivity are zero, the compression shock is infinitely thin (ideal surface of discontinuity). This leads to a well-known paradox of shock-wave theory: how are energy losses by a shock wave possible in a gas which is neither viscous nor heat conducting? This is explained by the fact that real gases always possess viscosity and thermal conductivity, however small these may be. It can be demonstrated theoretically that the existence of even extremely low thermal conductivity results in a compression discontinuity with a very small, but finite, thickess**.

Hugoniot Equation (Shock Adiabatic). The properties of a shock wave will now be discussed in more detail. Let us again consider the case of a piston traveling in a long cylindrical tube. Since compression waves moving through regions of denser gas travel more quickly than waves in a less dense gas, a plane shock wave is formed in front of the piston. This wave will travel to the right through the undisturbed gas with a certain velocity v_{sh}; and it will leave behind it disturbed gas, moving with a velocity of accompanying motion v, equal to the piston velocity u.

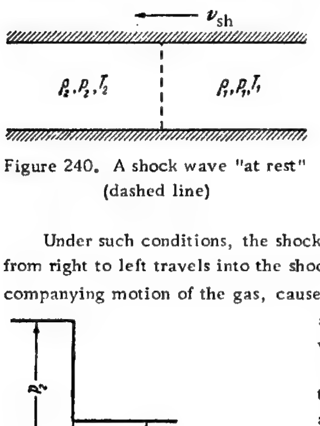


Figure 240. A shock wave "at rest" (dashed line)

The shock wave encounters in its motion undisturbed gas characterized by p, ρ and T, and leaves behind it disturbed gas with new constant disturbed values p', ρ', T'; therefore, the velocity of the shock wave $v_{sh} > v$ will be constant (without taking into account energy losses of the wave due to the viscosity and thermal conductivity of the medium).

By imparting to the whole tube, including the gas and the piston, a motion with a velocity v in the direction opposite to shock-wave propagation (see Figure 240), it is possible to "stop" the shock wave. Under such conditions, the shock wave stands in one place (Figure 240, dashed line), and the gas moving from right to left travels into the shock wave with a velocity $v_1 = v_{sh} - v$ (where v is the velocity of accompanying motion of the gas, caused by the shock wave). Quantities characterizing the state of the gas ahead of the shock wave have the subscript 1; those behind the shock wave have the subscript 2. A plane shock wave of this type is called a "normal compression shock" (Figure 241). In the case under consideration the gas (with pressure p_1, density ρ_1, and temperature T_1) traveling in a parallel manner from right to left with a velocity $v_{sh} = v_1$, undergoes a compression discontinuity as it passes through plane AA'. Its velocity drops to v_2, its pressure increases to p_2, and its density, to ρ_2. Let us determine the relation between quantities p_1, v_1, ρ_1, T_1 and p_2, v_2, ρ_2, T_2.

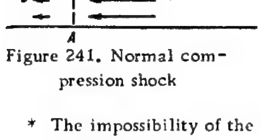


Figure 241. Normal compression shock

This problem can be solved by employing the fundamental laws of mechanics: the conservation laws for matter, energy, and momentum***. Less strictly, but more simply, this relation may be found in the following way. The velocity of propagation of sound is determined by the formula

$$c = \sqrt{\gamma \frac{p}{\rho}} = \sqrt{\frac{\Delta p}{\Delta \rho}},$$

* The impossibility of the formation of a rarefaction shock wave also follows from general thermodynamic considerations.

** The width of a shock-wave front is of the order of the mean free path of the molecules in the gas.

*** Cf. Prandtl, L. Fluid Mechanics. Russian translation. 1949.

where $\gamma=\frac{c_p}{c_v}$, Δp is a small increment of pressure, and $\Delta\rho$ is a small increment of the density of the medium in which the sound is traveling.

For the large disturbances of the gas caused by a shock wave,

$$\frac{\Delta p}{\Delta\rho}=\gamma\,\frac{\bar{p}}{\bar{\rho}},$$

where $\bar{p}=\frac{p_1+p_2}{2}$, $\bar{\rho}=\frac{\rho_1+\rho_2}{2}$ are mean values of p and ρ. Since

$$\Delta p=p_1-p_2,\quad \Delta\rho=\rho_1-\rho_2,$$

it follows that

$$\frac{p_1-p_2}{\rho_1-\rho_2}=\gamma\frac{p_1+p_2}{\rho_1+\rho_2}.$$

This equation can be written

$$\frac{p_2}{p_1}=\frac{(\gamma+1)\rho_2-(\gamma-1)\rho_1}{(\gamma+1)\rho_1-(\gamma-1)\rho_2};$$

this is called the Hugoniot equation (also called the Hugoniot adiabatic, or shock adiabatic).

For the continuous motion of an ideal gas, the relation between p and ρ is given by the Poisson equation (Poisson adiabatic):

$$\frac{p_2}{p_1}=\left(\frac{\rho_2}{\rho_1}\right)^{\gamma}.$$

Turning to the equation for the shock adiabatic curve, it is not difficult to conclude that for

$$\frac{\rho_2}{\rho_1}=\frac{\gamma+1}{\gamma-1}$$

the ratio $\frac{p_2}{p_1}$ approaches infinity; i.e., this adiabatic curve has a definite asymptote (the limiting value of $\frac{\rho_2}{\rho_1}$), shown in Figure 242 as a dashed line. Hence, if the amount of compression p_2 is indefinitely increased, the density of the ideal gas approaches the limiting value

$$\rho_2=\rho_1\frac{\gamma+1}{\gamma-1}.$$

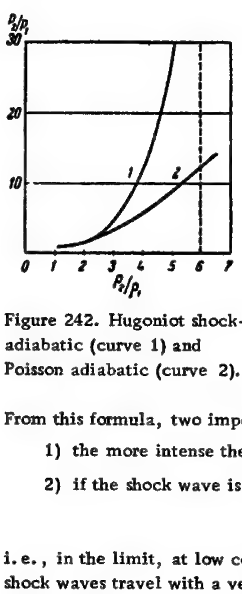


Figure 242. Hugoniot shock-adiabatic (curve 1) and Poisson adiabatic (curve 2).

This means, for instance, that for a diatomic gas (like air, $\frac{c_p}{c_v}=\gamma=1.43$), however great the suddenness of the discontinuity, the limiting increase in density during compression cannot be more than 6*; for a monatomic gas, the limiting bulk compression is 4. Thus, during intense compression the density of a gas increases comparatively slowly; but the volume V also increases slowly, which leads to a rapid increase in the product pV. Since the equation of state for an ideal gas is $pV=RT$, it is seen that pV determines the gas temperature, so that during intense compression the temperature of the gas increases rapidly.

Velocity of a Shock Wave. As shown by theory, the velocity of a shock wave is determined by the formula:

$$v_{\mathrm{sh}}=c\sqrt{\frac{\gamma-1}{2\gamma}+\frac{\gamma+1}{2\gamma}\frac{p_2}{p_1}}.$$

From this formula, two important conclusions follow:

1) the more intense the shock wave (the greater the ratio $\frac{p_2}{p_1}$), the greater is its velocity;

2) if the shock wave is a weak one $\left(\frac{p_2}{p_1}\approx 1\right)$, then

$$v_{\mathrm{sh}}\approx c,$$

i.e., in the limit, at low compressions, the shock wave travels with the velocity of sound, while powerful shock waves travel with a velocity far in excess of this. Thus, in an explosion the shock wave (often called the explosion wave) travels considerably ahead of the sound of the explosion.

Velocity of Accompanying Motion. From the continuity equation (law of conservation of matter), the mass of gas m_1 entering the shock wave is equal to the mass of gas m_2 leaving the shock wave. For a tube

* During strong compression of a gas, its temperature rises considerably. Since at high temperatures γ becomes less, the limiting bulk compression is somewhat increased. Calculations show the number 6 to be correct, provided the compression does not exceed a few hundred atmospheres.

1 cm2 in cross section, the following mass of gas will enter the shock wave during 1 sec,

$$m_1 = \rho_1 v_1,$$

while the following mass of gas will leave the shock wave,

$$m_2 = \rho_2 v_2.$$

Thus, in the present case (the motion of a gas in a cylindrical tube) the following mass of gas passes through a normal compression shock:

$$\rho_1 v_1 = \rho_2 v_2.$$

But we have already seen that

$$v_1 = v_{sh}, \quad v_2 = v_{sh} - v,$$

so that

$$\rho_1 v_y = \rho_2 (v_{sh} - v)$$

and

$$v = \left(1 - \frac{\rho_1}{\rho_2}\right) v_{sh}.$$

Substituting the value for $\frac{\rho_1}{\rho_2}$ from the Hugoniot equation, we obtain for the velocity of the accompanying motion caused by the shock wave the following expression:

$$v = \sqrt{\frac{2}{\gamma}}\, c \frac{\frac{p_2}{p_1} - 1}{\sqrt{(\gamma - 1) + (\gamma + 1)\frac{p_2}{p_1}}}.$$

Hence, it is seen that at $\frac{p_2}{p_1} \approx 1$ the velocity of accompanying motion is negligible. But with increasing intensity of the shock wave, i.e., for large compressions $\frac{p_2}{p_1}$, the gas-flow velocity increases rapidly. Thus, for a relative compression of the gas $\frac{\Delta p}{p_1} = 22.2$ ($\Delta p = p_2 - p_1$), the propagation velocity of the shock wave is nearly 5 times the velocity of sound; simultaneously, behind the shock wave there occurs a strong accompanying motion of the air, with a velocity more than 3 times the velocity of sound in still air. However, during weaker compression of air by a shock wave, a strong accompanying motion of the gas also occurs. For instance, at $\frac{\Delta p}{p_1} = 0.47$ (see table below) the velocity reaches 93 m/sec, i.e., it exceeds that of the most powerful hurricane.

The law of conservation of matter, and the fact that the density of the medium ρ_2 after the passage of a compression shock is greater than the density before the shock ($\rho_2 > \rho_1$), indicate that

$$v_1 > v_2,$$

i.e., the gas-flow velocity ahead of a normal compression shock is always greater than the velocity after the shock.

It can also be demonstrated that ahead of the shock the gas travels with a supersonic velocity, and after the shock with a subsonic velocity.

In the table the following quantities, evaluated using the previously given formulas, are given for different relative compressions: velocity of motion of the front of a plane shock wave; velocity of accompanying

Relative compression, $\frac{\Delta p}{p_1}$	Relative condensation, $\frac{\Delta \rho}{\rho_1}$	Velocity of motion of shock-wave front, m/sec	Velocity of accompanying motion of gas, m/sec	Temperature gradient
0	0	340	0	0
0.47	0.30	400	93	33
1.39	0.81	500	224	87
9.20	2.77	1000	739	465
22.2	3.74	1500	1181	1075
92.2	4.58	3000	2880	5940
165	4.72	4000	3300	7750
258	4.78	5000	4135	12 100

motion of the gas (air at normal atmospheric pressure and 15°C); relative change in density $\frac{\Delta\rho}{\rho_1} = \frac{\rho_2 - \rho_1}{\rho_1}$; and temperature gradient.

Any gas will become luminous at the temperatures indicated in the lower part of the table. This explains the light phenomena accompanying explosions; in many cases, the light is produced by the shock wave, not by the gases of the exploding substance. If the shock wave encounters an obstacle in its path, the luminosity increases. Figure 243 is a photograph of this phenomenon; the shock wave causes a paper sheet to glow (the latter remains stationary during the short exposure). There are grounds for assuming that the luminosity of meteorites is due to shock waves arising as the meteorites travel through the Earth's atmosphere at supersonic speeds.

For shock waves formed during the combustion of inflammable gaseous mixtures it is necessary to take into account the energy of the chemical reaction occurring during the explosion. Two cases should be distinguished: explosion and detonation. In the first case, rapid combustion takes place; in the second, ignition spreads through the gas due to the rise in temperature caused by the shock wave. The possibility of detonation (which occurs in gaseous mixtures, as well as in liquids and solids) depends upon the chemical and physical properties of the substance: the rate of chemical reaction, the thermal conductivity, etc. The velocity of propagation of detonation is quite considerable. For a detonating gas* at atmospheric pressure and room temperature it is 2800 m/sec, and for nitroglycerine, 7400 m/sec; immense pressures arise (100,000 kg/cm^2) for nitroglycerine.

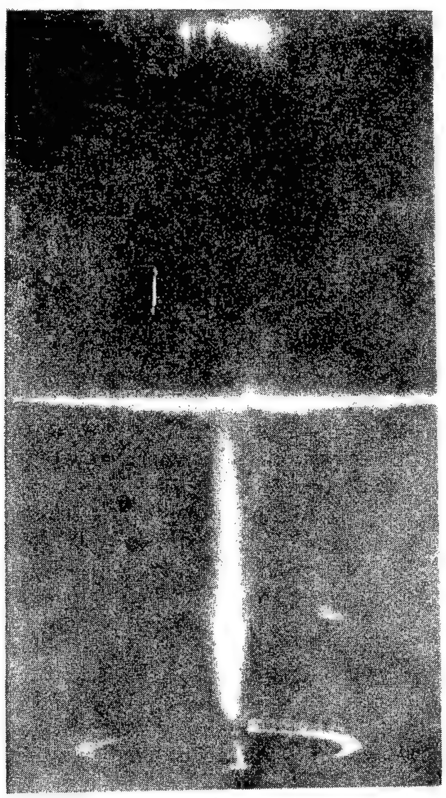
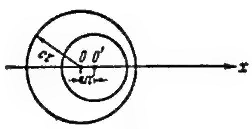


Figure 243. Luminosity caused by a shock wave

* [Stoichiometric water-forming hydrogen-oxygen mixture].

Shock Formation During Supersonic Flow past a Body. Let us first consider some kinematic properties of the propagation of sound from a source traveling at supersonic speed. If a point source of sound is stationary, it emits spherical waves, spreading with the velocity of sound and, in the course of time, filling up the entire space around the source. If the source is traveling at a uniform velocity u, then sound from the source is propagated in the direction of motion of the source at a velocity $c+u$, while in the opposite direction it spreads at a velocity $c-u$. In this case the distribution of the sound disturbance in space is no longer symmetrical (Figure 244a). Nevertheless, in this case also (if $u<c$) sound spreads to every point in space.



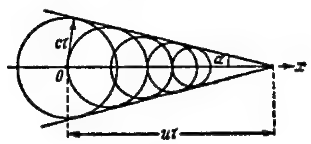


Figure 244. Motion of source in x direction with (a) subsonic and (b) supersonic velocity

The situation is changed substantially if the source travels at a supersonic speed ($u>c$). In this case the signal cannot reach points which lie in front of the source — the latter is not capable of sending sound signals ahead of it. The spherical waves are confined to a cone [the Mach cone], with its vertex at the source (Figure 244b). The region ahead of the source remains undisturbed.

It is not difficult to determine the angle between the cone generatrix (the front of the propagating wave) and its axis, which coincides with the direction of motion of the source. During a small time interval τ the radius of the spherical wave becomes $c\tau$; and the center of this wave is left by the source a distance $u\tau$ behind. Since the generatrix is tangent to the sphere, the following relation holds true:

$$\sin\alpha=\frac{c\tau}{u\tau}=\frac{c}{u}=\frac{1}{M},$$

where M, the Mach number, is greater than unity.

We know that, when objects move through a gas at a velocity greater than the velocity of sound (or, correspondingly, when gas flows about a body at a supersonic speed), compression shocks, or shock waves, are formed. A supersonic flow striking the nose of a projectile is decelerated to zero relative velocity at the point of splitting of the air stream. The transition from supersonic to subsonic velocities results in a shock wave, which is formed ahead of the nose of the projectile. This shock wave is called a "frontal" or "ballistic" wave (Figure 245). The higher the velocity

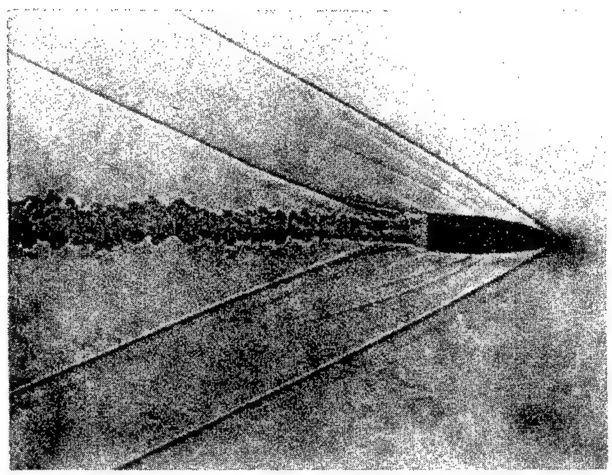


Figure 245. Photograph of a bullet in flight, obtained by the schlieren method

of motion of the body, the closer is this wave positioned with respect to it. If the speed of an airplane is less than the velocity of sound, but close to it, then on the surface of the wings and body, owing to local acceleration of the flow, zones are also formed where the flow velocity is greater than the speed of sound; obviously, in this case compression shocks also occur. The formation of the "frontal" wave presents in this case a more complicated picture, when the body has large dimensions or a blunt nose. In addition, an important role is played by rough spots on the object (e.g., the driving band of a bullet; see Figure 245, where additional compression shocks are seen about this band).

Let us attempt to determine the causes of a compression shock arising about a supersonic projectile.

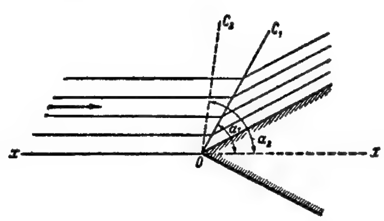


Figure 246. Supersonic flow over a wedge (formation of an oblique compression shock)

Let gas traveling at a supersonic velocity ($M_1>1$) encounter a wedge in its path (Figure 246). Only the part of the flow pattern above the xx axis will be considered; by symmetry, the nature of the flow will be the same in the lower portion. If the supersonic flow over the wedge is continuous, it is possible to reason as follows. The disturbance caused by the vertex O is communicated to the moving gas along a certain line OC_1, which makes with the direction of flow an angle α_1 corresponding to a Mach number M_1;

up to this line the flow continues without change. As the flow is deflected by the vertex O of the wedge, the gas is decelerated, and the flow velocity decreases; correspondingly, the Mach number M_1 also decreases to some value M_2. It is evident that a line of disturbance OC_2 forming an angle α_2 with the flow will correspond to the number M_2; where $\alpha_2 > \alpha_1$*. Reasoning in this way, we arrive at an impossible conclusion: that the line of disturbance OC_2 is located further downstream than the line OC_1, i.e., that the disturbances during the gas deceleration are communicated to the gas before deceleration has begun. It remains for us, then, to assume that continuous supersonic motion of the gas flowing around the wedge cannot occur, and that a line of discontinuity must be formed — a compression shock must occur (as it actually does). Contrary to the normal shock considered previously, in the present case an oblique compression shock is created, forming an acute angle with the direction of flow.

Theory shows that the equations describing the normal shock wave are also applicable to the oblique shock; however, instead of the flow velocity used in the former case, its projection onto the normal to the shock must be taken. Hence, an important conclusion follows: unlike the case of the normal shock, where behind the front the velocity always changes from supersonic to subsonic, across an oblique shock the flow velocity may remain supersonic**.

By measuring the angle in the schlieren photograph (Figure 245), it is possible to determine with great accuracy the velocity of motion of a projectile, provided the velocity of sound is known. A part of the wave which

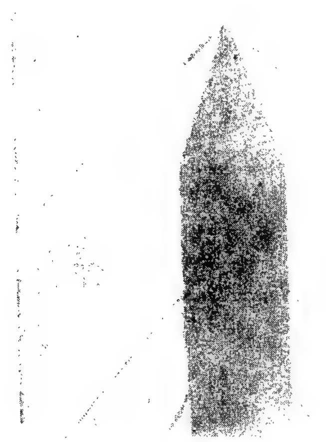


Figure 247. Shock (ballistic) waves spreading about a shell moving at supersonic speed

* $\alpha_1 = \arcsin \frac{1}{M_1}$, $\alpha_2 = \arcsin \frac{1}{M_2}$. Since $M_1 > M_2$, $\alpha_2 > \alpha_1$.

** This property is widely used in practice, e.g., to weaken the bad effect of the shock wave formed at the entrance to the nozzle of a jet aircraft engine. At the nozzle entrance a "needle" is placed, which causes the formation of a series of oblique shocks, thus making for smooth transition of the incoming supersonic flow to a subsonic flow.

is some distance from the bullet must be used, since in the immediate vicinity of the bullet the surfaces of discontinuity (shock waves) are not conical, due to the difference in the compression of the current over the bullet. The latter circumstance is evident from Figure 247, where the detailed structure of the shock waves in the vicinity of a shell has been recorded by means of schlieren photography. It is only at some distance from the shell that the shock wave becomes a regular cone, as we have mentioned above.

A shock wave is propagated in a direction perpendicular to the cone surface. The observer becomes aware of it as an abrupt, sharp sound, in many respects resembling the crack of a whip. The latter originates in the same way; actually, the tip of a whip moves through the air with a supersonic velocity.

Waves in a Free Gas Jet. When a gas jet traveling at supersonic speed flows out of an orifice, compression and rarefaction waves are formed in the free jet. Figure 248 is a photograph, obtained by the schlieren method, showing the wave pattern in an air jet issuing at supersonic speed from a circular nozzle orifice; the pressure in the jet is higher than in the surrounding medium. In this photograph, we see the compression waves or oblique compression shocks (dark portions) and the rarefaction lines (light portions). Figure 249 shows gas flow from the same nozzle, but at the velocity of sound. The appearance of the jet is sharply different from that in Figure 248.

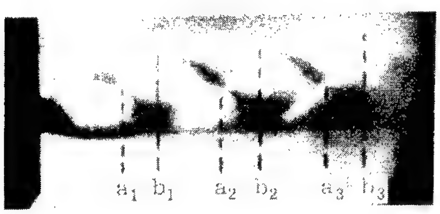


Figure 248. Supersonic flow of gas from a circular nozzle into a medium of lower pressure than the jet

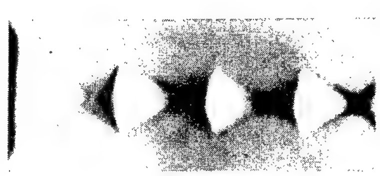


Figure 249. Flow of gas from a circular nozzle at the velocity of sound

Let us now consider briefly the physical picture of wave formation in a jet. First, we shall consider two cases of a gas flowing along a flat wall.

Let the gas move at a supersonic speed along a flat wall which ends at point O (Figure 250); also, let the pressure p_2 in the space to the right of point O be higher than the pressure p_1 in the jet. Employing the same reasoning as in the previous discussion of the formation of an oblique shock, we conclude that in this case, due to deceleration of the gas, an oblique compression shock is also formed (Figure 250a).

If the pressure p_2 in the space to the right of point O is less than in the jet, the situation is reversed. Instead of a compression shock, lines of rarefaction are formed, starting with line OC_2 (Figure 250b); after crossing rarefaction line OC_2, the flow will move rectilinearly and uniformly in a new direction, determined by the angle α.

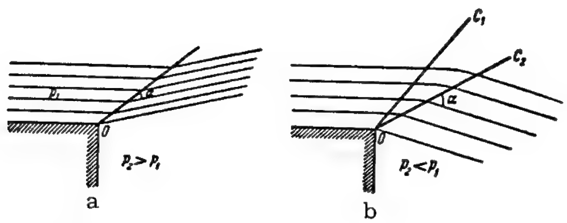


Figure 250. Formation of (a) an oblique compression shock and (b) rarefaction lines for supersonic flow parallel to a wall terminating at point O

These concepts make it possible to explain the wave pattern in a free jet of air, a photograph of which is given in Figure 248. There we see the oblique rarefaction lines and the compression wave pass through each other; and when they arrive at the free boundary of the jet they are reflected from it. The rarefaction lines reflected from the jet boundary are transformed into lines of compression, and vice versa. In the case of a supersonic.gas jet issuing from a circular orifice into a receiver where the pressure is lower than in the jet, rarefaction lines are formed as the jet leaves the orifice. These, spreading as a cone, reach the free boundary of the jet, are reflected by it, and return in a cone as oblique compression shocks.

Gas-Jet Ultrasonic Generator. The wave structure of the gas jet issuing from an orifice at supersonic speed can be utilized to generate powerful ultrasound in air. Figure 251 shows the distribution of pressure along a jet; the latter issues from the circular nozzle orifice at a minimum excess pressure of 0.9 atm. The intervals $a_1 b_1$, $a_2 b_2$, . . . (Figure 248), during which the pressure in the jet rises, are unstable states of the jet. By placing the mouth of a hollow cylinder (resonator) at these points, sonic and ultrasonic waves may be generated.

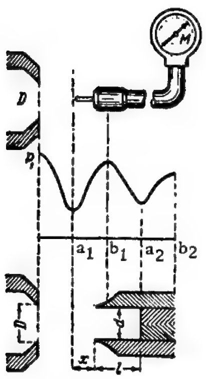


Figure 251. Diagram of the measurement of pressure distribution in a jet (curve p), using a manometer M and by positioning the nozzle and resonator of a gas-jet ultrasonic generator

Figure 252 represents the moment of formation of the vibrations of the compression shock in one of the intervals $a_1 b_1$, $a_2 b_2$, . . . between the nozzle and resonator. In Figure 252a the filling of the resonator with compressed air is shown; the pressure in the resonator exceeds that in the jet. Further (Figure 252b), the compression shock is seen to move from the edges of the resonator toward the left. In the next photograph (Figure 252c) the shock moves still more to the left, and in Figure 252d it stays in the region of instability, where it performs periodic vibrations in the direction of the axis joining the center of the nozzle outlet with that of the resonator. A cylindrical resonator with orifice diameter d and length l (Figure 251) resonates when the wave length satisfies the following equation:

$$\lambda_r = 4(l + 0.3d).$$

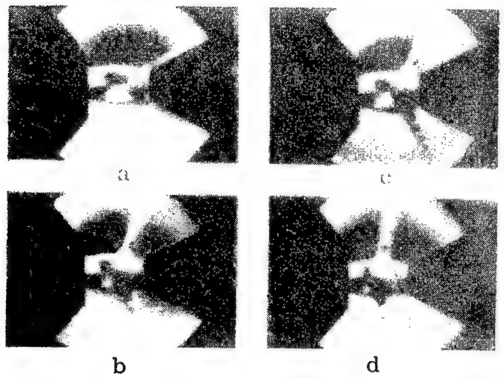


Figure 252. Formation of compression-shock vibrations in a gas-jet ultrasonic generator

In moving the resonator a distance x with respect to the nozzle orifice, it is found that the wave length of the ultrasound radiated by the gas-jet generator changes, becoming

$$\lambda = \lambda_r + kx,$$

where the constant k is 0.6 to 0.7. The most advantageous condition for radiation exists in the case when the diameter D of the nozzle orifice, the diameter d of the resonator orifice, and the resonator length l are all equal ($D = l = d$). For $l = 1$ mm, $\lambda_r \approx 65$ kc.

A gas-jet generator can generate acoustic intensities of several tens of watts. By using air (from a compressed-air cylinder or from a compressor) as the compressed gas blown through the nozzle, frequencies up to 120 kc can be obtained with such a generator. By using hydrogen (in which the sonic velocity is 1265 m/sec at 0°C) instead of air, frequencies up to 500 kc can be achieved.

Figure 253 shows a schematic drawing, and Figure 254 shows a photograph, of a gas-jet generator. In order to find the region in which compression-shock vibrations are generated, the distance between nozzle and resonator can be adjusted by means of a micrometer screw (it is convenient to design the micrometer in such a manner that its moving element is the resonator).

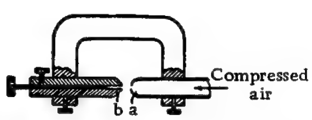


Figure 253. Diagram of a gas-jet ultrasonic generator

a) nozzle, b) resonator.

Shock Waves in Water. Water Hammer. Shock waves can originate and propagate not only in gases, but also in liquids* and solids. Unlike gases,

* Cole, R. Underwater Explosions. -Princeton University Press. 1948. Russian translation. 1950.

Figure 254. Gas-jet ultrasonic generator

the velocity of motion of bodies in liquids in real situations does not exceed the velocity of sound. Indeed, the velocity of sound in water is about 1500 m/sec, i. e., 4.5 times that in air, while the speeds of motion of objects in water so far attained are less than those in air. Hence, no shock waves have as yet been encountered which originate in the supersonic flow of a liquid around an object. However, during explosions in liquids, e. g., in water, as well as during other kinds of sudden changes in pressure, shock waves are also formed. Shock waves forming in water are of higher intensity, due to the high density of water (approximately 800 times that of air), and also due to the high sound velocity in water. When the flow of water in a pipe is stopped abruptly, or in hydraulic-turbine supply systems, as well as in many other cases, instantaneous jumps in pressure occur, and shock waves are formed. This phenomenon is known as water hammer. It may cause serious damage in various types of water systems.

N. E. Zhukovskii was the first (1899) to give a strict theoretical explanation of the water hammer.

In the propagation of sound in a pipe filled with water, it is necessary to take into account deformation of the walls. The velocity of sound in such a pipe is given by an expression derived by Korteweg, a Dutch scientist:

$$v_{sh}=\frac{1}{\sqrt{\frac{\rho_0}{E_w}+\frac{2R\rho_0}{d\cdot E_p}}},$$

where ρ_0 and E_w are, respectively, the density and the bulk modulus of elasticity of water; R is the radius of the pipe; d is the thickness of its walls; and E_p is the modulus of elasticity of the material of the pipe. This formula was shown by Zhukovskii to hold also for the great pressures arising in the case of water hammer and to determine the velocity of propagation of the shock wave in the pipe.

NIKOLAI EGOROVICH ZHUKOVSKII
(1847-1921)

Assuming the pipe walls to be free of deformation ($E_p = \infty$), we obtain from Korteweg's formula

$$v'_{sh} = \sqrt{\frac{E_w}{\rho_0}}.$$

This is the velocity of propagation of sound in a free liquid (see Chapter VII, § 1). On the contrary, if the liquid is assumed to be incompressible ($E_w = \infty$), we obtain

$$v''_{sh} = \sqrt{\frac{d \cdot E_p}{2R\rho_0}}$$

which is the propagation velocity of the change in pressure of an incompressible liquid along an elastic pipe.

Zhukovskii determined theoretically the large pressure increase p caused by a water hammer:

$$p = \frac{u_0 v_{sh} \gamma}{g},$$

where u_0 is the velocity of the liquid with respect to the shock; v_{sh} is the velocity of shock-wave propagation; $\gamma = g\rho_0$ is the specific weight of the liquid; and g is the acceleration of gravity. Thus the "increment" of pressure in the pipe due to the water hammer is directly proportional to the velocity of the liquid with respect to the shock and to the velocity of propagation of the shock wave in the pipe*.

* Zhukovskii, N. E. O gidravlicheskom udare v vodoprovodnykh trubakh (Water Hammer in Conduits). — Gostekhizdat. 1949.

In addition to theoretical calculations, Zhukovskii performed a large number of experiments at the Moscow Water Works at the village of Alekseevskoye, in order to confirm his theory. Also, he gave various practical instructions on safety measures for water systems, in order to minimize the effects of water hammer (gradual opening and closing of valves and water-gates, use of air and water bleeder valves, reservoirs, etc.). It should be noted that the theory of water hammer evolved by Zhukovskii has found practical use in many branches of technology.

Chapter IX

SOUND AND ULTRASOUND WAVES IN SOLIDS

Elastic waves are propagated not only in gases and liquids, but also in solids. Conditions for the propagation of elastic waves in homogeneous solids (such as the majority of metals: iron, steel, aluminum, etc.) are more favorable than conditions, for instance, in air; in metals sound travels great distances, with much less absorption.

In this chapter the fundamental problems related to the physics of the propagation of elastic waves in solids will be discussed, as well as some practical applications.

§ 1. Elastic properties of solids

We already know that the elastic properties of gases and liquids are determined only by their bulk elasticity and that, accordingly, the only kinds of waves which can travel in them are longitudinal elastic waves*. The elastic properties of solids are essentially different. They not only resist compression and extension, but also any effort to change their shape. Solids possess elasticity of form, which is possessed neither by liquids nor gases. Therefore, not only longitudinal waves, but also other types of waves, can be propagated in solids.

Types of Deformation. Before examining the elastic properties of solids, it is necessary to become acquainted with the fundamental concepts of a more general branch of physics — the theory of elasticity.

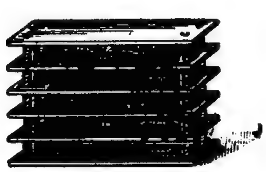


Figure 255. "Shelves", made of boards on springs

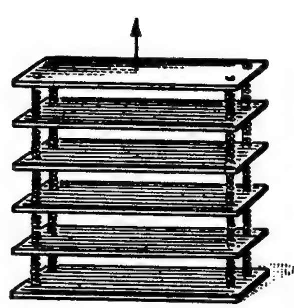


Figure 256. Extension deformation

* Some very viscous liquids (e.g., polyisobutylene) are similar to solids in having elasticity of form, and in being capable of transmitting transverse waves, as well as longitudinal waves.

Under the action of forces, solids change shape; e.g., a rubber bar can be stretched, compressed, bent, or twisted. All the various deformations of solid bodies can be reduced to two basic types, and these can best be understood by means of a model. Let us fasten a few similar boards together one over the other by means of similar springs (Figure 255). These "shelves" serve as a rough model of a solid, where the separate boards are analogous to the layers of a solid.

With the bottom board attached to a table, let us pull the top board upward; the distances between every pair of adjacent boards will increase by the same amount (Figure 256). Similarly, if the top board is pressed down, then the distances between adjacent boards will all decrease by an equal amount (Figure 257). This type of deformation is called *uniform extension or compression*.

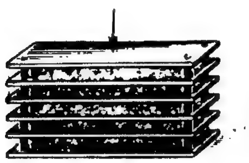


Figure 257. Compression deformation

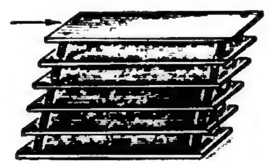


Figure 258. Uniform, or pure, shear deformation

Let us now apply a force to the top board, but parallel to the plane of the board (Figure 258). The distances between the boards will not change, but the boards will become shifted with respect to each other; points previously lying on the same vertical are displaced the same amount with respect to those on adjacent boards. This deformation is called *uniform, or pure, shear*.

The theory of elasticity demonstrates that all the various deformations of a solid can be reduced to these two basic types: extension or compression, and shear.

Figure 259. Flexural deformation

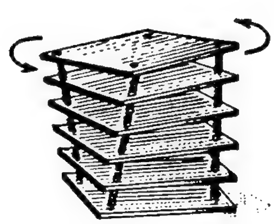


Figure 260. Torsion deformation

For instance, if one side of the top board is pressed down, a deformation is produced which is called *bending or flexure* (Figure 259); the distances between adjacent boards becomes greater on the left and smaller on the right. Various extensions and compressions will occur in different parts of the system. This deformation is a type of *nonuniform extension and compression*. When the top board is twisted in its own

plane (Figure 260), torsion deformation is produced. This is evidently a nonuniform shear deformation; the distances between the boards will not be changed, but points previously lying along the same vertical are mutually shifted through various distances. Points near the edges of the boards undergo the greatest displacement, while points in the middle of the boards are not shifted at all.

Hooke's Law. Deformations [strains] of Extension and Compression. Young's Modulus. Let us now find the quantitative relation between the forces applied to a solid and the resulting deformations (or strains). For example, what is the elongation of a steel rod if the stretching force is P kg. The solution of such problems in elasticity is based on Hooke's law.

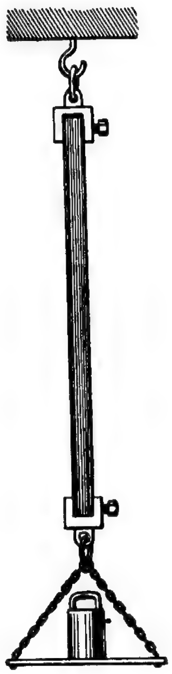


Figure 261. Stretching of a rod

Consider a cylindrical rod of length L_1 and diameter d; its cross-sectional area $S = \frac{\pi d^2}{4}$. One end of the rod is fixed, while a stretching force P is applied to the other end (e.g., a weight is attached to it, Figure 261). The stretching force per unit area $F = \frac{P}{S}$ is called the stress. Under the action of the force P, the rod is deformed (stretched); its length becomes L_2, and the change in length is L_2—L_1.

How is the elongation of the rod, $\Delta L = L_2 - L_1$, related to the stress causing it? Experiment shows that the greater the stress F, the greater is the elongation of the rod. This is Hooke's law, the fundamental law of elasticity, which states: for small deformations, strain is proportional to stress. For large strains, Hooke's law is not obeyed strictly. If the stress is sufficiently large, the elastic limit may be passed. In this case, permanent deformations occur in the rod; the rod will remain somewhat elongated (in general, deformed) even when the stress is no longer applied. If the stress is further increased, the rod can break or rupture.

So, according to Hooke's law, the elongation of the rod ΔL must be proportional to the applied stress $\frac{P}{S}$. Obviously, the longer the rod, the larger is ΔL, all other conditions being equal. Hence, the elongation ΔL must be proportional to $L \cdot \frac{P}{S}$, i.e.,

$$\Delta L = kL \cdot \frac{P}{S}.$$

This equation relates the strain and stress for a rod of length L; the coefficient of proportionality k depends upon the properties of the material composing the rod. It is more customary to deal with its reciprocal $\frac{1}{k}$, usually denoted by E, so that

$$\Delta L = \frac{1}{E} \cdot L \cdot \frac{P}{S},$$

and

$$E = \frac{L}{\Delta L} \cdot \frac{P}{S}.$$

The quantity E is called Young's modulus, the modulus of longitudinal elasticity, or simply the modulus of elasticity. As long as the elastic limit is not exceeded, the modulus of elasticity is constant for a given material; it is a characteristic quantity determining the elastic properties of the material*. Solid bodies, especially metals, offer very strong resistance to changes in their length. Young's modulus E, usually expressed by engineers in kilograms of force per square millimeter, is 22,000 kg/mm^2 for steel, 20,790 kg/mm^2 for iron, 1800 kg/mm^2 for lead, and about 1100 kg/mm^2 for wood (for spruce, with a stress parallel to the fibers). The value of varies with the grade of steel or iron, and given values may vary, within certain limits, even for any one material. The modulus E is also defined in the absolute (cgs) system of units, where it is expressed in dynes per square centimeter. Since 1 kg = 981,000 dyne and 1 cm^2 = 100 mm^2, then in order to express E in the absolute system of units, the values given above should be multiplied by $0.981 \cdot 10^8$; e.g., for steel $E = 2.16 \cdot 10^{12}$ dyne/cm^2.

The greater Young's modulus E, the greater is the stress which must be applied to a rod to stretch it by a given amount. Indeed, since $F = \frac{P}{S}$ and $\Delta L = \frac{1}{E} L \frac{P}{S}$, we obtain

$$F = \frac{\Delta L}{L} E.$$

Here, $\frac{\Delta L}{L}$ is the relative elongation (strain) of the rod and F is the force per unit area (stress). This formula makes it possible to formulate Hooke's law for an extension deformation as follows: the relative elongation is proportional to the stress, and the proportionality coefficient $k = \frac{1}{E}$ is the reciprocal of Young's modulus.

We have discussed only stretching; however, if the force is reversed, compression occurs, and the reasoning is identical.

Poisson's Ratio. When a rod is stretched under the action of an applied force P, its cross section contracts. If the diameter d before stretching decreases by Δd, then, as has been established experimentally, the ratio of relative decrease in rod diameter $\frac{\Delta d}{d}$ to relative elongation $\frac{\Delta L}{L}$ is constant for a given material. This ratio, usually designated by σ, is called the lateral contraction modulus, or Poisson's ratio:

$$\sigma = \frac{\frac{\Delta d}{d}}{\frac{\Delta L}{L}} = \frac{E}{F} \frac{\Delta d}{d}.$$

For metals σ is approximately 0.25, and for rubberlike substances $\sigma = 0.5$. Contrary to Young's modulus E, the value of σ for materials of the same type is approximately the same.

* The following illustration may clarify the meaning of Young's modulus E. Let the stretching force P be sufficiently great to extend the rod to twice its original length. In reality, this cannot be done for such materials as metals, wood, etc., since the elastic limit would be exceeded and the rod broken. Still, let us assume Hooke's law to be applicable to such a stretching action. If the cross-sectional area of the rod $S = 1$, then from the preceding formula we obtain $E = P$. Hence, the modulus of elasticity E represents a force which, when applied to the end of a rod of unit cross section, extends it to exactly twice its original length.

Shear Deformation. Shear Modulus. As shown by the theory of elasticity, Young's modulus E and Poisson's ratio σ are sufficient for a complete description of the elastic properties of any homogeneous, isotropic solid*. However, in many cases it is convenient to introduce as well the so-called shear modulus. As mentioned above, not only are deformations [strains] of extension or compression possible in solids, but also shear deformations. Let us now examine in more detail the manner in which a shearing strain occurs in the "shelves" made of boards and springs. In the case of uniform, or pure, shear, the stack of "shelves" will assume the shape of an oblique parallelepiped, but without changing in height (Figure 263). The top board is shifted parallel to itself, all its points being displaced, with respect to points in the bottom board previously lying on

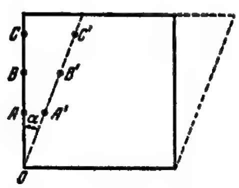


Figure 262. Pure shear

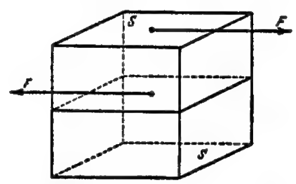


Figure 263. Shear in a cubical body, caused by the action of a couple

the same verticals, by the same distance and in the same direction. Points on intermediate boards and on the joining springs are displaced less (e.g., AA') than points nearer to the top board (e.g., CC'). The shearing strain can be defined by any of the displacements AA', BB', CC', etc., which represent the amount of absolute shear. But it is better to consider the relative shear, in the same manner as the relative elongation was used in the discussion of extension deformation. Though distances AA', BB', CC', ... are different, their ratios to distances OA, OB, OC are equal, i.e., the ratio

$$\frac{AA'}{OA} = \frac{BB'}{OB} = \ldots = \gamma$$

remains unchanged**. The quantity γ is called the relative shear. For extension or compression deformations we introduced a proportionality coefficient (Young's modulus E) between the applied stress and the corresponding strain. The same can be done in the case of shear deformation. Let us consider a cubical body with face area S (Figure 263). For uniform shear, caused by a force F acting in the plane of one face, we have, according to Hooke's law,

$$\frac{F}{S} = \mu\gamma.$$

If the stress $\frac{F}{S}$ is denoted by τ, then

$$\tau = \mu\gamma.$$

* Bodies whose physical properties (density, modulus of elasticity, etc.) are the same in all directions are called isotropic. Some bodies may be homogeneous, yet not isotropic; and they are said to be anisotropic (e.g., crystals). Thus, crystals of quartz and Rochelle salt have different piezoelectrical properties and different Young's moduli along their X, Y, Z axes.

** $\gamma = \mathrm{tg}\,\alpha$, where the angle α (measured in radians) is called the angle of shear. For small angles of shear, $\mathrm{tg}\,\alpha \approx \alpha$, and the relative shear is simply the angle of shear.

Here, μ is the coefficient of proportionality between the applied stress and the resulting relative shear. Just as Young's modulus E, the coefficient μ depends upon the physical properties of the solid but not upon its size and shape.

The coefficient μ is called the shear modulus. From the formula it is seen that the greater the shear modulus μ, the greater is the force which must be applied to the body to produce a given shear. The shear modulus, as well as E, is measured either in kg/mm^2 (practical units), or in dyne/cm^2 (cgs units). For cast steel, for instance, $\mu = 8.4 \cdot 10^{11}$ dyne/cm^2, for cast iron $\mu = 4.4 \cdot 10^{11}$ dyne/cm^2, for brass $\mu = 3.5 \cdot 10^{11}$ dyne/cm^2. The theory of elasticity indicates that the quantities E, σ, and μ are not independent, but are related by the equation

$$\mu = \frac{1}{2} \frac{E}{1+\sigma}.$$

§ 2. Types of elastic waves. Measurement of elastic moduli of solids by acoustical methods

The discussion in the preceding section gives us a clear notion of the way in which solids differ from liquids and gases in their acoustical properties.

Longitudinal Waves. We will begin with those features of wave motion which are similar for both solids and fluids, and then we will pass to processes which are essentially different, occurring only in solids. In solids, as in liquids, longitudinal elastic waves may be propagated, in which movements of the particles are performed in the direction of propagation of the wave. The mechanism of formation of an elastic longitudinal wave in a solid is in no way different from that of the formation of an elastic (or sound) wave in a liquid or gas. In fluids, an elastic wave originates due to the elasticity of the medium and the inertia of its particles, and the same is true in a solid.

Let us suspend a long metal rod on a thread and hit it sharply on the end with a hammer (Figure 264). As a result of the blow, compression is caused at the end of the rod, and the particles of the solid body are displaced in the direction of the blow. Because of the elasticity of the body and its inertial properties, this compression is communicated to adjacent layers of the rod; an elastic pulse will run along the rod, similar to a sound pulse propagating through the air. This elastic pulse, or compression wave, reaches the opposite end, is reflected from it, returns, is again reflected, and so on. The coefficient of reflection of an elastic wave from the end of a rod, at the metal-air interface, is close to unity, as already mentioned. Therefore, the elastic pulse formed as a result of the hammer blow will travel back and forth in the rod, and will undergo a large number of reflections. After the blow, a sound is heard, the intensity of which gradually decreases. Attenuation of the elastic wave propagating in the rod occurs because part of the energy of the blow is radiated by the rod as sound waves; also, the pulse energy is diminished due to the forces of internal friction in the metal; the imperfect elasticity of the metal causes vibrational energy to be transformed into heat.

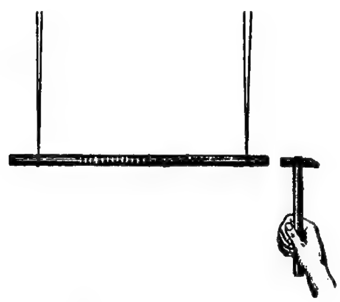


Figure 264. A hammer blow on the end of a suspended rod

Measurements of Young's Modulus by an Acoustical (Dynamical) Method. If one end of a rod is in some way subjected to periodic longitudinal compressions and extensions, standing longitudinal waves are formed in the rod. This may be done in the following way. The rod is attached at its midpoint to a holder and placed together with the holder in a special stand

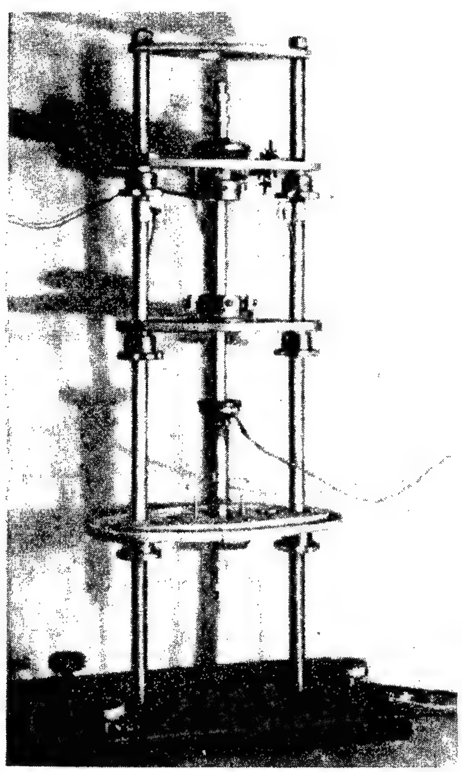


Figure 265. Special stand with rod, exciting coil, and receiver, for measurements of Young's modulus and shear modulus of solids

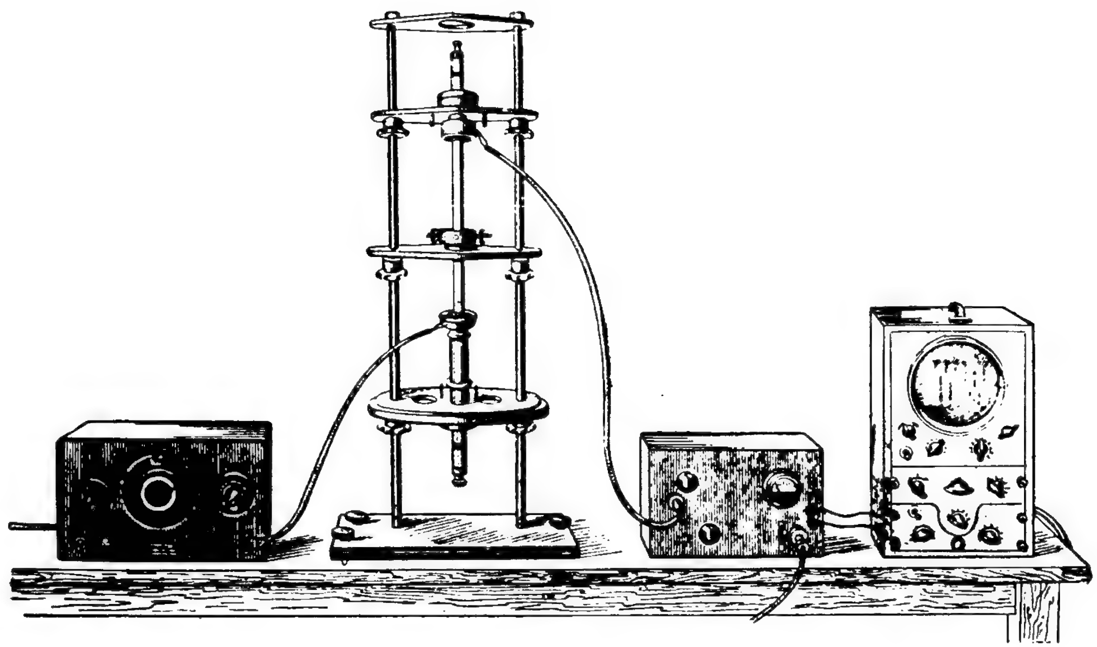


Figure 266. Equipment for measurements of Young's modulus and the shear modulus of solids

(Figure 265). An electromagnetic exciter, e.g., the coil of an ordinary electromagnetic telephone*, is adjusted to touch one end of the rod, by means of a micrometer screw. If voltage from an audio oscillator (Figure 266, left) is applied to the telephone coil, then a periodically changing attractive force will act upon the end of the rod, just as upon a telephone diaphragm. Thus, longitudinal elastic waves are continuously excited in the rod. In order to make this possible, the rod should be made of a magnetic material: steel, iron, nickel, etc. In a nonmagnetic rod (glass, ebonite, aluminum, brass, etc.), longitudinal vibrations can be produced by cementing a thin sheet of transformer iron to the end of the rod.

If, by means of the micrometer screw, a similar electromagnetic receiver is brought close to the upper end of the rod, then an alternating emf is generated in it during the vibrations of the rod. By amplifying this alternating voltage and applying it to an electronic oscilloscope, the vibrations of the rod can be observed (Figure 266).

If the frequency of the audio oscillator is gradually varied, then at a certain value f_1 the oscilloscope will detect a sharp increase in vibration amplitude (especially for a steel or aluminum rod). The rod begins to emit sound, indicating resonance conditions. Due to the vibrations of the rod, standing longitudinal waves are formed in it. Since it is fixed at its midpoint, there is no displacement of the particles at this point; here a displacement node always exists and also a velocity node. The principal resonance of the rod vibrations, i.e., maximum vibration, is observed when the length L is equal to one half wave length of the longitudinal elastic wave λ, i.e., when $L=\frac{\lambda}{2}$. In this case, the ends of the rod will have displacement antinodes. If the oscillator frequency is further increased, a second resonance point is observed at a certain frequency f_2, where $f_2=3f_1$. The rod evidently vibrates in its odd harmonics. A diagram of the vibrations of the rod at its fundamental frequency and its first overtone is given in Figure 267.

If the frequency f_1 of fundamental resonance of the rod and the rod length L are known, it is easy to find the propagation velocity of longitudinal waves in the rod. Indeed, since $\lambda=\frac{c}{f}$ and since at resonance $L=\frac{\lambda}{2}$, then $c_{\text{long}}=\lambda f_1=$ $=2Lf_1$, where c_{long} is the velocity of longitudinal waves in the rod.

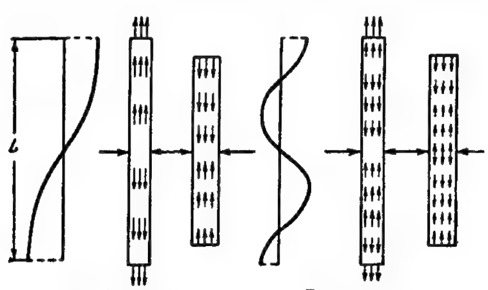


Figure 267. Vibrations of a rod, fixed at its midpoint, in its fundamental and first overtone

* The clearance between the ends of the telephone magnets and that of the rod should be a fraction of a millimeter; they must not touch, so as to permit the rod to vibrate freely.

Thus, measurement of the propagation velocity of longitudinal waves is actually reduced to measurement of the resonance frequency of vibrations of the rod and of the rod length. Modern electronic methods make it possible to perform frequency measurements with a very high degree of accuracy — up to 0.01 % and more.

For a rod made of steel with a length $L = 50$ cm, such measurements showed that primary resonance occurred at 5100 cycles. Hence,

$$c_{\text{long}} = 2 \cdot 50 \cdot 5100 = 510{,}000 \text{ cm/sec} = 5100 \text{ m/sec}.$$

The propagation velocity of a longitudinal wave in a rod with a diameter considerably less than the wave length* is

$$c_{\text{long}} = \sqrt{\frac{E}{\rho}},$$

where E is the Young's modulus of the rod material and ρ is its density. If in this formula E is in dynes per cm^2 and ρ is in g/cm^3, then c is in cm/sec.

In this way, since the density of the rod material is known, and the propagation velocity c_{long} of longitudinal waves in it was measured, Young's modulus $E = \rho \cdot c^2_{\text{long}}$ can be calculated. This is one of the most precise methods of measuring Young's modulus, and is known as the method of dynamical measurement (since measurements are made during vibration). It should be noted that the values of E found from dynamical and from statical measurements (by measuring the elongation under constant load) may differ considerably for some materials, especially for porous and plastic materials.

For materials which cannot be molded into a rod (e.g., pitch, animal fat, rubber, etc.) or of which only short rods can be made, thus making it necessary to deal with very high resonance frequencies (calling for crystals of quartz or Rochelle salt), the so-called method of the composite rod is used. A small rod of the material in question is cemented onto a relatively long basic rod made of metal (20 to 30 cm long). If the resonance frequency of the basic rod is known, then from the resonance frequency of the composite rod, the primary resonance frequency of the sample can be found. If the length and density of the sample are measured, then its Young's modulus may be determined.

Observation shows that for a steel or aluminum rod very sharp resonance is obtained, so that a slight change in oscillator frequency is sufficient to make the vibration amplitude, as seen on the oscilloscope, fall sharply. For a lead or ebonite rod, the resonance is much less sharp. The reason for this is that in steel and aluminum longitudinal waves are propagated with very low attenuation, while in lead and ebonite the attenuation is quite large. Different solids possess different capacities for absorbing elastic waves. A bell made of copper, steel, or cast iron continues sounding a long time after it has been struck. No such sounding is obtained with, for example, a lead bell; while a rubber bell would give no sound whatsoever. Of metals, aluminum has the lowest sound absorption. In iron, the absorption of longitudinal elastic waves is approximately 12 times as great as in aluminum; in steel, 50 times; in copper, 70; in glass, 130; in cork, 8500; and in rubber, 13,000 times that of aluminum.

* If the length of the rod L is comparable to its diameter d (e.g., if L is only about 7 or 8 times greater than d), a correction must be introduced for the lateral expansion and contraction of the rod during its longitudinal vibration, and the formula for c_{long} becomes more complicated.

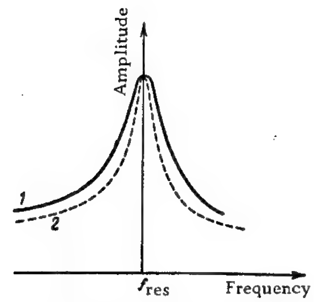


Figure 268. Resonance curves for vibrations of an ebonite (curve 1) and a textolite (curve 2) rod

Length of textolite rod, 35 cm; $f_{res} = 3300$ cycles, $c_{long} = 2310$ m/sec. Length of ebonite rod, 40 cm; $f_{res} = 2010$ cycles; $c_{long} = 1600$ m/sec.

Once the resonance curve for the vibrations of a rod is obtained, it is possible to calculate the absorption coefficient for longitudinal waves in the rod material. Figure 268 shows the resonance curve for longitudinal vibrations in ebonite and textolite rods, obtained using the equipment of Figure 266. The curves show sharper resonance for textolite than for ebonite. This means that the absorption of longitudinal waves in ebonite is greater than in textolite. Resonance curves for such metals as steel and aluminum are extremely sharp, indicating very low elastic-wave attenuation in these metals.

Velocity of Longitudinal Waves in a Continuous Medium. We have discussed longitudinal elastic waves propagating in a rod with transverse dimensions much less than the wave length. For the case of longitudinal waves propagating through an infinite solid body, the velocity of propagation is

$$c^*_{long} = \sqrt{\frac{E(1-\sigma)}{\rho(1+\sigma)(1-2\sigma)}},$$

where, as before, E is Young's modulus, σ is Poisson's ratio, and ρ is the density of the medium.

For many solids, $\sigma = 1/4$, in which case*

$$c^*_{long} \approx \sqrt{1.21\frac{E}{\rho}} = 1.1\sqrt{\frac{E}{\rho}}.$$

Thus, the propagation velocity of longitudinal waves in an infinite solid is somewhat greater than in a rod. This is true because the rigidity of the continuous medium is effectively greater than that of a thin rod. The sides of a rod are free and have no surrounding medium which restrains their deformations; but if a similar rod is imagined to be placed in a continuous medium, its sides must be in contact with the rest of the medium.

Transverse, or Shear, Waves. Every solid body resists changes in its shape. Let us again strike a rod suspended by a thread, but along the

* For $\sigma = \frac{1}{3}$ (brass, aluminum, and many other solids)

$$c^*_{long} \approx 1.2\sqrt{\frac{E}{\rho}}.$$

side rather than at the end; i.e., the blow is normal to the lateral surface. The layer of particles at the point which is struck is displaced from its equilibrium position — it is subjected to a shearing strain. Due to the elastic binding between particles of the rod, this shear is communicated to the adjacent layer. Elastic forces and the inertia of the particles bring about the formation in the rod of a shear wave. The wave itself travels along the rod, as in the preceding cases, but the motion of the particles is now directed normally with respect to the direction of propagation of the wave; and so the wave is t r a n s v e r s e*.

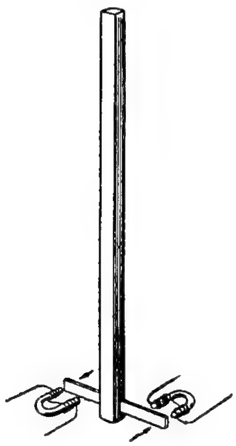


Figure 269. Excitation of transverse vibrations in a rod

Quite pure transverse, or shear, elastic waves can be obtained in a rod by means of an electromagnetic exciter. This method is shown in Figure 269. Voltage from an audio oscillator is applied to the exciter, producing transverse vibrations in the rod. The end of the rod is acted upon by a torsional couple, twisting the rod about its axis (torsion is a combination of simple shear deformations without any extension or compression). A detector, identical to the exciter and mounted at the upper end of the rod, makes it possible to detect shear vibrations and to find the resonance frequencies for transverse vibration of the rod. The propagation velocity of transverse waves in a rod is given by the formula:

$$c_{\text{trans}} = \sqrt{\frac{\mu}{\rho}},$$

where μ is the shear modulus of the rod material. This velocity c_{trans} can be determined by the same method as the velocity c_{long} for longitudinal waves.

Since there is a relation between the moduli E and μ and Poisson's ratio (see end of § 1 of this chapter):

$$\mu = \frac{1}{2}\frac{E}{1+\sigma},$$

we obtain for steel $\left(\sigma = \frac{1}{4}\right)$:

$$\mu \approx \frac{E}{2.5} = 0.4\,E \quad \text{and} \quad c_{\text{trans}} = \sqrt{\frac{\mu}{\rho}} \approx 0.63\sqrt{\frac{E}{\rho}} = 0.63\,c_{\text{long}}.$$

Thus, the propagation velocity of transverse waves in a steel rod is almost one half that of longitudinal waves.

In an infinite solid medium, the velocity of shear waves is expressed by the same formula as that for shear waves in a rod, so that $c_{\text{trans}} = c^{*}_{\text{trans}}$ unlike longitudinal waves, for which the velocity in a continuous body is greater than in a rod. Although the compressional rigidity of a continuous medium is greater than that of a rod, for shear vibrations the rigidity is the same in both cases.

* In reality, when the rod is struck pure transverse waves are not produced; and the picture is more complicated. In order to produce pure shear waves by striking the rod as described, the rod must be very thick and short (otherwise it bends).

The table below lists the velocities of longitudinal and transverse waves in some solids, as well as the elastic moduli (E, μ, and σ) and the acoustic impedance ρc for these solids. All the quantities are expressed in cgs units. The acoustic impedance is given in terms of the velocity c_{long} in a rod; for a continuous medium, the velocity c^{*}_{long} should be used.

Density, elastic moduli, sound velocity, and acoustic impedance for some solids

Substance	Density ρ, g/cm^2	Young's modulus, $E\times10^{11}$, dyne/cm^2	Shear modulus, $\mu\times10^{11}$, dyne/cm^2	Poisson's ratio σ	Velocity, cm/sec: Longitudinal waves in a rod $c_{long}\times10^5$	Longitudinal waves in a continuous medium $c^{*}_{long}\times10^5$	Transverse waves $c_{trans}\times10^5$	Acoustic impedance $\rho c_{long}\times10^6$
Steel	7.8	21.6	8.4	0.28	5.05	6.1	3.3	3.93
Cast iron	7.7	11.5	4.4	0.27	3.85	4.5	2.4	2.96
Copper	8.93	12.3	5.55	0.35	3.58	4.6	2.26	3.20
Brass	8.5	10	3.5	0.37	3.42	4.25	2.0	2.90
Aluminum	2.65	7.05	2.63	0.32	5.25	6.4	3.16	1.39
Lead	11.4	1.62	0.56	0.45	1.2	2.2	0.7	1.37
Quartz (along the X-axis)	2.65	7.9	-	-	5.45	-	-	1.44
Wood (oak)	0.8	1.3	-	-	4.05	-	-	0.32
Paraffin wax	0.9	0.192	-	-	1.46	-	-	0.13
Rubber	0.9	0.0001	-	0.5	0.03	-	-	0.003

Consequently, in an infinite solid medium two types of waves are possible, longitudinal and transverse; the velocity of the former is approximately twice that of the latter.

Dispersion of Ultrasonic Longitudinal Waves in a Rod. As already mentioned, the velocity of longitudinal elastic waves in a rod (provided the wave length is greater than the rod diameter) is

$$c_{long}=\sqrt{\frac{E}{\rho}}.$$

On the other hand, when the wave length is considerably less than the rod diameter, the rod may be considered to be an infinite solid body; and the velocity of longitudinal waves is (for steel, $\sigma=1/4$):

$$c^{*}_{long}\approx 1.1\sqrt{\frac{E}{\rho}}.$$

Consequently, for wave lengths comparable to the diameter of the rod, there is a dispersion region, in which the velocity of longitudinal waves depends upon wave length. This is confirmed by experiment.

Figure 270 represents the velocity of propagation of longitudinal waves in a hollow nickel rod as a function of frequency. For low frequencies, velocity $c_{long}=\sqrt{\frac{E}{\rho}}=4925$ m/sec. As the frequency increases, c_{long} decreases, at first slowly, then rapidly drops to zero. Then there is a narrow frequency band within which waves do not propagate at all. With a further increase in frequency, c_{long} assumes a very large value, which then quickly decreases to 5170 m/sec, corresponding to the velocity of propagation in

a continuous nickel medium. Thus, in a range between about 150 and 250 kc, where the wave length is comparable to the rod diameter, sound dispersion occurs (we shall call it "geometrical dispersion", since it is due not to the inner structure of the medium, but to external factors).

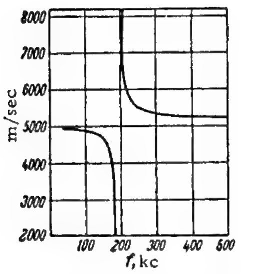


Figure 270. Velocity of propagation of longitudinal waves in a nickel tube

Length of tube, 100 mm; outside diameter, 8.6 mm; inside diameter, 8.3 mm

Group Velocity. Wave-Front Velocity. Signal Velocity. Dispersion of elastic waves does not only occur in rods; we have already encountered it while discussing the propagation of ultrasonic waves in polyatomic gases and organic liquids. Ultrasonic dispersion should also be expected in metals, when the wave length is comparable to the size of the crystalline grains*.

When measurements of the velocity of sound are made at a single fixed frequency (see Chapter IV, § 2 and previously in this section), the phase velocity of propagation is measured. Strictly speaking, this is the velocity of a particular monochromatic (sinusoidal) wave of constant amplitude which is infinite in time.

In reality, of course, such waves do not exist, and it must be decided to what extent the infinite wave approximates an actual process. If the number of wave periods during the time interval under consideration is great, it would not introduce great error to replace a sinusoid which is limited in time by an unlimited one. Thus, as a result of measurement, the phase velocity of a sinusoidal wave may be found: $c = \lambda f$. In practical problems, however, we are always interested in the transmission of some certain signal. For this purpose, it is necessary to introduce into the sinusoidal wave some distinctive feature or mark, for it is not the sinusoid itself which is important, but the telltale mark which it carries. This is the situation, for instance, in radio transmission, where high-frequency oscillations are modified (modulated) in amplitude (or in frequency) with a low, audio frequency. In this case it is essential that the disturbances in the regular sine wave occur quite slowly ($\Omega \ll \omega$; where Ω is the signal, and ω is the carrier frequency), and that the frequency spectrum occupied by the signal should be sufficiently narrow. In considering the phenomenon of beats (Figure 39), we saw that the addition of two sinusoidal oscillations with very close frequencies produces a resultant oscillation with a slowly changing amplitude. This oscillation consists of groups of waves or wave packets with a very narrow frequency spectrum. A pulse in the form of a "section" of a sinusoidal wave train is also a wave group; if the duration of the wave pulse $T \gg \tau$, where τ is the period of the sine wave, then the spectrum of the wave-train section is sufficiently narrow. In all the indicated cases the wave group may be called a quasi-monochromatic group.

How will such a signal or quasi-monochromatic wave group be propagated in a medium?

If the medium is free of dispersion, the answer is simple: all the frequencies are propagated with one and the same velocity (for instance, that

* Such dispersion has not yet been discovered.

of sound in air), and the velocity of propagation of a wave group or signal is equal to the phase velocity c.

The situation becomes rather complicated, however, in the presence of dispersion. In this case, the propagation velocity of a quasi-monochromatic wave group is determined by the interference of all the sinusoidal waves composing the group, each of which is traveling at its own phase velocity, determined by its frequency. As a result of interference, the quasi-monochromatic group as a whole travels at a velocity u, which is called the group velocity*. Group velocity has meaning only when the wave group or packet is composed of elementary waves lying within a very narrow frequency range, i. e., the frequencies must differ very little from each other (quasi-monochromatic conditions). In other words, the concept of group velocity cannot be applied unless the wave packet is not spread out to any appreciable extent during its motion.

If the frequency range $f_2 - f_1$ becomes wider — i. e., scattering of the phase-velocity values becomes greater due to dispersion — the packet rapidly becomes "diffuse", and the group velocity as the velocity of the packet as a whole becomes meaningless. However, in a medium where dispersion is not too great, a wave packet may travel a long distance without becoming "spread" appreciably.

Since energy is localized in the region occupied by the packet, the velocity of transport of this energy must be equal (at least, approximately) to the group velocity. The group velocity u may be either less than the phase velocity c (for media with normal dispersion; i. e., c increases with an increase in λ, as in waves on water, where $u = \frac{c}{2}$), or greater than c (for media with anomalous dispersion; i. e., c diminishes with an increase in λ, as, for instance, during sound propagation in a polyatomic gas).

Let us consider the situation in a more complicated case, when the pulse spectrum is not as narrow and so does not represent a quasi-monochromatic wave group.

In this case the concept of group velocity is inapplicable; during its motion, the pulse will "spread" appreciably, especially in a medium with considerable dispersion.

Let a signal [wave pulse] in the form of a portion of a sinusoidal wave train (see Figure 89) be propagated in a dispersive medium. It is required to find the time necessary for the pulse to travel a given distance. Experiment shows that the starting point of the pulse travels through the medium with a velocity corresponding to that of the highest frequencies and always equal to the velocity of sound in a continuous medium free of dispersion. This velocity of the initial point of the wave pulse is called the wave-front velocity. Thus, a plane wave front always remains flat for the highest frequencies, whether there is dispersion in the medium or not; and for the moment of initial establishment of the signal the laws of geometrical acoustics will apply.

For light propagation in a dispersive medium, the wave front (the so-called ultraviolet precursor) always travels at a velocity equal to the velocity of light in a medium free of dispersion.

* For two sinusoidal waves, of frequencies ω_1 and ω_2, differing only a little from each other, this velocity is

$$u = \frac{\omega_2 - \omega_1}{k_2 - k_1} = \frac{\Delta\omega}{\Delta k}, \quad \text{where} \quad \omega = 2\pi f \quad \text{and} \quad k = \frac{2\pi}{\lambda}.$$

In optics, this "precursor" has only theoretical significance, for its intensity is extremely low and the sensitivity of measuring instruments does not permit its detection. If ultrasonic pulses from a quartz plate are sent into a metal rod, the shortest waves (with a wave length considerably shorter than the rod diameter), will arrive first; this ultrasonic "precursor" is detectable*.

After the arrival of the precursor, which is the first to reach the observed point in the medium, the main part of the signal arrives; this part is measurable with ordinary instruments.

The velocity of propagation of the main part of the signal is known as the signal velocity.

In a dispersion-free medium, the phase velocity, the group velocity, the wave-front velocity, and the signal velocity all have the same value.

We have till now neglected, however, the attenuation of sound in the medium. If this is large, then, in addition to dispersion effects the pulse will become spread out due to attenuation, since various frequencies composing the pulse spectrum become attenuated differently (sound absorption is proportional to f^2). When attenuation is taken into account, there are additional limitations to applying the concept of group velocity**.

The Acoustical Method Applied to Investigations of Ferromagnetic Metals. Of special interest is the use of the dynamical methods described in the preceding section in order to study materials, the elastic properties of which depend upon their internal structure. Among such materials are, for instance, the ferromagnetic metals used to make magnetostrictive generators and ultrasonic receivers.

In ferromagnetic metals the attenuation of elastic waves is much more pronounced than in nonferromagnetic metals with similar elastic and plastic properties. When ferromagnetic metals are magnetized, the attenuation of elastic waves is sharply decreased. This effect is a result of the properties of the internal structure of ferromagnetic metals.

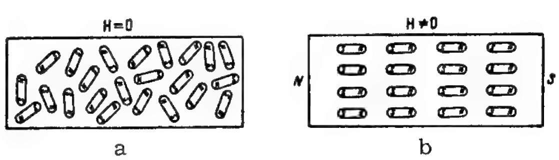


Figure 271. (a) Random and (b) ordered arrangement of the microscopic magnets in a ferromagnetic body

Every ferromagnetic metal consists of a large number of domains of local magnetization, which possess definite magnetic moment. The domains resemble microscopic magnets, which in the absence of an external magnetic field H are disoriented, so that the ferromagnetic body as a whole is nonpolarized (Figure 271a); the resultant vector of the magnetic moment of such a body is zero. When a ferromagnetic body is placed in a magnetic field, an arrangement of the domains takes place; their magnetic-moment

* A perfect analogy cannot be drawn between this case and that of a dispersive medium, inasmuch as the dispersion in the rod is of a geometrical nature, as already pointed out (see end of preceding subsection).

** Generally speaking, the sound velocity c depends upon the attenuation of waves in the medium. However, the effect of viscosity upon the velocity of sound is negligible.

vectors, under the influence of the external magnetic field, become aligned with the field, and the ferromagnetic body becomes polarized (Figure 271b).

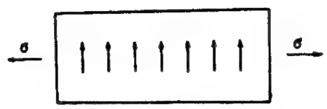


Figure 272. Orientation of magnetic moments (represented by vertical arrows) under the action of an elastic stretching force σ.

A redistribution of the regions of local magnetization may be caused by other means than the action of an external magnetic field. If a ferromagnetic body is subjected to an external elastic stress (for instance, if it is stretched), then magnetic effects also occur in it; there is a redistribution of the domains of local magnetization.

Thus, if nickel is acted upon by an elastic stretching force σ, the domain vectors become oriented in a plane perpendicular to the direction of the applied stress (Figure 272), while under the action of an external magnetic field (in the absence of elastic stresses) these vectors become oriented in the direction of the external field.

Let us now consider the propagation of longitudinal elastic waves in a ferromagnetic metal. During the periodic extensions and compressions caused by the passage of the wave, a periodic reorientation of the magnetic-moment vectors in each domain takes place; every "extension-compression" cycle is accompanied by a change in the orientation of these vectors (this circumstance results in energy losses in elastic waves traveling through a ferromagnetic metal). This phenomenon, known as magnetoelastic hysteresis, results in a greater attenuation of elastic waves in ferromagnetic metals.

During magnetization of a ferromagnetic body, the vectors of the local magnetic moments turn toward the direction of the magnetic field and become fixed in this direction. As a result, during propagation of elastic waves in a polarized ferromagnetic no vector reorientation occurs and attenuation of the waves is sharply cut down; the vibration amplitude of the ferromagnetic metal (e.g., a nickel rod) increases.

Another characteristic of ferromagnetic metals is the increase in Young's modulus when they are magnetized; this is called the ΔE effect. Both these phenomena — the decreased attenuation of elastic waves when a magnetic field is applied and the ΔE effect — have a common physical nature and have been investigated by means of the dynamical (acoustical) method.

In the statical method of determining Young's modulus for ferromagnetic metals, relatively large mechanical strains and stresses must be dealt with, leading to distortion of the results of measurement. Determinations of Young's modulus by the dynamical method are free of these disadvantages, since in this case only negligibly small strains and stresses are involved.

However, the above electromagnetic dynamical method of excitation and recording of longitudinal waves is not applicable to studies of wave attenuation and the ΔE effect in ferromagnetic metals, because the magnetized cores of the exciting coil and the receiver introduce distortions of the

magnetic field in the rod. Therefore, in studying these phenomena, it is preferable to employ methods of excitation and recording of vibrations which do not require changes in the magnetic state of the sample. For instance, it is possible to make use of *L*-cut Rochelle salt crystals, cemented onto the ends of a rod of the ferromagnetic metal being studied*. If one plate is connected to a generator of electrical vibrations, the other one is connected to an amplifier, and the rod is attached at its midpoint (similar to Figure 266), it is possible to measure, using an electronic oscilloscope, the resonance frequency of the rod and the width of the resonance curve. The data obtained make it possible to determine Young's modulus and the attenuation of longitudinal waves in the rod. If the rod is longitudinally placed in a uniform magnetic field and the intensity of the field is varied, it is possible to follow changes in the Young's modulus of the sample and in the amplitude of vibration of the rod, whereupon the attenuation of longitudinal waves in the sample is easily determined.

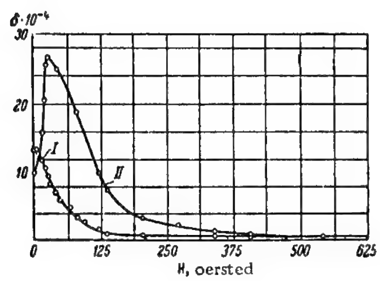


Figure 273. Attenuation decrement for a nickel rod as a function of the intensity of the magnetic field

Curve I – annealed sample. Curve II – compressed sample (by hammering)

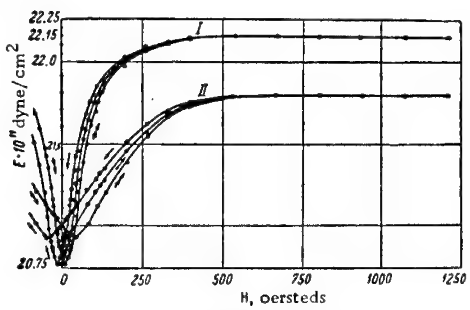


Figure 274. Young's modulus of a nickel sample, as a function of the magnetic-field intensity

Curve I – annealed sample. Curve II – compressed sample (by hammering)

Figure 273 shows the curves for elastic-wave attenuation in a rod of pure nickel, subjected to various thermal and mechanical treatments, as a function of the intensity of the external magnetic field. Curve I corresponds to lower internal mechanical stress (annealed sample); curve II corresponds to considerable internal stress (sample compressed by hammering).

Figure 274 shows the dependence of Young's modulus on the intensity of the external magnetic field. Curves I and II correspond to the same conditions of the sample as in Figure 273. The arrows show the direction of field change.

The curves in Figures 273 and 274** confirm the qualitative reasoning given above; quantitative results are in agreement with the theory describing magnetoelastic hysteresis and the ΔE effect.

The acoustical method of measurement is of essential importance in investigations of the above phenomena.

Surface Waves. In Chapter I we became acquainted with gravitational waves on a water surface, formed by the effect of the force of gravity and the inertia of the particles. The water particles in such waves were seen

* The weight of each Rochelle salt plate should be considerably less than the weight of the rod; but this is easily achieved.

** Measurements performed by V.P. Sizov at Moscow State University.

to perform movements in circular paths, where the amplitude of vibration rapidly decreased with the depth. Somewhat similar waves, called surface waves, originate on the free surface of an elastic solid. These are also rapidly attenuated as they travel from the surface into the interior of the body; the paths of motion of the solid particles again form circles (to a first approximation), the planes of which are normal to the surface and coincide with the direction of wave propagation. Displacements of the particles, therefore, have components which are parallel and normal to the surface. But gravitational waves on the surface of water arise due to the forces of gravitation and particle inertia, while elastic surface waves are caused by elastic and inertia forces. These waves are often called Rayleigh waves, since the famous English scientist Rayleigh was the first to point out the possibility of their formation. The velocity of surface waves in solids is approximately 0.9 times the velocity of propagation of transverse waves.

RAYLEIGH
(1842-1919)

The velocity of propagation of surface waves is independent of wave length, i.e., the waves are not subject to dispersion (provided they propagate throughout a homogeneous, isotropic half-space); while for gravitational waves on water the velocity increases with the wave length. Figure 275 represents diagrammatically the types of waves which are possible in solids: longitudinal (compression or expansion) waves; transverse (shear) waves; bending [flexure] waves; Rayleigh surface waves; and pure transverse (Love) surface waves. The last type of wave is formed when at least two media form contacting layers. Then, under certain conditions, waves may

be formed in the upper layer, the nature of which is represented in Figure 275. Their velocity of propagation is somewhat greater than that of Rayleigh waves, and depends upon the wave length.

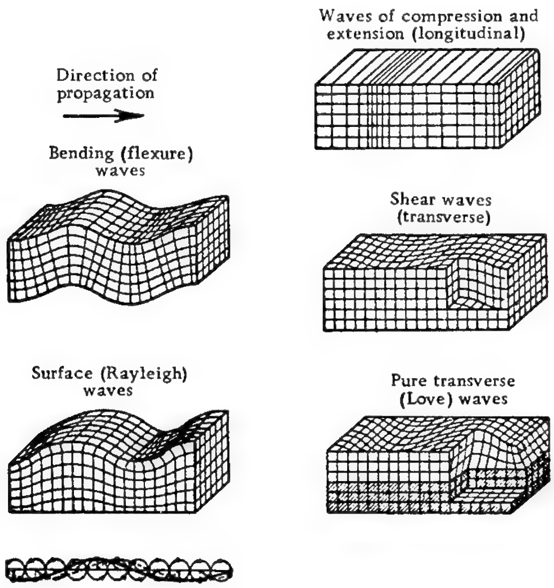


Figure 275. Types of elastic waves in solids

Deviations from Hooke's Law. Plastic Waves. Waves of Finite Amplitude. We have discussed the elastic properties of solids in the presence of small stresses and strains, when Hooke's law is obeyed fully. The situation becomes much more complicated if there is not direct proportionality between stress and strain. Figure 276 shows a typical relation between the relative change in length $\Delta l/l$ of cylindrical rods of cast iron and of copper (room temperature) and the applied stress $\sigma = \frac{P}{S}$, where S is the cross-sectional area of the rod at a given moment and P is the applied force. Up to point A in the upper half of the figure (region of tension) there is direct proportionality between stress and strain; this point is called the proportional limit. Beyond point B (the elastic limit) the stress-strain curve has a sharp break; deformations become greater with smaller loads. Region BC is called the region of yield; beyond point C the curve again turns upward (region of strengthening of the material), until breaking of the rod occurs. During compression

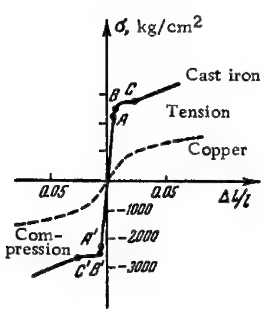


Figure 276. Stress-strain relation for cast iron and for copper

(lower half of the diagram), the pattern of the deformation is similar, but the stresses corresponding to points A', B', and C' are greater (by a very small amount) than those during tension (points A, B, and C).

The situation is approximately the same for other metals with relatively high rigidity. The behavior of ductile metals (aluminum, copper, tin lead, etc.) is different.

If the load is removed from the rod in the above example which has been subjected to tension, the unloading curve ABC does not pass through point C (Figure 277); the segment OC represents the residual strain. Subsequent application of a load will produce a strain described by the curve CDE, which together with the unloading curve forms a loop of elastic hysteresis (hysteresis means lag). The stress-strain diagram typical of plastic materials is

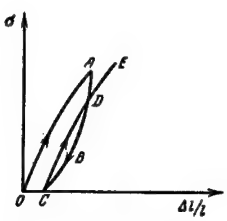


Figure 277. Hysteresis loop

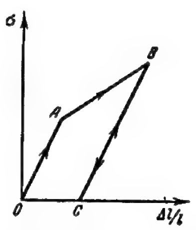


Figure 278. Idealized dependence of stress upon strain for a plastic material

shown in an idealized form in Figure 278; the hysteresis loop, by the way, is not shown. Segment OA corresponds to the region of linear elasticity; if the stress is removed, the deformation again becomes zero. At point A the slope of the stress-strain curve changes, but proportionality between stress and strain is maintained. As the stress is decreased, beginning from point B, the relation between stress and strain follows the straight line BC, which can be considered to be parallel to the straight line OA. When the load is removed, a residual strain appears, characterized by segment OC. After this, the material may be considered to be elastic, provided the stress does not exceed the value at point B.

Consider a long rod made of a plastic material, which is stretched at a constant rate. As shown by experiment, this produces a so-called plastic wave, the velocity of which is less than that of an ordinary elastic wave, since the elasticity of the material along segment AB is less than that along segment OA. If the load is suddenly removed from the rod, an elastic wave ("unloading wave") will run along the rod; the elastic-wave-front velocity is determined by the slope of the straight line BC. This wave has a greater velocity and so overtakes the plastic wave, which started earlier; consequently, there is interaction between the two, consisting, for instance, in the elastic wave being reflected from the rear surface of the plastic wave. These effects are complicated considerably by the reflection of waves from the ends of the rod*.

The majority of solids, in contrast to fluids, have moduli of elasticity

* Many important results in the theory of plastic waves have been obtained by Kh. G. Rakhmatullin in the U.S.S.R. and by Taylor, von Kármán, and Biot abroad.

which decrease for large strains; and instead of the shock waves encountered in gases and liquids, plastic waves are produced.

On the other hand, in a body whose modulus of elasticity increases with the deformation (this is the case for rubber, as well as for certain polymers in which the elasticity is increased due to the appearance of directivity in long chain molecules), it is possible, in principle, for waves to be formed which are similar to shock waves. However, all these problems have not yet been dealt with adequately.

§ 3. Propagation of elastic waves. Ultrasound in solids

Reflection and Refraction of Waves at Interfaces. When we discussed the laws of reflection and refraction of sound waves arriving from air to the surface of a liquid or a solid, we noted that in the reflection of sound waves from a solid wall practically all the energy is concentrated in the reflected wave, because the acoustic impedance ρc of a solid (for instance, metal) is immeasurably greater than ρc for air. When sound waves traveling in a liquid encounter a solid, the amount of energy transmitted into the solid is already appreciable. In liquids and gases only longitudinal sound waves can be propagated, so that when waves arrive at the interface between two media neither of which is a solid both the reflected and refracted waves are also longitudinal. The situation in solids is much more complicated, since both longitudinal and transverse waves can be propagated in them.

Indeed, if a rod suspended by threads is struck at its end, then along the length of the rod longitudinal compression-rarefaction waves are produced. If the rod end is struck normally with respect to its lateral surface, transverse shear waves are produced. Now let us see what type of waves are produced if the blow is struck at an angle.

Such a blow produces both compression and shear, in the form of two elastic pulses: a compression pulse (longitudinal wave) and a shear pulse (transverse wave).

Similarly, if a longitudinal elastic wave arrives at a solid normally to its surface, then the wave penetrating into the solid and that reflected from it are longitudinal waves.

When a transverse wave arrives normally to the surface of a solid, then the wave penetrating into the solid and the reflected wave are both transverse.

In other words, when an elastic wave arrives normally to the surface of a solid, then both the reflected and refracted waves are of the same type as the incident wave.

The situation becomes much more complicated if the elastic wave passes from one solid to another at an angle with respect to the interface. Just as an oblique blow at the end of a rod produces waves of two types, so does the oblique incidence of waves cause the waves to separate, or undergo transformation. However, before we discuss in more detail the effects occurring during reflection and refraction of longitudinal and transverse waves at a flat interface between two solid media, it should be noted that transverse waves are polarized. Suppose that transverse waves arise during the shear vibrations of a Y-cut quartz plate (see Chapter V, § 1), and that the direction of the displacements, made by the plate and transmitted into a solid,

are those shown in the upper part of Figure 279a. Now suppose that an identical plate, used as a receiver, is placed as shown in the lower half of Figure 279a. The receiving plate will not detect any vibration, since the direction of the displacements in the propagating transverse wave is perpendicular to the direction of the shear vibrations of the receiving plate which cause its electrodes to become charged. The correct mutual position of source and receiving plates, a position which ensures the detection of transverse waves with the indicated direction of displacements, is shown in Figure 279b. This example demonstrates what the polarization of transverse

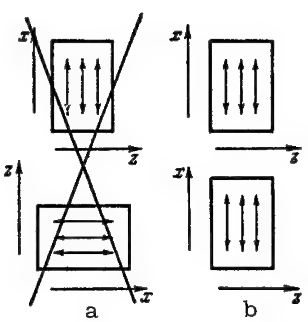


Figure 279. Polarization and shear vibrations of a Y-cut quartz plate. In position a the lower (receiving) plate does not detect vibrations of the upper (source) plate. The correct position of the plates is shown in b.

waves really signifies, and shows that it is the direction of the displacements in the transverse wave which are important. This type of polarization is called linear polarization; transverse waves are said to be linearly polarized (as distinguished, for instance, from circular polarization). Since transverse waves are polarized, three distinct cases are possible in the reflection and refraction of longitudinal and transverse waves. Let us consider the first of these: a longitudinal wave arriving at the interface between

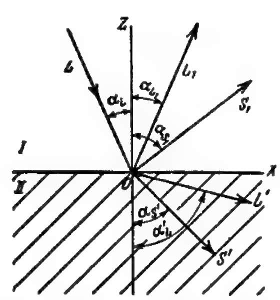


Figure 280. Splitting of a plane longitudinal wave arriving at the interface between two media

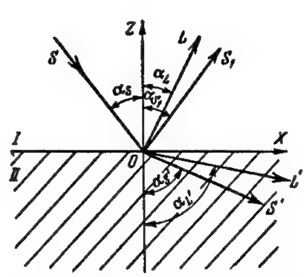


Figure 281. Splitting of a plane transverse wave arriving at the interface between two media

two solids. If a longitudinal wave α_L approaches the surface of solid II from solid I at an angle L (Figure 280), then two waves are produced in the second body — a refracted longitudinal wave L' and a refracted transverse

wave S' (the angles of refraction are $\alpha_{L'}$ and $\alpha_{S'}$, respectively). The latter wave is said to be vertically polarized, meaning that the plane of the displacements caused by it is the incidence plane (plane XOZ in Figure 280).

Two waves are also reflected from the interface — a longitudinal wave L_1 and a transverse wave S_1, with angles of reflection α_{L_1} and α_{S_1}; the reflected wave is also vertically polarized. For the incident and reflected longitudinal waves, the law stating that the angle of incidence equals the angle of reflection holds true: $\alpha_L = \alpha_{L_1}$.

If a vertically polarized transverse wave arrives at the interface between two solids (Figure 281), then four waves also originate; this case reduces to the previous one — the refracted and the reflected waves S' and S_1 are vertically polarized and $\alpha_S = \alpha_{S_1}$.

As already mentioned, in the case of oblique incidence four waves are produced. As the angle of incidence increases, there comes a point where the longitudinal wave is no longer transmitted into the second medium, but begins traveling along the interface (total internal reflection, Figure 282a). The corresponding angle of incidence is called the first critical angle. For a further increase in the angle of incidence, a point is reached where the transverse wave S' is no longer transmitted into the second medium either (Figure 282b). The angle of incidence at which this wave also begins traveling along the interface is called the second critical angle (total internal reflection of the transverse wave). For an angle of incidence greater than the second critical angle, no waves at all are transmitted into the second medium.

The critical angles depend upon the ratio of wave velocities in the first and second media*. Total internal reflection does not occur unless the velocity of wave propagation in medium II is greater than that in medium I.

Finally, there is a third case, in which a horizontally polarized transverse wave arrives at the interface (SH-type wave); in such a wave the direction of vibration of the particles is perpendicular to the incidence plane XOZ. This is the simplest case; from purely geometrical considerations it is clear that here only two waves arise, one reflected and one refracted. Therefore, if horizontally polarized transverse waves (SH waves) are

* The law of wave refraction for oblique incidence is given by the formula

$$\frac{\sin \alpha_L}{c_L} = \frac{\sin \alpha_{L'}}{c_{L'}} = \frac{\sin \alpha_{S'}}{c_{S'}},$$

where c_L is the longitudinal-wave velocity in the first medium; $c_{L'}$ is that in the second medium; and $c_{S'}$ is the transverse-wave velocity in the second medium. The meaning of angles α_L, $\alpha_{L'}$ and $\alpha_{S'}$ is evident from Figures 280 and 281.

The first critical angle is obtained for $\alpha_{L'} = 90°$, i.e., when $\sin \alpha_{L'} = 1$; the angle of incidence $\alpha_L = 1$ can then be calculated from the formula

$$\sin \alpha_L = \frac{c_L}{c_{L'}}.$$

The second critical angle, for which $\alpha_{S'} = 90°$, is calculated from the formula

$$\sin \alpha_L = \frac{c_L}{c_{S'}}.$$

When the velocities of wave propagation in the first and second media are known, the angles of the refracted waves can be determined, as well as the critical angles.

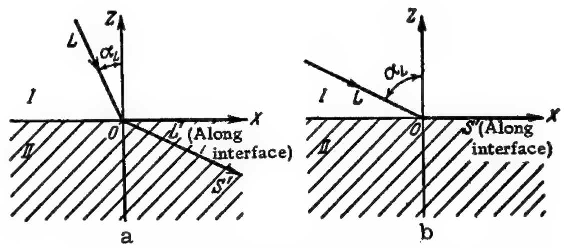


Figure 282. Total internal reflection of a plane longitudinal wave.

a) First critical angle; b) second critical angle.

propagated in a limited solid in the form of a plate with a thickness considerably greater than the wave length, then no transformation will occur in reflections from interfaces and the type of the wave remains the same.

Velocity Measurement and Absorption of Ultrasound. We have limited ourselves so far to discussion of elastic vibrations in solids with frequencies below a few kilocycles. Now the propagation in solids of elastic waves of higher frequencies (ultrasonic waves) will be discussed.

In the preceding chapters, we became acquainted with the properties of the propagation of ultrasonic waves in gases and liquids; we have seen the various uses of ultrasound in different branches of technology and in scientific experiment. It remains to investigate the properties of ultrasound propagation in solids, as well as the most important and interesting ultrasonic applications related to this.

In the preceding chapters, the principal methods for precise measurement of sound and ultrasound velocities in gases and liquids were described — the interference method and the pulse method. Moreover, the interference method was subdivided into use of the traveling-wave and the standing-wave interferometers. These methods also made it possible to determine the absorption of sound and ultrasound waves.

Use of the traveling-wave interferometer to measure the ultrasonic velocity and absorption in solids encounters great difficulty. An ultrasonic receiver cannot be moved through a solid body, although this was easily done in gases and liquids. Furthermore, it is often impossible even to make velocity measurements by the interference method, where a fixed distance is maintained between the source and the receiver and the frequency is varied, since in a piece of metal or other solid standing waves will always be formed, due to its finite dimensions and to the low attenuation of ultrasound.

We have already mentioned the measurements of the velocities and absorption of sound waves in solids by means of the vibrations of a rod. This method of measurement, which is actually a standing-wave-interferometer method, can be used not only for audible, but also for ultrasonic, frequencies. In addition to this method of measurement (to be discussed below), the pulse method is used for velocity and absorption determinations in solids which are opaque to light, and at high frequencies (about 1 to 10 mc).

In order to generate longitudinal waves in solids, an X-cut quartz plate is employed, which operates like a piston, while for transverse waves

Y-cut plates are used, which produce shear vibrations. The surface of a solid, however well polished, always has some roughness, which leads to breaks in the "acoustic contact" during transmission of ultrasound from the plate to the solid. To obtain good contact between plate and sample, the surface of the latter is usually wetted with transformer oil, or else special coatings are used.

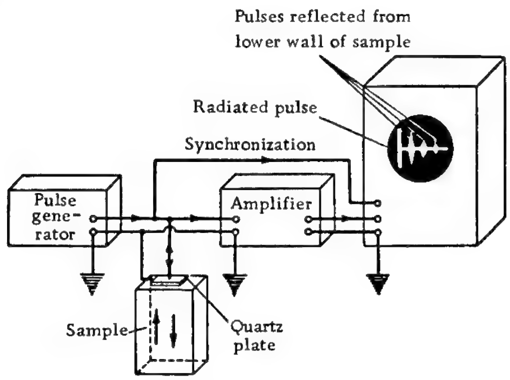


Figure 283. Measurement of the velocity of ultrasound in a solid sample (pulse method). The emitter plate doubles as receiver

Figure 283 represents schematically one of the simplest methods of ultrasonic-velocity measurement in a metal sample. An ultrasonic pulse produced by an X-cut quartz plate arrives at the lower interface of the sample and is reflected, after which it returns to the same quartz plate; the latter now operates as an ultrasonic receiver. The alternating voltage generated between the plate electrodes is amplified and applied to the vertical-deflection plates of an electronic oscilloscope; the oscilloscope sweep is synchronized with the pulse, i.e., the beginning of the sweep coincides with the moment of sending out the pulse.

The pulse repetition rate is usually a few tens of cycles per second, so that during the interval between two pulses the ultrasonic waves will undergo several reflections from the walls of the sample and will have sufficient time to die out. If the dimensions of the sample and the sweep time are known, it is not difficult to determine the ultrasonic velocity by observing the pattern on the oscilloscope. The absorption is determined by the decrease in amplitude of successive reflections. Figure 284 is a photograph of the multiple reflection of ultrasonic wave pulses in a quartz plate at an ultrasonic frequency of 10^8 cycles (with the waves traveling along the optical axis of the crystal).

The pulse method of absorption measurement in the simple form just described is an approximate method and so cannot be used for solids with good sound conductivity, i.e., low wave attenuation. Errors are chiefly connected with the problem of the transmission of the elastic vibrations from the piezoelectric source to the solid sample under investigation, i.e., with the acoustic contact. The imperfect nature of the majority of connecting materials — liquids in the case of longitudinal waves and solids in the case of transverse waves — causes large conversion losses during transmission of shear waves into a solid sample, and this error is difficult to estimate.

Moreover, there are additional losses due to secondary reflection; and these must be corrected in accurate measurements by subtracting them from the total loss.

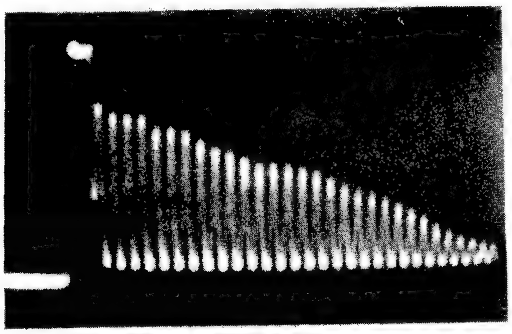


Figure 284. Multiple reflection of ultrasonic wave pulses in a quartz plate at an ultrasonic frequency of 10^8 cycles

A strict account of the acoustic losses due to secondary reflection can be made by providing a sufficiently thick liquid layer (e.g., water) between the piezoelectric transducer and the sample. For this purpose, special apparatus has been developed, using a liquid bath; the apparatus is shown in Figure 285*.

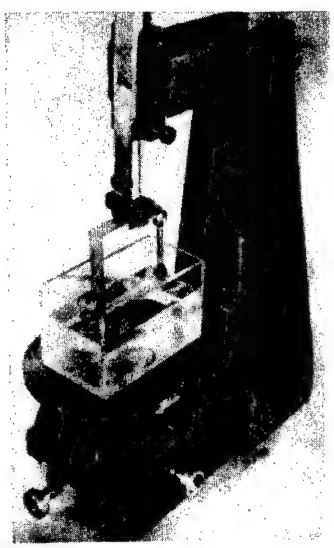


Figure 285. General view of apparatus for measurements by means of the liquid-bath method

* This method was first proposed by the American physicist W. Roth. The equipment in Figure 285 was developed by V.P. Sizov at Moscow State University.

A rather full description of the liquid-bath method of velocity and absorption measurements for ultrasonic waves will be given, since it demonstrates some characteristic features of ultrasonic measurements in solids.

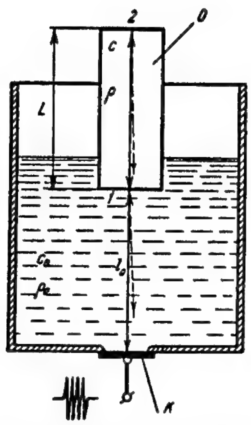


Figure 286. Arrangement of sample in a liquid bath for measurements with longitudinal waves

K–quartz plate, O–sample

The liquid bath with the piezoelectric quartz plate K (Figure 286) is mounted on a leveling platform. The latter is a large metal disk supported by three leveling screws with ball supports. By means of the screws, the height of each of the three supports can be altered, in order to align the plane of the table with respect to the axis of the piston-rod, to the end of which the sample is clamped. The table in turn rests on a platform, which can be smoothly shifted in the horizontal plane in two mutually perpendicular directions. In this way the required precise alignment of the plane of the source with respect to the plane of the sample is achieved.

The displacement of the piston-rod with the sample attached to it also causes a change in the path length of the pulse in the liquid. One surface of the piezoelectric plate is in contact with the liquid (its electrode is "grounded") and radiates into it ultrasonic pulses 1-2 μ sec in duration, along with the carrier frequency. The sample under investigation is dipped into the liquid some distance from the quartz plate and in such a way that its upper end is outside the liquid (Figure 286). The ultrasonic pulse arrives at the end of the sample, where it is partially reflected and partially transmitted into the sample. A series of reflected pulses arrives at the quartz plate, which after sending of the pulse operates as a receiver. The first incoming pulse, arriving after an interval $\frac{2l_0}{c_0}$ (c_0 is the sound velocity in the liquid), corresponds to reflection from the end of the sample (Figure 286). The following pulses in the train arrive after multiple crossings of the sample due to reflections from its upper and lower ends, since during each arrival of the ultrasonic pulse at the lower wall some of the ultrasound penetrates into the liquid. The time interval between the pulses forming the pulse train is $2L/c$, where L is the length of the sample and c is the velocity of longitudinal waves in it. A pulse reflected from the lower end of the sample and again arriving at the sample after reflection from the quartz plate results in a second train of pulses. If a liquid which is a good sound conductor is used, there may be several such pulse trains. Figure 287 shows a typical picture of such a series (using a sample of a magnesium-aluminum alloy). From the foregoing reasoning, as well as from Figure 288, it may be concluded that the retardation time for any pulse in any given series (with respect to the main pulse) is determined by the equation

$$t_{N,n} = 2N\frac{l_0}{c_0} + 2(n-1)\frac{L}{c},$$

where $N = 1, 2, \ldots$ is the number of the pulse train; and $n = 1, 2, \ldots$ is the number of the pulse in the train. Under certain conditions superposition of pulses in adjacent trains may occur, complicating the measurements. This is easily avoided by moving the sample in the vertical direction.

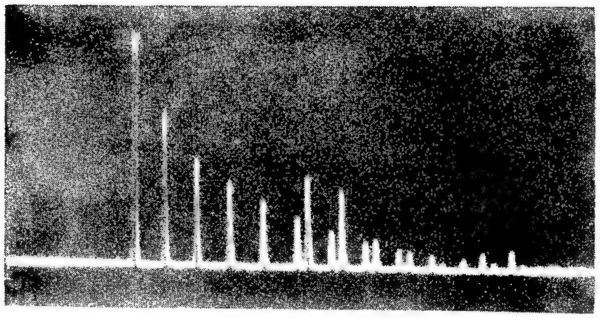


Figure 287. A pulse train on the screen of an oscilloscope—liquid-bath method

Magnesium sample, frequency 10 mc

In order to improve the accuracy of velocity measurements for longitudinal waves in a sample, it is useful to measure the time intervals not between two consecutive pulses, but between as large a number of pulses in the pulse train as possible. Then the velocity of longitudinal waves c_l is calculated from the formula

$$c_l=(n-1)\frac{2L}{T},$$

where n is the number of pulses in the train and T is the time between pulses. This method of measurement of ultrasonic velocities in solids can

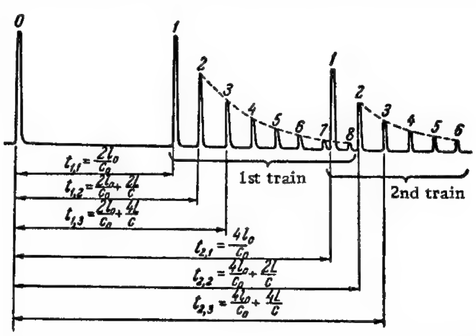


Figure 288. Sequence in time of two trains of reflected pulses in liquid-bath measurements

be used not only for longitudinal, but also for transverse (shear), waves. In this case the sample is shaped as shown in Figure 289. The angle φ, determined by the ratio c_t/c_l (c_t = velocity of transverse waves) or by Poisson's ratio, is selected when the condition is satisfied that the propagation path of the reflected transverse wave is parallel to the lateral faces of the sample.

The ultrasonic beam in the segment AB is propagated as a longitudinal wave. At point B the longitudinal wave is reflected and converted to a transverse wave, which travels further in the sample along the path BC.

Returning, after reflection at point C, along the same path, the transverse wave is once more transformed at point B — into a longitudinal wave, which then emerges from the sample into the liquid.

Thus, along segment BC = L a transverse wave is propagated. The retardation time for the pulses in the pulse train under observation is determined chiefly by the propagation velocity c_t of transverse waves in the sample. c_t must be further corrected for the propagation of a longitudinal wave in the sample over the short segment 2AB = d. The absorption coefficient can in principle be found from the attenuation of the pulse amplitude in traveling a given distance; for instance, from the ratio of the amplitudes of two adjacent pulses (i. e., over a small distance equal to twice the length of the sample). However, the accuracy which can be thus attained is not high, especially if attenuation in the sample is low. Therefore, for a more accurate measurement of the absorption coefficient one should compare the amplitudes of pulses (by calculating their ratio) of higher orders with the amplitude of the first pulse reflected within the sample (i. e., with pulse number 2, see Figure 288). Thus, if the amplitude of the 2nd pulse is compared with that of the n-th pulse, then the length of the path traveled by the pulse in the interim is $2L\,(n-2)$.

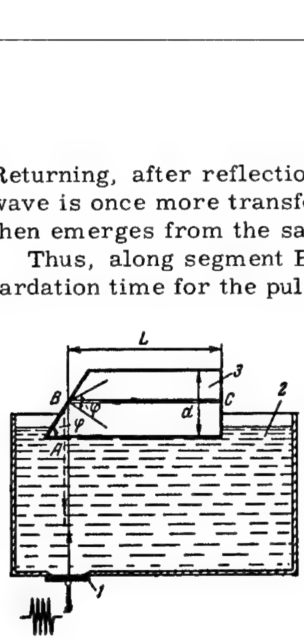


Figure 289. Position and shape of sample in velocity and absorption measurements for transverse waves

1) Quartz plate; 2) liquid (water); 3) sample

During reflection from a solid-liquid interface, a pulse is partly reflected and is partly transmitted into the liquid; hence, in calculating the absorption coefficient it is necessary to correct for energy losses caused in this way. However, losses at a solid-air interface may be neglected, since the acoustic impedance of air, compared to that of a solid, is negligibly small. An accurate correction for the losses due to reflections is made possible by pulse-trains discrimination through vertical movement of the sample. If A_2 is the amplitude of the 2nd pulse, A_n is the amplitude of the nth pulse, $Z_0 = \rho_0 c_0$ is the acoustic impedance of the liquid, and $Z = \rho c$ is that of the sample, then the attenuation coefficient for longitudinal waves is given by the formula

$$\alpha_{\text{long}} = \frac{1}{2L}\left[\frac{A_n - A_2}{n-2} - \alpha_{\text{ref}}\right] \text{db/m},$$

where

$$\alpha_{\text{ref}} = 20 \lg \frac{Z - Z_0}{Z + Z_0}.$$

In measuring the absorption coefficient for transverse waves* (see Figure 289), in addition to reflection losses it is necessary to correct for energy losses during the transformation** from longitudinal to transverse waves and back (during reflections), and also to correct for the part of the path in the sample over which the waves are propagated as longitudinal

* Method developed by K.V. Goncharov at Moscow State University.

** Complete transformation of a longitudinal wave into a transverse wave in a solid sample is possible for values of Poisson's ratio $\sigma \leqslant 0.26$. For example, for a majority of metals $\sigma > 0.2$, and no angle of complete transformation exists. Thus, when a longitudinal wave arrives at a free boundary there will always exist two reflected waves, one longitudinal and one transverse.

waves. This method of measuring the absorption coefficient in solids permits correction for the losses during multiple reflections of ultrasonic pulses in the sample, thereby improving considerably the measurement accuracy. Obviously, all possible types of error are not eliminated, e.g., improper alignment of the sample, inaccurate determination of the pulse amplitudes, etc. Diffraction effects and the influence of the side walls of the sample play an important part, especially with low-attenuation samples.

Absorption of Elastic Waves in Solids. The problem of elastic-wave absorption in solids is much more complicated and less studied than for fluids.

If a solid were perfectly elastic and homogeneous, there would be no losses in the propagation of elastic waves in it. However, real bodies are never perfectly elastic and homogeneous, and when elastic waves are propagated in them, part of the wave energy is somehow transformed into heat or otherwise lost. There are several general mechanisms of this transformation, ultimately resulting from two dissipative processes: losses due to internal friction and losses due to heat conduction. Solids are distinguished by their variety of types. There are amorphous, more or less homogeneous, solids, such as glass, fused quartz, various plastics, etc. Metals in their usual state are polycrystalline bodies and may be considered

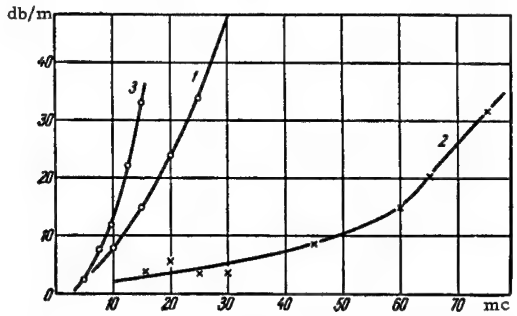


Figure 290. Attenuation of longitudinal waves in an MA-2 magnesium-aluminum alloy (curve 1), in fused quartz (curve 2) (measurements by the liquid-bath method), and in aluminum with an average grain size of 0.13 mm (according to Mason, curve 3)

to be acoustically homogeneous only at low audio frequencies, when the length of the elastic wave is considerably greater than the crystalline grains of which the given metal sample is composed. Beside amorphous solids and polycrystalline bodies, there are monocrystalline bodies: crystals of minerals (e.g., piezoelectric quartz, tourmaline, and so on), single crystals of metals (e.g., individual metal crystals or large, specially prepared single crystals of various metals). Finally, in the fourth group of solids, from the acoustical viewpoint, are included extremely variegated substances of a mixed type (i.e., all solids that cannot be included in the first three groups); these are the most widespread under natural conditions and include stones, granulated media (e.g., sand), etc.

Let us first consider the simplest case of an amorphous homogeneous solid, in which the elastic properties do not depend upon the direction of

propagation — the isotropic case. Such is the case, for instance, of well-prepared optical fused quartz.

Figures 290 and 291 represent the attenuation of longitudinal and transverse elastic waves in optical fused quartz and in magnesium-aluminum alloys (measured by the liquid-bath method), as a function of frequency.

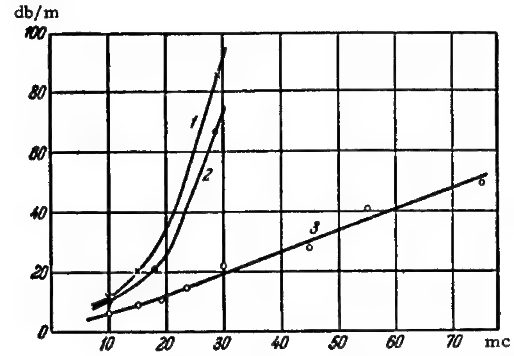


Figure 291. Attenuation of transverse waves in Mg-Al alloys types MA-3 (curve 1) and MA-2 (curve 2), and in optical fused quartz (curve 3)

From the data shown, it may be concluded that the attenuation coefficient for longitudinal waves in fused quartz is proportional to the frequency up to approximately 35 to 40 mc; at higher frequencies, deviations occur. For transverse waves in fused quartz, the proportionality of attenuation and frequency, according to results of measurements, holds for a range from 10 mc up to at least 75 mc. Numerous solids show this same dependence of absorption on frequency for low ultrasonic and for audible frequencies. Thus, from a few kilocycles up to 100 kc, measurements by the method described in § 2 of this chapter (Figure 268, measurements of the width of the resonance curve for the sample rod) for lead, nickel, copper, and glass rendered the same frequency dependence for the absorption coefficient.

Here is an essential difference in the nature of the absorption of elastic waves in solids, as compared with that in liquids and gases, in which absorption is proportional to the square of the frequency. This is usually explained by hysteresis losses arising during the propagation of an elastic wave in a solid, the elasticity of which is not perfect. In Figure 277, a curve was shown, giving the intensity as a function of the deformation; from this curve it is seen that the deformation is not exactly repeated in the course of a cycle, and that a loop is formed — the hysteresis loop. The area of this loop represents the mechanical energy lost as heat*. In the figure, the size of the loop is exaggerated. Indeed, had we attempted, by any static method (i. e., by applying a load to the sample, removing it, and then measuring the amount of strain), to observe a difference in the behavior of the stress-strain curve, we would have discovered no hysteresis loop whatsoever for such good "conductors" of sound as fused quartz, glass, etc. This effect is extremely small for the small strains usual during the

* This is similar to the magnetization of a ferromagnetic substance, where the area of the hysteresis curve represents losses due to heating.

propagation of elastic waves. However, for elastic waves of sufficiently high frequency, when a compression pulse passes, the above cycle is performed in turn by every single layer of the material; and the number of such cycles may become millions per second for ultrasonic frequencies. Therefore, although the hysteresis loop itself may have negligible area, the cycle is repeated so many times in one second that the cumulative effect becomes appreciable. From this discussion, it should be clear that the hysteresis losses must be proportional to the number of cycles per second, i. e., the absorption of elastic waves must be proportional to the frequency, and this agrees with the experimental results given above.

The physical absorption mechanism just discussed is not the only one. Velocity gradients produced by stress waves may lead to losses of a second kind, just as in fluids. By this is meant viscosity losses, which are the main losses in fluids; in this case the absorption proves to be proportional to the square of the frequency. This type of loss (when mechanical motion depends on the rate of strain) is said to be viscous in nature. It should be noted, however, that in general the concept of the viscosity of a solid is, from the viewpoint of molecular theory, at present far from being clear, inasmuch as the types of molecular processes causing the transformation of mechanical energy into heat have not yet been explained. These losses, characteristic of fluids, are the subject of a thoroughgoing theory, but play an insignificant part in homogeneous solids with good elastic qualities (such as fused quartz, glass, etc.)*; they may become important, however, for such materials as certain polymers. While for such bodies as quartz it is possible to limit the discussion to hysteresis-type losses, here we still ignore scattering by inhomogeneities of the medium (to be discussed below); this is far from being the case for other bodies, e. g., metals.

Let us now discuss the absorption of elastic waves in metals. We commonly deal with metals which are a conglomerate of small grains consisting of minute single crystals, oriented randomly with respect to one another. Such crystals may be observed with the naked eye, for instance, in a break in the surface of a duralumin or steel rod. If the surface of any metal is polished and then etched, the polycrystalline structure is quite distinctly visible. Figure 292 is a photograph of the etched surface of pure (99 %) aluminum, where individual, randomly arranged, crystalline grains of various sizes are visible.

The absorption (attenuation) of elastic waves in polycrystalline substances (pure metals, alloys, etc.) depends in general upon a large number of factors, all of which are not always possible to account for. Thus, if a metal sample is cast, it has pores and cracks, entrained gas, and various other impurities, especially oxides on the boundaries of single crystals. As a rule, cast metal has lower sound conductivity than rolled metal. On the other hand, in a rolled metal residual stresses occur; they are present to an even greater degree after surface hardening, e. g., after forging. During rolling, some anisotropy of the elastic properties is produced; for instance, the attenuation of waves along the direction of rolling proves to be somewhat greater than parallel to this direction.

* However, the English physicist Lamb showed experimentally that, within a frequency range of 100 to 1000 mc, the attenuation in fused quartz is proportional to the square of the frequency; this apparently points to the presence of "viscosity" losses within this range.

Thus, the history of the metal, the means of its production, and other properties are of great importance. Suppose, however, that a given metal sample does not contain blisters or cracks, and also that it only contains an insignificant amount of gas. Consider, that is, that we are dealing with a "pure" polycrystalline isotropic metal sample. Let us now determine the principal causes of attenuation during the propagation of elastic waves in such a sample.

Figure 292. Etched surface of a sample of pure (99%) aluminum with coarse grains (3-4 mm)

There are several such sources. First, there is the imperfect elasticity of the metal, resulting in hysteresis effects (in the same way as in amorphous solids). As long as the wave length of the elastic wave is considerably greater than the size of the individual grains composing the crystalline grain, this will be the principal cause of absorption. In this case there is direct proportionality between the absorption coefficient and the frequency.

As the frequency increases the wave length (though still greater than the size of the crystalline grains) becomes comparable to the grain size, and scattering appears. This in itself does not cause losses (i.e., transformation of energy into heat), but it attenuates the transmitted beam and ultimately lowers the sound conductivity of the material (actually it increases the coefficient of attenuation).

Under these conditions, when $\lambda \gtrsim l$, where l is the grain size, the scattering may be considered to approximate Rayleigh scattering (see Chapter VII, § 3), the intensity of which is proportional to the fourth power of the frequency.

The attenuation of elastic waves in a polycrystalline solid may therefore be expressed by the relation

$$\alpha = B_1 f + B_2 f^4,$$

where the first term is governed by hysteresis losses, and the second by scattering*. Here B_1 and B_2 are certain constants, depending on the type of wave (longitudinal or transverse) and on the properties of the material (size of the minute crystals, type of crystal lattice, and values of the elastic parameters in different directions). This qualitative formula may be said to agree with experimental results provided the wave length is at least 10 times the size of the grains**.

The general course of the attenuation is represented by the curves in Figures 290 and 291.

Substantial discrepancy is encountered with respect to the quantitative agreement of theory and experiment; this appears to be explained by the fact that not all the essential factors influencing the results are taken into account by the approximate theory proposed by Mason. For instance,

* Rayleigh's theory of scattering was used to explain the attenuation of elastic waves in polycrystalline substances (metals) by the American physicist Mason. Cf. Mason, Piezoelectric Crystals and Their Application in Acoustics. Russian translation. Moskva. 1952.

** According to L.G. Merkulov.

during the scattering of waves by crystallites there must occur a transformation of waves from one type (e.g., longitudinal) to another (e.g., transverse), resulting in additional losses. A more general scattering theory, taking into account wave transformation as well, yields considerably better agreement between theoretical and experimental values for the attenuation coefficient of many metals*.

Complete scattering of a diffuse nature occurs in metal samples consisting of large single crystals, the average linear dimensions of which are greater than the wave length of the wave being propagated. Thus, if transverse waves in the form of a train of pulses are sent through such a sample, then instead of a series of reflected pulses following each other at intervals of $2L/c_t$, random noise is observed in the sample; the noise is similar to reverberation effects, and its level decreases according to an exponential law. Figure 293 is an oscillogram of a characteristic signal with a carrier frequency of 10 mc in a conglomerate of single aluminum crystals a fraction of a centimeter in size (for a wave length of about 0.1 mm).

Neglecting the absorption at the boundaries between individual crystalline grains, i.e., assuming absorption to exist only within the grains, we may consider the attenuation in time of the reverberational noise to be described by the same law as the attenuation of a plane wave in a single crystal. Consequently, from the steepness of the exponential drop of the residual noise the mean absorption of shear waves in a single crystal of metal is determined by the formula

$$\alpha_{abs} = \frac{\Delta A}{L} = \frac{\Delta A}{c_t \tau} \text{ db/m},$$

where ΔA is the drop in the reverberational-noise level in decibels, and τ is the observation time interval. Here c_t represents the average velocity of propagation of transverse waves in different directions and with different polarizations.

Measurements performed by this method yield, for the attenuation of transverse waves in a single aluminum crystal and at a frequency of 10 mc, the value of 1.6 db/m, which is in agreement with measurements performed according to other methods.

While heat conduction does not play much of a part in the propagation of elastic waves in such solids as fused quartz, it may be significant in metals. Relaxation processes of thermal origin may be involved in this. During the bending [flexural] vibrations of a metal plate, changes in the volume of the plate cause changes in its temperature: during compression the plate is heated, during extension it is cooled.

When the plate is bent, the inner (compressed) side is heated and the outer side is cooled; as a result, during vibration there is a continual heat flow from one side of the plate to the other (across the plate). For very slow vibrations, the process must be isothermal; and in this case, it is reversible, with no energy losses. For very rapid vibrations, the process must be adiabatic, so that heat will have no time to flow across the plate, and no losses will occur, just as in the first case. But when the period of the flexural vibration is comparable to the time necessary for heat to flow across the plate (relaxation time for the given process), an irreversible transformation of mechanical energy into heat will occur, as evidenced by a greater attenuation of the vibrations of the plate. A theory constructed by the American physicist Zener, based on these assumptions,

* This theory was developed by I.M. Lifshits and G.D. Parkhomovskii. Experiments to test the theory for several metals were performed by L.G. Merkulov at the Leningrad Electrotechnical Institute.

is found to be in good agreement with experimental data for thin metal plates, at frequencies of several cycles per second.

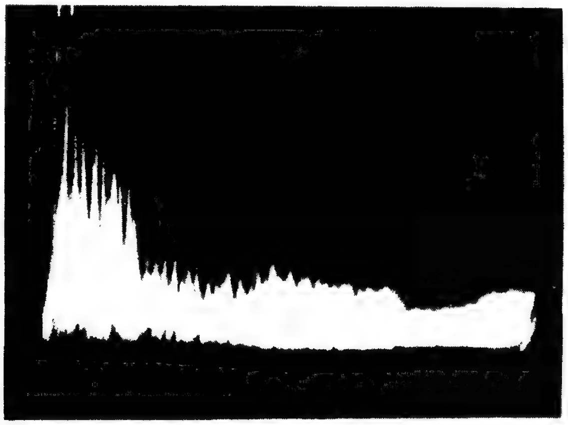


Figure 293. Residual sounding in an aluminum sample (average grain size 4-5 mm) after transmission of a pulse train of transverse waves ($f = 10$ mc)

To return to the propagation of elastic waves in metals, we see that, in addition to losses due to hysteresis and to scattering by crystalline grains, it is in general necessary to consider the absorption which may occur due to thermal processes of a relaxational nature. The minute crystals differ widely from one another in shape and size, as well as in the orientation of their crystallographic axes. Therefore, for a given acoustic excess pressure exerted by the wave, the deformation experienced by every individual tiny crystal is nonuniform — in different parts of the crystal the deformation has a different value and direction. During compression strains a crystal is heated (different crystals are heated differently), and the temperature difference between individual crystals will not be the same. Owing to heat conduction, local heat flow will occur through crystal boundaries. Just as in the case of the flexural vibrations of a plate (discussed above) we are dealing here with a relaxation process. The coefficient of absorption will depend upon frequency and it will be maximum when the wave period coincides with the time necessary for the equalization of temperature throughout the volume of the crystalline grain, i.e., with the relaxation time. The same condition may be alternatively expressed as a condition of equality between the length of the thermal wave (Chapter VI, § 50) and the average size of a crystal*.

Although such thermal relaxation is possible in a polycrystalline metal

* The theory describing this process was given by Zener. According to this theory, the relaxation frequency ω_r is

$$\omega_r = \frac{3\pi\kappa}{\rho c_p l^2},$$

where κ is the coefficient of thermal conductivity; ρ is the density; l is the average linear size of the grains; and c_p is the specific heat at constant pressure. At very high frequencies, heat flow may occur directly at the grain boundaries. This situation has been discussed by M. A. Isakovich.

body, the main energy losses by elastic waves in the ultrasonic range are determined by the phenomena of hysteresis and scattering.

In ideal single crystals the three-dimensional lattice has a geometrical structure depending on the type of crystal (cubic, hexagonal, or other). There cannot be any microscopic cracks, foreign inclusions, or separate grains, as in real single crystals. However, there may be residual stresses and defects in the crystal lattice; these are commonly termed dislocations (regions of "disorder" in the crystal). In other words a single crystal is an anisotropic body, but it is far more homogeneous than a polycrystalline and even (to a certain extent) than an amorphous body, since in the latter some inhomogeneities always exist.

At present, we do not yet have the complete data which would enable us to determine the principal causes for the attenuation of elastic waves in single crystals; and it is not known whether an essential part is played by hysteresis and by losses related to thermal conduction and "viscosity". The role played by dislocations during elastic-wave propagation is not yet completely clear. On the one hand, wave scattering by dislocations is possible; on the other hand, dislocations themselves sometimes seem to come into existence through the action of a passing wave; and this may have the nature of a relaxation process.

Thus, even the study of elastic-wave propagation in a single crystal is rather complex; yet this case is more simple than that of polycrystalline bodies. As shown by theory, three plane elastic waves may be propagated in an unlimited, homogeneous, anisotropic medium in an arbitrary direction; and in general none of these is purely longitudinal or purely transverse. One of the three waves is called the quasi-longitudinal wave; however, the displacement due to this wave does not coincide with the direction of propagation, but makes a certain angle with it. The two other waves are quasi-transverse, and their directional angle is larger. Several properties of the propagation of these waves are known; however, this field has not yet been investigated sufficiently, and a discussion of this subject would lead too us far afield*. We will only mention here that there exist preferred directions in crystals, so that the situation is simplified; and the nature of wave propagation in these directions does not differ from that in a homogeneous medium. Waves traveling along these directions are ordinary "pure" longitudinal and transverse waves. These special directions coincide with the axes of symmetry of the crystals.

In studies of the laws describing elastic-wave propagation, a single-crystal sample must be strictly oriented with respect to the crystallographic axes; therefore, acoustical studies of single crystals must be accompanied by precise X-ray investigations.

Propagation of Elastic Waves in a Granular Medium. The problem of elastic-wave propagation in a granular medium is very interesting from the acoustical viewpoint. One example of this type of medium is sand, both dry and water-saturated. The study of wave propagation in such a medium is of great importance in general and applied seismology and in hydroacoustics. In metals, individual crystalline grains are in some manner "welded" to one another during the process of melt crystallization, and

* Several interesting results of the properties of elastic-wave propagation in crystals have been investigated by Masgrave and in the U.S.S.R. by K. S. Aleksandrov at the Institute of Crystallography of the U.S.S.R. Academy of Sciences.

spaces between grains are filled by smaller crystals or else by oxides (due to the presence, however insignificant, of impurities); sand, on the contrary, is a porous medium. Spaces between individual grains of sand (sand grains are fragments of quartz, shaped during geological eras by surf or the movement of rivers) are filled with air (dry sand) or water (when the sand lies under water or deeper down near ground-water).

Figure 294 represents photographs (enlarged about 50 times) of two types of sea sand, differing in the size of their grains. Attention is drawn to Figure 294b, where the contact between two grains of sand is evident.

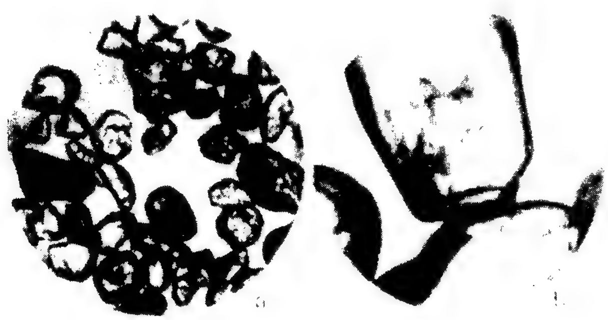


Figure 294. Microphotograph of two types of sand: a) No. 1; b) No. 2

Studies of velocity and absorption during the propagation of elastic waves in sand at frequencies of several tens of kilocycles per second can be performed in the laboratory using the pulse method. For such experiments, sand may be poured into a steel pipe 40 to 50 cm long; the pipe radius and the length of the sand column should be considerably greater than the wave length*. The piezoelectrical transducers are mounted in such a way that their surfaces are flush with those of the pistons, through which pressure is exerted upon the sand by means of a hydraulic press. When the sand has just been poured in and there is no pressure on it, then at a frequency of 30 kc and a distance of 20 cm no wave propagation can be detected between source and receiver. It is sufficient to apply a very small pressure to the sand column, less than 1 kg/cm^2, in order for a detectable signal to be transmitted. With a further increase in pressure, the attenuation decreases rapidly; at the same time, the velocity of longitudinal waves increases.

Figure 295 shows the velocity of longitudinal waves as a function of pressure for the two types of dry sand in Figure 294; the increasing pressure simulates the conditions at increasing depth in a sand deposit bearing its own weight. It is possible to construct a theory of propagation of elastic waves in a granular medium like sand, assuming each grain to have, as an average, the same size and spherical shape. In the case of sand which is subject to the gravitational effect of its own weight, elastic longitudinal waves which travel vertically downward have a velocity of propagation c''_{long} (where "

* Experiments on the propagation of elastic waves in sand were performed by N.V. Tsareva at the Geophysical Institute of the U.S.S.R. Academy of Sciences.

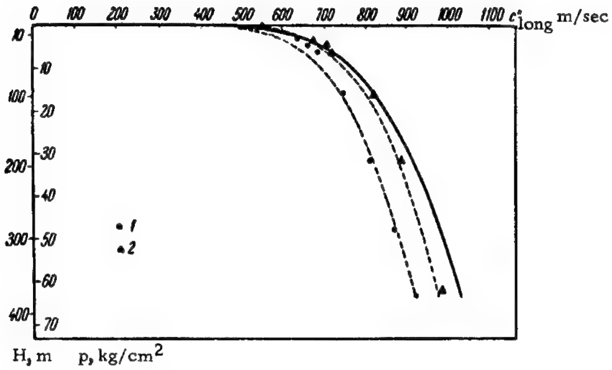


Figure 295. Velocity of longitudinal waves c''_{long} as a function of pressure p, or of the underground depth H, for two types of sand (see Figure 294)

Frequency, 30 kc. 1) sand No. 1; 2) sand No. 2. The solid curve corresponds to theory

indicates that the direction of the pressure coincides with that of wave propagation) which may be represented by the expression (similar to that for an isotropic medium):

$$c''_{long} = \sqrt{\frac{\bar{K}}{\rho}},$$

where $\bar{K}$ is the average bulk modulus of elasticity of the granular medium. Calculation shows that $\bar{K}$ can be represented as the sum of two terms, one corresponding to the bulk elasticity of the granular medium and the other corresponding to the elasticity of the grain contacts*. At low pressure, the

* It can be shown that

$$\bar{K} = \frac{K_1 K_2}{K_1 \frac{V_1}{V} + K_2 \frac{V_2}{V}} + K_3,$$

where K_1 is the bulk modulus of the medium filling in the pores; K_2 is the bulk modulus of the grains; V_1 is the volume of the material filling in the pores; V_2 is the volume of the grains; and K_3 is the elasticity of the grain contacts.

It was shown by Hertz in his theory of the contact of two solid balls, acted upon by a force p the direction of which coincides with the line through the centers of the balls, that the [incremental] approach δ of the centers of the balls is

$$\delta = 1.65 \sqrt[3]{\frac{p^2(1-\sigma^2)}{rE^2}},$$

where σ is Poisson's ratio; E is Young's modulus for the material of the balls; and r is their radius. From this formula, K_3 can be calculated (for a medium composed of similar balls), and hence c''_{long} may be found. As shown by calculation, the formula for K_3 as a function of pressure (i.e., of the underground depth H) is

$$K_3 = 5.78 \sqrt[3]{H\left(\frac{V_1 - V_2}{V_1 V_2}\right)\frac{E^2}{(1-\sigma^2)^2}}.$$

As seen from this formula, K_3 is proportional to the cube root of H, and consequently, to the cube root of the pressure. The theory of propagation of elastic waves in sand was developed by V. S. Nesterov at Moscow State University, and also by White and Sengbush in the U.S.A.

first term is the important one; since it is very small for dry sand, the velocity of propagation is also very small. With increasing pressure, the main influence begins to be that of the elastic contacts. An elastic wave propagated in sand which is under pressure may be said to travel mainly over the coupling paths represented by grain contacts. The elasticity of the latter is proportional to the cube root of the pressure; consequently, the velocity of propagation is proportional to the sixth root of the pressure, or of the underground depth. The principal conclusions of this theory are in satisfactory agreement with experiment, as is evident from the curves in Figure 295. A similar theory has been developed for transverse waves; in this case the relation between the velocity of transverse waves c''_{trans} and the pressure proves to be the same as that for longitudinal waves. Absolute values of c''_{trans} are less than those of c''_{long} by a factor of 1.6 to 1.7.

This problem of the absorption of elastic waves in a granular medium like sand has not yet been solved theoretically. Experiment shows the attenuation to be proportional to the frequency, which points to hysteresis effects.

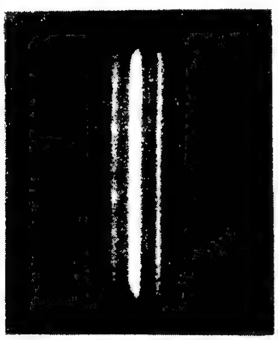


Figure 296. Diffraction of light while passing through a vibrating quartz plate

Frequency of vibration of plate, $2.98 \cdot 10^8$ cycles (1117th harmonic!)

If the sand is water-saturated, c''_{long} is approximately equal to the sound velocity in water and is equal to 1500-1600 m/sec; this value varies only a little with changing pressure (at least, up to pressures of 65 kg/cm^2). The role of the elastic contacts in this case can be said to be small, and the main influence is that of the bulk elasticity of the sand-water mixture. Transverse waves behave in the same manner in water-saturated sand as in dry sand, which can be explained by the fact that they cannot travel in water.

Photography of Ultrasonic Waves. Light Diffraction. Ultrasonic waves can be photographed in transparent solids, such as quartz, many other crystals, and various types of glass, by the schlieren method (see Figure 104) just as successfully as in liquids.

Diffraction of light by ultrasonic waves can also be observed in transparent solids. Such an observation is especially successful in a quartz plate which itself serves as a source of ultrasonic waves. Figure 296 represents a photograph (by S. Ya. Sokolov) of light diffraction by ultrasonic waves in a quartz plate vibrating at $2.98 \cdot 10^8$ cycles per second. Its fundamental frequency is $2.67 \cdot 10^5$ cycles, and so the plate is vibrating in its 1117th harmonic. Diffraction of light in a quartz crystal has been observed up to frequencies of 10^9 cycles. K. N. Baranskii succeeded in attaining an even greater frequency of longitudinal ultrasonic vibration in piezoelectric quartz — $2 \cdot 10^9$ cycles. A carefully polished X-cut quartz plate, selected from among several specimens, $12 \times 50 \times 20$ mm in size and with a fundamental frequency of vibration along the X axis of 230 kc, was excited by a radio oscillator with a decimeter-wave range. Special methods were employed in order to adjust one of the antinodes of the electric field to the point where the

electrode plates of the quartz were attached, thus achieving a maximum field intensity (about 200 v/cm for a power of 5 to 10 watts)*. High-order harmonics (from 430 to 8500) were excited in the plate, with continual variation of the vibration frequency. At frequencies above 10^9 cycles, the quartz plates lost their resonance properties, and both odd and even harmonics were equally well excited. This is explained by the fact that at such high frequencies the attenuation of elastic waves in quartz begins to play an essential part, and traveling waves begin to be propagated in the plate instead of standing waves.

To detect elastic vibrations in quartz, the method of light diffraction was used. The length of a longitudinal ultrasonic wave in quartz at a frequency of $2 \cdot 10^9$ cycles is $3.5 \cdot 10^{-4}$ cm, i.e., it corresponds to a wave length in the near infrared region of the spectrum. As a light source, mercury lamps were used, and from their radiation monochromatic light at 5460 Å and 5780 Å was selected. With a comparatively thick plate and a short wave length of the elastic waves, the spatial nature of a diffraction grating is clearly expressed, since diffraction phenomena are characterized by selective reflection of light from ultrasonic waves (see Chapter VII, § 3). For instance, at a frequency of about $3.33 \cdot 10^8$ cycles it is possible to observe only alternate spectra with orders +1 or -1 (as the sign of the Bragg angle is reversed); this is shown in the diffraction-spectrum photographs (Figure 297a, b).

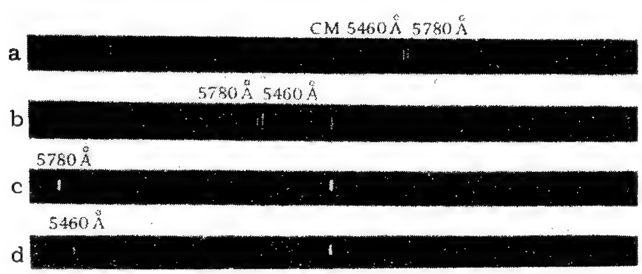


Figure 297. Diffraction spectra for ultrasonic waves in quartz

a, b) $f = 3.33 \cdot 10^8$ cycles; c, d) $f = 1.16 \cdot 10^8$ cycles. CM — central maximum

The periodic structure made up of ultrasonic waves resolves the diffracted light into monochromatic components. We can see this in the two lower photographs. Both spectra c and d in Figure 297 were obtained at the same ultrasonic frequency, but at different Bragg angles; consequently in one photo only the 5780 Å line is seen and not the 5460 Å line, while in the other the reverse is true. It may therefore be said that ultrasound at

* At frequencies from $1 \cdot 10^8$ to $8 \cdot 10^8$ cycles, the quartz plate was clamped between metal electrodes, which were connected to a coaxial cable. One end of the cable was connected to the oscillator; the other was short-circuited through a piston located some distance from the quartz. Stationary electromagnetic waves were produced in this cable. By moving the piston, an antinode of the voltage wave could be produced at the electrode of the quartz. At frequencies above $8 \cdot 10^8$ cycles, the same effect was achieved with hollow coaxial resonators, also tuned by a piston.

frequencies of about 10^9 in quartz serves as a monochromator in the visible part of the spectrum.

When discussing hypersound in liquids, we mentioned that the origin of hypersonic vibration is related to thermal motion in the medium. Vibration at a frequency of $2 \cdot 10^9$ cycles in quartz proves to be closely related to hypersonic vibration and may be called artificial hypersound.

Thermomechanical Vibration (Fluctuation) in Piezoelectric Crystals. As we have seen, hypersonic vibration, both thermal and artificial, can be studied by optical methods, making use of the diffraction of light by hypersound. However, thermal motion and the elastic waves generated by it, while limited in the direction of high frequencies, extend far into the low-frequency region, the lowest frequency being determined by the geometrical dimensions of the body. If the body under investigation possesses a piezoelectric effect, then its relatively low-frequency, thermomechanical motion may be studied by other methods.

Consider a bar of some hard solid substance. The surfaces of the bar will be in a state of continual random motion, due to the action of Debye elastic waves, which are always present in it and in the spectrum of which there will be found wave lengths comparable to the dimensions of the bar. These displacements are very small, and their direct measurement is extremely difficult.

Suppose, however, that the slab is cut out of a crystal which exhibits a piezoelectric effect. In this case, the piezoelectric effect, together with the resonance properties of the bar, leads to particular conditions in the bar. Because of the piezoelectric effect and the effects of Debye elastic waves, random alternating electromotive forces are generated at the electrodes of the piezoelectric bar; and these can be detected by electronic methods, in the same way as thermal noise is detected in electric conductors. Thermoelectrical fluctuations in electrical conductors have a continuous frequency spectrum ("white" noise); such electrical fluctuations are investigated by means of high-gain amplifiers. Also, special methods are employed of minimizing all external noise, including the thermal noise of the amplifier input circuits, in order to isolate the useful signal (in this case, the random electromotive forces generated at the terminals of the electrical conductor under investigation). If the electrodes of a piezoelectric bar are connected to this type of apparatus in order to investigate thermoelectrical fluctuations, then at frequencies close to the resonance frequencies of the bar (odd harmonics, when the thickness of the slab is equal to $n/2$ times the wave length of the elastic wave, where $n = 1, 3, 5, \ldots$), sharp surges of voltage can be observed. The vibrations on the electrodes of the bar are random ones, due to the randomness of the thermomechanical motion; while at the same time certain frequencies predominate and the spectrum contains very sharp inhomogeneities. These specific characteristics of the spectrum of thermoelectrical vibrations of a piezoelectric bar are the result of the piezoelectric effect in the bar, as a consequence of which the resonance of the thermomechanical vibrations (Debye waves) has a corresponding electrical aspect*. Figure 298 is an oscillogram of the spectrum of the thermomechanical motion in a piezoelectric bar of 45° X-cut Rochelle salt, obtained with an acoustic spectrograph, using a cathode-ray

* This phenomenon was studied by K.V. Goncharov and the author.

tube with high persistence. To the right of the thin vertical line (the 20 kc marker) and against the background of amplifier noise, a voltage peak is observed, corresponding to the first resonance frequency of the bar for longitudinal elastic waves.

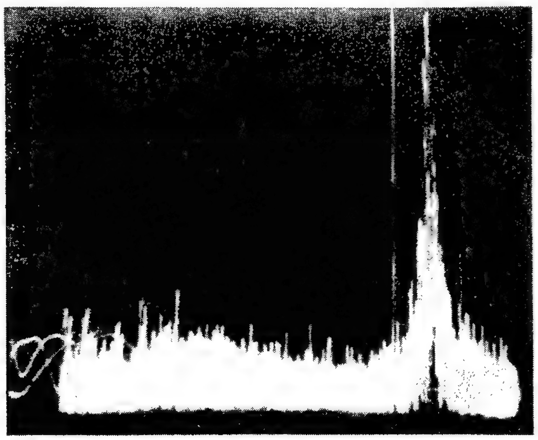


Figure 298. Thermal noise in a 45° X-cut Rochelle salt bar

Thin vertical line indicates 20 kc.

With a wide-band spectrum analyzer, it is possible to analyze the spectral composition of the thermomechanical vibrations of a piezoelectric element, and to obtain a series of resonance voltage peaks, each of which corresponds to a certain resonance of longitudinal or transverse waves in the piezoelectric element or transducer under investigation. This method of investigation of thermomechanical vibrations in piezoelectric crystals may find application in the study of the frequency characteristics of complex piezoelectric transducers, which is of definite practical value. It is evident that the thermal-noise level for piezoelectric and magnetostrictive receivers also determines their sensitivity limit, and limits the possibility of observation of weak sounds. By applying in this connection general fluctuation theory, it is possible to calculate the thermal-noise spectra for piezoelectric resonators and to establish sensitivity limits for piezoelectric receivers.

Ultrasonic Flaw Detection in Metals and Alloys. The ability of high-frequency ultrasonic waves to travel over long distances in metals without undergoing much absorption can be utilized for ultrasonic examination of specimens of various manufactured products in order to ascertain their quality. In casting processes and the subsequent working of metal, blisters, cracks, and various other nonuniformities may appear. If these remain unnoticed, the eventual breakdown of a machine part may be caused. In essential parts of machines and mechanisms — crankshafts, piston-rods, airplane propellers, etc. — such defects obviously cannot be tolerated. X-ray flaw detection makes it possible to examine metals to small depths only, while ultrasonic examination penetrates to a depth of over 10 m.

Sokolov in 1927 was the first to point out the possibility of ultrasonic flaw detection. At first he suggested a method of examining the specimen using continuous irradiation with ultrasonic waves. Consider a specimen in the form of a bar (Figure 299). If a radiating quartz plate is moved along one side of the bar, while a receiving quartz plate is moved along the other side, the amplitude of the signal at the receiver plate falls sharply if there is a defect in the bar. When they arrive at the boundaries of a blister, ultrasonic waves are reflected and change direction; the blister acts as a sort of barrier to ultrasound.

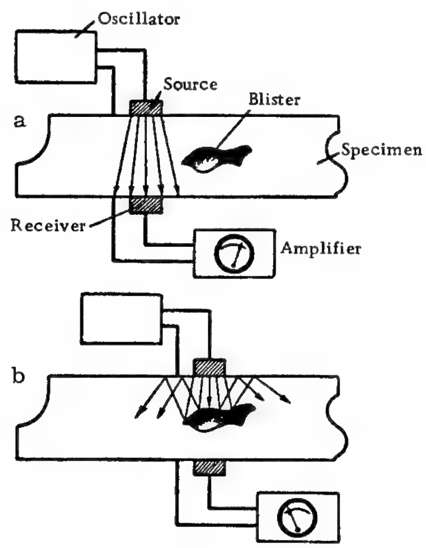


Figure 299. Ultrasonic flaw detection in a metal specimen

a) Ultrasound is picked up by the receiver (no "shadow");
b) a blister in the specimen impedes the passage of ultrasound to the receiver (receiver in "shadow" zone),

This method gives good results in many cases, yet it also suffers from serious disadvantages. For example, small blisters do not produce a sharp "shadow" and cannot be detected. Also, during continuous wave radiation reflection from the boundaries of the sample causes a complicated ultrasonic field to be generated in it, and the obtained results may be difficult to interpret. Finally, with the source and receiver operating continuously, it is difficult to eliminate electrical inductive coupling between the oscillator and the receiver amplifier.

The pulse method of ultrasonic flaw detection, also suggested by Sokolov, has proved to be more valuable. Figure 300 shows diagrammatically a method for detecting faults in a metal bar by means of the Sokolov flaw detector. The principle of the pulse-type flaw detector is as follows: various defects in the specimen — fissures, blisters, hairline cracks, etc. — will reflect ultrasonic pulses, which are then detected by the receiver plate and recorded (after suitable amplification) by an oscilloscope. Figure 301

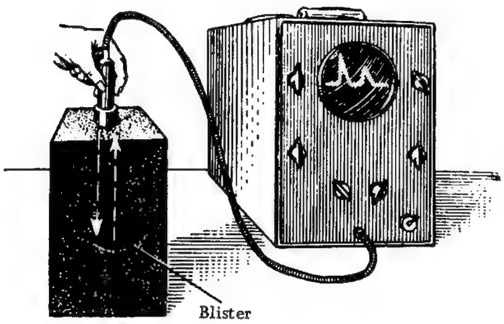


Figure 300. Flaw detection using the Sokolov ultrasonic flaw detector

represents a typical oscillogram from the screen of a flaw detector, while Figure 302 shows the corresponding defects. The examination of metal specimens is usually conducted at ultrasonic frequencies from one to several megacycles. The wave length of longitudinal ultrasonic waves of these frequencies (for instance, in iron) is from 5 to 1 mm (at a frequency of 5 mc).

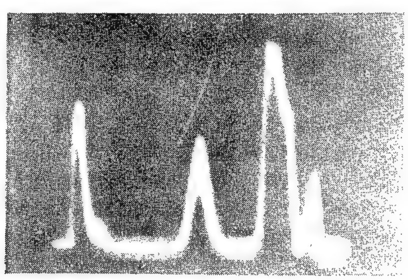


Figure 301. Typical oscillogram on the screen of a pulse-type flaw detector

If a flaw is smaller than the wave length, no reflection will occur, only scattering of the ultrasonic waves. Appreciable reflection is to be expected only when the dimensions of the specimen are considerably greater than the wave length; and then the flaw acts as a mirror for ultrasonic waves. Consequently, in order to detect minute flaws the ultrasonic frequency must be raised. However, it is impossible to continue far in this direction; as soon as the ultrasonic wave length becomes comparable to the size of the metal grains (on the average, from a fraction of a millimeter to a millimeter), the absorption and scattering of ultrasound are much increased.

Due to the high propagation velocity of elastic waves in metals, the duration of an ultrasonic pulse during flaw detection must be very short. Indeed, radiation of the pulse should terminate long before the arrival at the quartz plate of a pulse reflected from a flaw. Otherwise the reflected pulse will

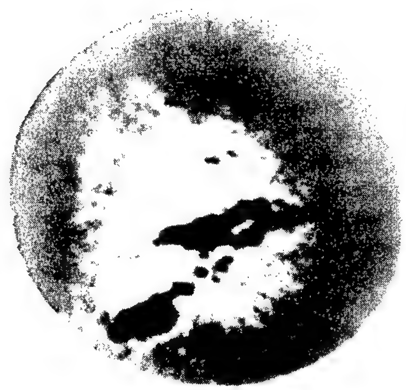


Figure 302. Defect in a specimen, corresponding to the oscillogram in Figure 301 (photographed after sawing the specimen in half)

be superposed upon the transmitted pulse, and it will be impossible to take measurements. The width of the transmitted pulse must be especially small when it is necessary to detect flaws close to the surface of the specimen. For this reason, the duration of ultrasonic pulses in ultrasonic flaw detectors is only from a fraction of a microsecond to a few microseconds. For a frequency of 5 mc and a pulse width of 1 μ sec only 5 vibrations will be contained in the pulse. Mechanical pulse delays are also used in the examination of surface layers (see Figure 303).

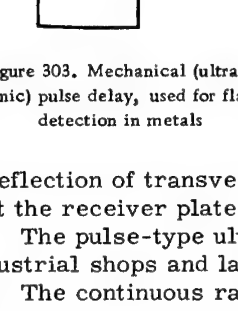


Figure 303. Mechanical (ultrasonic) pulse delay, used for flaw detection in metals

Since the propagation velocity of transverse waves is less than that of longitudinal waves, the former are sometimes used, especially to examine layers near the surface of a specimen.

In working with a flaw detector, the possibility of the appearance of types of waves other than those used in measurements must be considered. Thus, in examining the specimens by means of longitudinal waves, care must be taken in observing the pattern on the oscilloscope; it must be established that the observed reflection is not a reflection of transverse waves, or a result of the arrival of surface waves at the receiver plate.

The pulse-type ultrasonic flaw detector is finding increasing use in industrial shops and laboratories.

The continuous radiation of ultrasonic waves has at present found another application: by means of standing ultrasonic waves, it is possible to make accurate measurements of very small thicknesses. It is often necessary to measure the wall thickness of some manufactured product without harming the product. If a quartz plate is placed against the wall of the product, and the frequency of the exciting voltage is varied, then when the

thickness of the wall becomes equal to one-half the ultrasonic wave length, standing waves are formed (i. e., resonance occurs). By determining the frequency f at which resonance is produced, and provided the velocity of ultrasound in the material is known, it is possible to determine the wave length λ and, consequently, the wall thickness d:

$$d=\frac{\lambda}{2}=\frac{c}{2f}.$$

Such devices, called "ultrasonic thickness gauges", find application in the solution of many practical problems.

Ultrasonic Delay Lines. Ultrasonic delay lines are commonly used in electronic computers, and especially in radar equipment. Such lines make it possible to produce a precise time standard, or to delay for a specified period one electrical pulse with respect to another. This delay line is an acoustic waveguide made of some material with low ultrasonic attenuation; mercury, magnesium-aluminum alloy, fused quartz, etc., are used for this purpose.

An ultrasonic pulse, radiated, for instance, by a quartz plate situated at one end of the delay line (the carrier frequency used is from a few megacycles to a few tens of megacycles, depending on the purpose of the line), arrives at the opposite end of the line, where it is received by another quartz plate, connected to an amplifier. The time for passage of the ultrasonic pulse over the delay line is determined by the distance to be traversed in the line, as well as the velocity of ultrasound in the material of the acoustic waveguide. The time for passage of an electric pulse through the wires of the circuit is extremely small, since the velocity there is approximately that of light, while in the delay line the ultrasonic pulse travels at 1460 m/sec (in mercury) or 5750 m/sec (in fused quartz, longitudinal waves). Therefore, by means of an ultrasonic delay line one electric pulse may be retarded with respect to another by periods of time ranging from a few tens of microseconds to ten milliseconds; this cannot be achieved by normal electronic methods using inductive and capacitive delay lines.

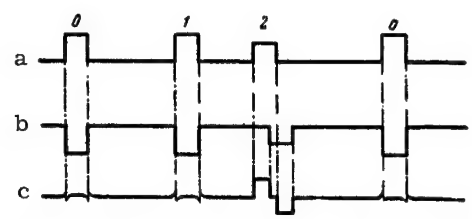


Figure 304. The addition of a signal (a) reflected from a target and the same signal (b) (of opposite sign) passed through a delay line

0) Sounding pulse; 1) pulse from a stationary object; 2) pulse from a moving target. The resultant signal (c) does not show a reflection signal from the stationary object (interference).

Ultrasonic delay lines are mostly utilized in radar, where they facilitate the location of moving targets against a background of diverse natural and

artificial interference. Figure 304 explains the principle of an ultrasonic delay line. In Figure 304a an oscillogram is shown schematically of a signal received by the radar after reflection from a moving object and a stationary object, when the radar is operating without a delay line. Pulse 0 is the sounding pulse; pulse 1 corresponds to reflection from some stationary object; and pulse 2 is reflected from a moving object. In reality, during the operation of radar equipment, the pattern on the cathode-ray tube (indicator), is very complicated, since the incoming signal consists of a large number of pulses, including random pulses (interference); and it is sometimes very difficult for the operator to select the signal corresponding to reflection from the target.

Figure 304b shows the same oscillogram as in Figure 304a; only the sign of the signal has been reversed. This oscillogram is obtained by passing the incoming signal through a delay line in order to retard it by a time exactly equal to τ, the interval between pulses; reversal of the signal is easily achieved by electronic methods.

If the two oscillograms a and b are now added together and if the amplitudes are equal, then the pulses corresponding to reflection from the stationary object are cancelled out, leaving only the pulse from the moving object, since during the delay time τ this pulse has shifted somewhat in time (Figure 304c). In this way, by means of an ultrasonic delay line, the accuracy of radar equipment is improved and it becomes more proof against interference.

The necessity for detecting moving targets at great distances requires the production of ultrasonic delay lines with long delay times. Under these circumstances, special attention must be paid to the material filling the acoustic waveguide and to the type of ultrasonic waves used, because of the attenuation of the ultrasonic pulse in the material of the acoustic waveguide.

Until recently, ultrasonic delay lines made use of liquid acoustic waveguides filled with mercury. Mercury is used because of its good acoustic conductivity and its comparatively low ultrasonic velocity which permits long delays with relatively short acoustic waveguides. However, mercury ultrasonic delay lines suffer from many disadvantages, which create difficulties for their practical application. Consequently, in practical systems mercury delay lines have been replaced by solid ultrasonic delay lines, which are more stable (and also more interesting from the viewpoint of the physical phenomena observed).

Research has shown that various types of waves may be utilized for ultrasonic delay lines — longitudinal, transverse, or surface waves, depending upon the purpose of the radar installation. The choice of the type of waves depends upon the conditions of minimum attenuation per unit delay time. Besides this, other factors are important, for example, the possibility of efficient excitation or detection of a certain type of wave. In many cases, transverse waves are preferred, provided they have low attenuation per unit delay time. In other cases, e.g., when alternating delays are required, surface waves may be used.

The shape of the acoustic waveguide has an appreciable effect upon the nature of the propagation of a wave packet such as an ultrasonic pulse. In solid acoustic waveguides with limited cross sections, transverse waves are commonly used, since the side walls have no effect on their propagation.

Indeed, as already mentioned, a transverse wave striking an interface at an angle which is larger than the critical angle is not divided, so that when a transverse wave travels along a free boundary [total internal reflection] it is not accompanied by transformation.

However, wave transformations in ultrasonic delay lines are not always undesirable. Transformations between types of waves find practical application in cases where the generation of longitudinal waves is more efficient than generation of transverse waves (e.g., the use of barium titanate ceramic transducers as compared to Y-cut piezoelectric quartz plates), but transverse waves are used in the acoustic waveguide. In this case, at the beginning of the acoustic waveguide longitudinal waves are transformed into transverse waves by causing the longitudinal wave to approach the free reflecting surface at an angle; and the transverse component of the reflected wave is transmitted into the acoustic waveguide. At the end of the latter, the reverse transformation occurs; and at the reception end is placed a piezoelectric transducer which is sensitive to longitudinal waves. With short delay lines, special measures are required to prevent a pulse reflected from the receiving plate from reaching it again after a secondary reflection; this would cause distortion of the radar information during passage through the delay line. In ultrasonic lines designed for large delays, a great number of reflections of the ultrasonic pulse are used and a long path length between reflections. Figure 305* shows a design of an ultrasonic-line acoustic waveguide with a stellar configuration, utilizing 31 crossings

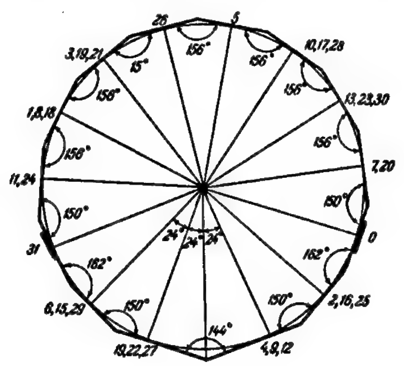


Figure 305. Diagram of a 31-crossing ultrasonic delay line of fused quartz

Numbers outside the figure indicate the numbers of the reflections.

(30 reflections). Lines of this type are usually made of optical fused quartz and permit delays of about 1 to 2 msec, for a quartz-disk diameter of about 10 cm. In such lines, in addition to the choice of a material with good acoustic conductivity for the type of wave employed, consideration must be given to the necessity of eliminating possible false signals. These signals usually appear due to the passage of ultrasonic pulses along some other

* The configurations of the ultrasonic delay lines shown in Figures 305 and 306 are given more detailed consideration in an article by Arenberg (Convent. Rec. I.R.E., Part 6, 63. 1954).

path in the acoustic waveguide or due to transformation to some other type of wave. Figure 306 shows some designs of ultrasonic delay lines, with acoustic waveguides made of magnesium-aluminum alloys* and optical fused quartz.

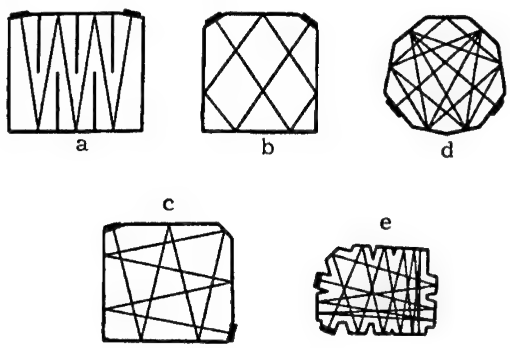


Figure 306. Some designs of ultrasonic delay lines

Very interesting also are surface waves, the velocity of propagation of which is less than that of longitudinal and transverse waves, and the formation of which can also be observed at interfaces between two media. For instance, surface waves can be obtained as a result of transformations from other types of waves — longitudinal or transverse — incident obliquely at an interface, provided the velocity of the longitudinal or transverse wave in the first medium is lower than that of the surface wave in the second. Certain other properties of surface waves are also of interest, for example, the way in which they propagate along curved surfaces or in plates, where a traveling surface wave alternately passes from one face to the other.

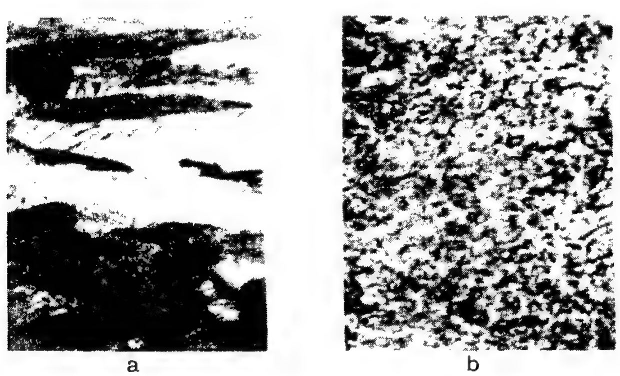


Figure 307. Microphotograph of polished cadmium surface

a) Crystallization under normal conditions; b) crystallization with ultrasonic irradiation of the molten metal

* See, for instance, the article by Metz and Andersen (Electronics, No. 7. 1945).

Effect of Ultrasound on Solids. The effects of powerful ultrasound upon solids lead to many interesting phenomena. Sokolov discovered that molten metals subjected to ultrasound solidify more rapidly and have a finer grain and more uniform structure (Figure 307). S.N. Rzhevkin and E.P. Ostrovskii demonstrated that by means of ultrasound solids can become dispersed in liquids (e.g., lead in water). Ultrasound makes it possible to obtain very thin layers of deposited metal during electrolysis, to remove gases (degassing) from certain solids, and so on. Research on ultrasound discloses more and more such effects, and many of these are certain to find important practical applications.

Reflection of Sound by Elastic Shells and Plates. Anomalous (Nonspecular) Reflection and Transmission of Sound. Most of the objects surrounding us are elastic solids. Some of them, from the acoustical viewpoint, may be regarded as elastic shells or plates; for instance, walls and floors in buildings, the hulls of ships or submarines, etc.

Until recently, such elastic solids were thought to scatter sound like absolutely rigid, immobile bodies. It was also thought that if the dimensions of a body are large compared with the acoustic wave length and if its surface consists of relatively flat or slightly convex parts, then it would reflect sound as does a mirror. However, it has now been established that such a simplified conception of elastic bodies, including elastic shells and plates, does not represent reality. It has been found that the vibrations of shells, plates, and elastic bodies can in general bring about appreciable additional scattering of sound, and also lead to the appearance of considerable reflection in the direction opposite to that of the incident wave. This reflection has been termed anomalous or nonspecular reflection (Figure 308).

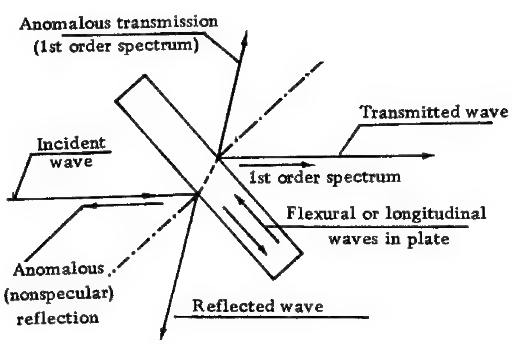


Figure 308. Anomalous reflection and transmission of waves during the incidence of ultrasonic waves on a thin plate of finite length

Also, an anomalous transmission of sound through plates and shells is observed. Anomalous passage of sound through a plate in a liquid was observed in 1940 by S.N. Rzhevkin and S.I. Krechmer, using schlieren photography. They discovered another wave beyond the plate, in addition to the normal transmitted wave the direction of which coincides with that of the incident wave; the direction of the new wave was opposite to that of the wave reflected from the plate (see Figure 308). Rzhevkin explained this anomalous transmission of sound through the plate as being due to the

presence of standing flexural vibrations, generated in a plate of finite dimensions as a result of the effects of incident waves. Later, L. M. Lyamshev at the Acoustics Institute of the U.S.S.R. Academy of Sciences and also the American acoustician Finney discovered the nonspecular reflection caused by longitudinal vibrations of the plate*.

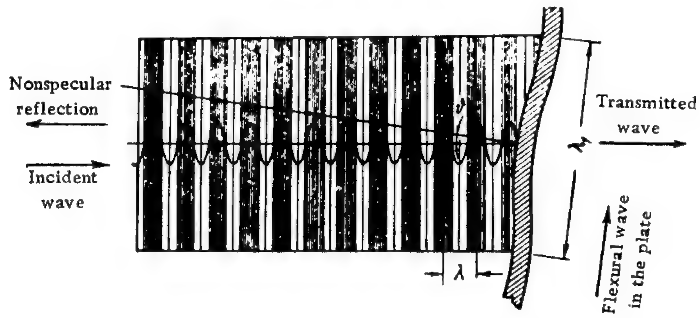


Figure 309. Flexural resonance of a plate

The essential nature of nonspecular reflection is easily understood from the example of nonspecular sound reflection by a thin finite plate. When a plane sound wave arrives at an angle ϑ at a thin plate of finite length (Figures 309 and 310), flexural waves and longitudinal waves are generated in the plate. The waves in the plate may be simple flexural or simple longitudinal waves; for the latter, the thickness of the plate equals one or more half waves, and periodic thickening and narrowing take place at the faces of the plate (on both sides). When a sound wave arrives at the plate at such an angle that the propagation velocity of the waves generated in the plate is equal to that of free longitudinal or flexural waves, a type of resonance occurs**, and the vibrating plate radiates intensively. The portion of the radiation the direction of which is opposite to that of the incident wave represents anomalous reflection of the sound, while the portion of the radiation which passes through the plate in the direction opposite to the reflected ray gives rise to anomalous transmission.

During the reflection of sound by a thin plate, anomalous reflection and transmission of sound are observed for a series of discrete angles of incidence, corresponding to the excitation of flexural or longitudinal waves of different types in the plate. The direction of nonspecular reflection is determined from the condition of coincidence (when the phase velocity in the incident wave coincides with the velocity of free flexural waves in the plate):

$$\sin\vartheta = \frac{c}{c_1},$$

where ϑ is the angle of incidence; c is the velocity of sound in the medium surrounding the plate; and c_1 is the velocity of flexural or longitudinal vibrations in the plate.

* Lyamshev, L.M., Otrazhenie zvuka tonkimi plastinkami i obolochkami v zhidkosti (Reflection of Sound by Thin Plates and Shells in Liquids). —Izdatel'stvo Akademii Nauk S.S.S.R. 1955.

** This resonance, the importance of which was stressed by the German acoustician Krämer, has been called coincidence resonance and is also known as spatial-frequency resonance.

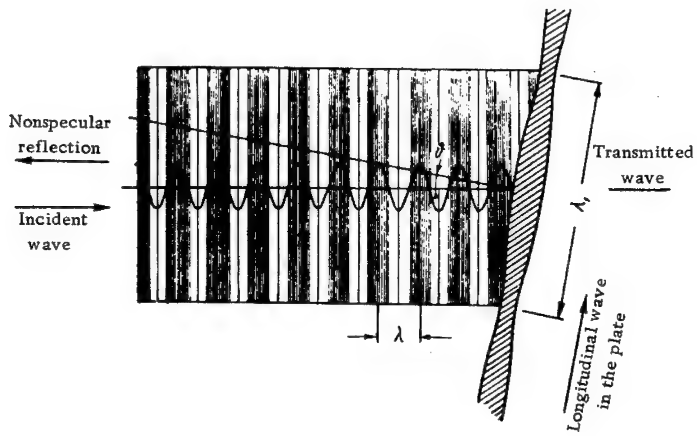


Figure 310. Longitudinal resonance of a plate

Anomalous reflection and transmission of sound at a plate can be clearly illustrated in the following way (Figure 308)*. Let a plane sound wave arrive at the plate from the surrounding liquid; when coincidence occurs (e.g., for flexural waves), the plate begins to radiate intensively. A plate in which a system of standing waves has been produced (see Figures 309 and 310) can be considered a plane diffraction grating, composed of two moving sinusoidal gratings, corresponding to the waves being propagated in the plate in opposite directions. Since the plate does not perform piston vibrations (does not pulsate along its whole surface with the same amplitude), therefore no zero-order spectrum (plane wave, normal to the grating) is formed behind the plate. At the same time, the sinusoidal flexural wave, moving along the plate in one direction, yields a secondary spectrum of order +1, while the wave traveling in the opposite direction yields a spectrum of order -1, and the corresponding angles satisfy the condition

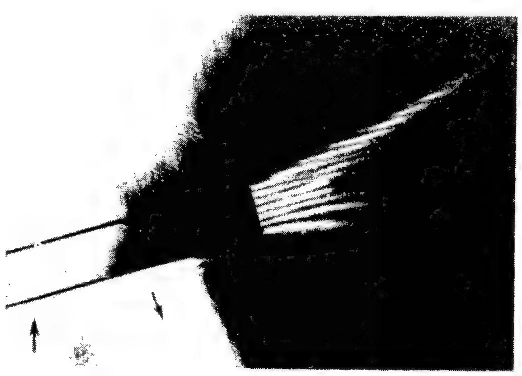


Figure 311. Radiation at the end of a glass plate in water (plate thickness, 10 mm; frequency $f = 1.81$ mc) during the incidence of waves upon the plate

* The explanation was suggested by S. N. Rzhevkin.

$\sin \vartheta = \pm\frac{\lambda}{\Lambda}$, where Λ is the wave length of the vibration traveling along the plate.

It is evident that the spectrum of order +1 will correspond to the ordinary wave transmitted beyond the plate, the direction of which coincides with that of the incident wave; while the spectrum of order -1 will correspond to the anomalous transmitted wave, the direction of which is opposite to that of the reflected wave. If the radiation of the plate is observed from the side of the incident wave, it is also easy to see that it must produce an ordinary mirror-reflection wave and also a wave the direction of which is directly opposite to that of the incident wave (nonspecular, or anomalous, reflection).

Figures 311 and 312 represent two photographs*, obtained by the schlieren method. The first of them clearly shows radiation at the end of a plate, during the arrival of an ultrasonic beam (shown by an arrow) at the plate. In the plate standing waves of a high order** are generated, which produce on the end surface a vibratory motion, similar to a sinusoidal

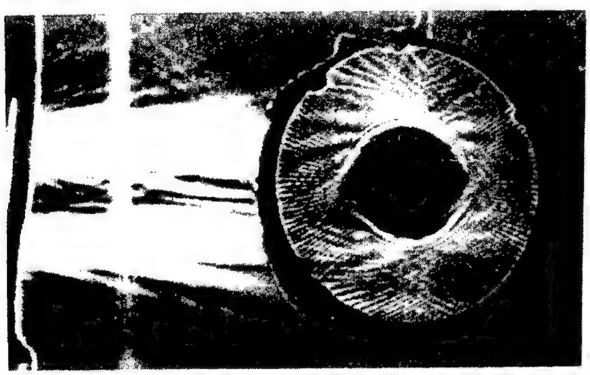


Figure 312. Sound field inside a cylindrical shell $f = 3.1$ mc, $D = 30$ mm. Two ultrasonic beams are incident on the shell.

grating of finite size (with a small number of rulings). The bands in the figure are a result of the interference of the two waves produced by this grating, traveling with equal angles to the axis of the plate and corresponding to spectra of orders +1 and -1. The very interesting photograph in Figure 312 shows an interference pattern for a system of standing waves, formed in a cylindrical steel shell by the action of two ultrasonic beams entering from outside. These standing waves are produced by the propagation along the shell (in opposite directions) of two waves generated by the two ultrasonic beams. Every (nearly flat) element of the shell may be considered to produce spectra of orders +1 and -1. The sum total of these spectra yields the interference bands seen in the photograph and directed normally to the shell (along the radii). Superposition of the rays (spectra) radiated

* Obtained by V. I. Makarov at Moscow State University.

** The plate is several half waves thick.

by different elements situated about the circumference causes considerable increase in intensity along an inner ring corresponding to the envelope of these rays (caustic surface). It is of special interest to note that inside this envelope there is no sound. An acoustic shadow zone is formed.

Figure 313 shows a polar characteristic of reflection from a thin brass plate*. The polar characteristic of reflection was measured in the direction opposite to that of the incident wave. During measurement, the frame

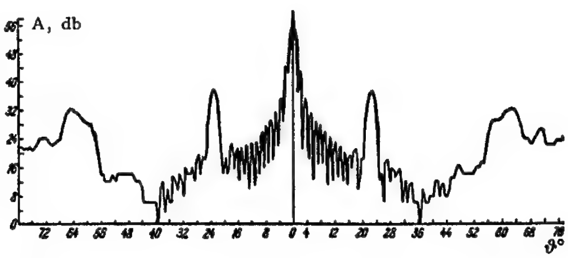


Figure 313. Polar characteristic of reflection at a brass plate, area $15 \times 30\,mm^2$, thickness 0.8 mm, in water (frequency, 10^6 cycles)

with the plate was rotated, the angles of rotation being recorded; while the amplitude of the reflected pulse was measured using the scale on the oscilloscope screen. The amplitude of the reflection is plotted (in db) on the polar characteristic along the vertical axis, with respect to a standard level, while the angle of incidence in degrees is plotted on the horizontal axis. Maximum reflection ($\vartheta = 0°$) corresponds to mirror [specular] reflection during normal incidence of the sound wave at the plate.

In the polar characteristic of reflection at a plate represented in Figure 313, the reflection maxima correspond to nonspecular reflection produced by longitudinal (angle of incidence $\vartheta = 23°$) and flexural (angle of incidence $\vartheta = 63°$) vibrations of the plate.

If the vibration frequency or the thickness of the plate (rod) are increased, additional directions appear of intense nonspecular reflection. Indeed, a thick plate (thick rod) forms an elastic layer. It is found that different vibrations can travel along the layer with definite propagation velocities, differing from one another. These velocities are determined by the elastic constants of the layer and depend as well upon the thickness of the layer and the frequency of vibration. Every vibration traveling along the layer at a given velocity is a so-called normal wave. Nonspecular reflection of sound from a thick plate (rod) is always observed when the phase velocity of the incident sound wave in the liquid along the plate coincides with the velocity of one of the normal waves in the plate.

* The polar characteristic was obtained by L.M. Lyamshev at the Acoustics Institute of the U.S.S.R. Academy of Sciences.

Chapter X

PROPAGATION OF ELASTIC WAVES IN THE EARTH'S CRUST

The conditions and nature of elastic-wave propagation in solids have been discussed, and so we may now pass on to a study of the propagation of elastic waves in the solid crust of the earth.

§ 1. Earthquakes as a source of elastic waves in the earth

We live at the bottom of an air ocean, and our hearing is adapted to receive sound waves traveling through the air. However, not only in the surrounding atmosphere, but also underneath, in the earth, a vast world of sound exists. On the one hand, sounds are transmitted into the earth from its surface. A thudding blow at the surface of the earth can be heard over a considerable distance by placing an ear against the ground. Also, in the earth itself, there are many sources of elastic waves, which may be immeasurably more intense than the familiar sound sources in the air.

With the solid earth under us, we have become accustomed to considering its interior to be an immobile and inert mass, devoid of any change; this is the more true because most of the interior of the earth is still inaccessible to man, who, on the other hand, has traveled to great heights in the atmosphere and to great depths in the ocean. The deepest that man has so far penetrated into the earth is a little over 7 km (by well-drilling).

In reality, however, a great number of diverse physicochemical processes are taking place within the earth. The most familiar proof of activity within the earth's crust is that given by earthquakes — one of the most terrible natural phenomena. The cause of earthquakes is not yet completely clear*, but there is no doubt that most earthquakes are the result of movement of matter in the deep layers beneath the earth's crust (the crust thickness is estimated to be several tens of kilometers). The earth's crust reacts to these movements from beneath it, and is strained, producing cracks and shifts. This horizontal and vertical shifting of rock due to processes below the earth's crust, and often in the crust itself, slowly dies out. Nevertheless, rapid shifting of layers with respect to one another may also occur, causing earthquakes. Nearly all the earthquakes observed at the surface are caused by this shifting of rock, and only a tiny fraction are caused by volcanic activity.

During strong earthquakes, an immense amount of energy is liberated. For instance, according to Golitsyn's calculations, the energy of the

* On this subject, see Bonchkovskii, V.F. Zemletryaseniya i metody ikh izucheniya (Earthquakes and Methods for Studying Them). —Izdatel'stvo AN SSSR. 1949.

Issyk Kul earthquake (1911) was $4.6 \cdot 10^{17}$ mkg. This would be sufficient to heat 10^{10} tons of water (i.e., $10\,km^3$) from 0 to boiling. It should not be supposed, by the way, that earthquakes are rare natural phenomena; it is only extremely powerful earthquakes which are rare. Not all quakes are perceived by us as shocks and not all can even be recorded by the special measuring instruments which will be described below. The number of earthquakes which may in any way be recorded reach several hundred thousand per year, and an even greater number of tremors remain unobserved. It may be assumed that, on the average, every few minutes an earthquake which may be recorded by instruments occurs at some point on the globe. Thus, the earth is a body which continuously undergoes vibration.

BORIS BORISOVICH GOLITSYN
(1862-1916)

The vibration occurring at the focus of an earthquake is the source of longitudinal and transverse elastic waves of tremendous power, which spread out from the focus as spherical waves. High-frequency waves (short waves) are comparatively rapidly absorbed, or scattered by nonuniformities in the earth's crust, while low-frequency elastic waves (long waves) are propagated over long distances.

Even though we are unable to penetrate directly deep into the earth, several important conclusions about the internal structure of our planet result from observations of the propagation of elastic waves through the earth. Golitsyn has said graphically that every earthquake may be likened "to a torch which is lighted for a moment to illuminate the interior of the earth that we may see what is taking place there". The investigation of the propagation of elastic waves in the earth's crust and interior is the task of seismology, an important branch of geophysics. The eminent Russian scientist Golitsyn may properly be regarded as the founder of this science.

§ 2. Detection and recording of seismic waves*. Seismographs

How is it possible to detect elastic, or seismic, waves traveling through the earth? The question naturally arises whether an ordinary carbon microphone cannot be used for this purpose. A microphone, however, is hardly suitable. When a microphone operates in air, its diaphragm vibrates, while the body of the microphone, which has considerable inertia, remains at rest. If a microphone were buried in the ground, then during the passage of an elastic wave which causes the soil particles to vibrate, the entire microphone would be shifted, together with its diaphragm; for such applications the inertia of the microphone case is too low for it to remain at rest.

It is also possible simply to place the microphone plus its diaphragm against the ground, providing good contact between the two (not an easy task in itself); but in this case, during shifting of the soil particles in a horizontal direction no pressure will be applied to the carbon powder. During vertical displacements of the soil particles some pressure variations are transmitted from the diaphragm to the carbon powder, i.e., an elastic wave is detected; but the sensitivity of such an instrument will be very low. This is explained chiefly as follows.

Seismic waves arriving from distant earthquakes possess comparatively long periods, up to several seconds. Because of the high velocity of propagation of elastic waves in solids, the wave lengths attain several kilometers. For instance, for a period of 5 sec and an average velocity of propagation of longitudinal waves in the upper parts of the earth's crust of 5 km/sec, the wave length is 25 km! Ordinary microphones show poor sensitivity to such low frequencies and long waves. Moreover, the displacement of the particles of a solid body during the passage of an elastic wave is exceedingly small, and the amplitude of vibration of the diaphragm will be insignificant. Still another circumstance should be taken into account: elastic waves in solids may be either longitudinal or transverse; even if a microphone detects the waves, it is not capable of distinguishing between types of waves.

For the detection and subsequent recording of seismic waves detectors of another type are used, called seismographs. There are very many different types of seismographs, but they all have essentially the same working principle.

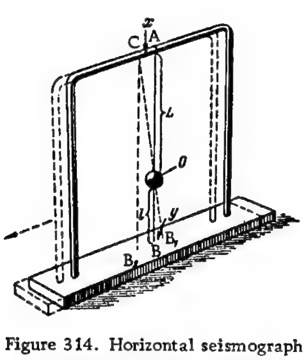


Figure 314. Horizontal seismograph utilizing a vertical pendulum

Consider a heavy weight suspended on a long fiber. As indicated in Chapter I, such a vertical pendulum, once it is displaced from equilibrium, will perform vibrations. Let the suspension point of the pendulum be rigidly attached to some surface (Figure 314), relative to which the position of the weight O can be recorded. If, as a result of a passing elastic wave, the suspension point is suddenly shifted (during a time which is much shorter than the period of natural vibration of the pendulum) from point A to point C (horizontal shift), then, due to the large mass of the weight and to its correspondingly high inertia, the weight will

* ["Seismic waves" are also called "earthquake waves" or "earth waves".]

remain at rest, while the recording device of the pendulum moves to B_1. Hence, if the pendulum support is shifted by an amount $x = AC = BB_2$, then the recording tip of the pendulum is shifted a distance $y = BB_1$. From the similar triangles AOC and BOB_1, it is easily seen that

$$x:y = L:l,$$

where $L = OA$ and $l = OB$; thus,

$$x = y\frac{L}{l}.$$

The ratio $K = L:l$ is called the magnification of the pendulum. In the manner shown, a vertical pendulum may be used to record horizontal movements of the ground.

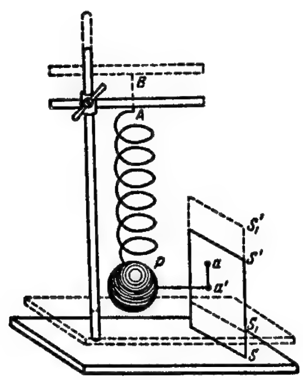


Figure 315. Operation of a vertical seismograph, consisting of a weight suspended on a spring

If the movement of the ground is vertical, it may be recorded using a heavy weight suspended on a spring (Figure 315). A sudden vertical movement of the ground, which shifts the suspension point of the spring from A to B, is not transmitted to the weight instantly; because of its inertia, the weight will lag behind the movement of the support and the ground upon which the support stands. During this time, the stylus which is fixed to the weight will draw a line aa', indicating the displacement of the suspension point, upon the surface SS', which is rigidly attached to the support. For small vertical displacements of the ground, weight P may be considered to remain essentially at rest.

In reality the situation is, of course, not as simple as this, since neither the horizontal nor the vertical motion of the ground occurs instantaneously; and the pendulum and the weight on the spring are shifted somewhat during this period of time. This undesired shift introduces some error into the recording of ground movements.

In order for a seismograph using the vertical-pendulum principle and recording horizontal ground movements (horizontal seismograph) to be sufficiently sensitive to long-period vibrations, the pendulum must be very long. It is possible, however, to use instead of the vertical pendulum a horizontal pendulum with a special suspension, which has very high sensitivity for a comparatively small pendulum length. This type of horizontal

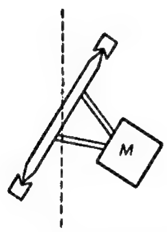


Figure 316. Horizontal pen-dulum with a skew suspension

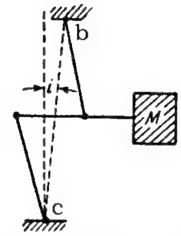


Figure 317. A horizontal pendulum with an improved skew suspension

The axis of the pendulum passes through points b and c.

seismograph operates in approximately the same manner as a door which is hinged on a slant. If a weight is attached by means of a horizontal steel rod to an ordinary door, suspended vertically by hinges at its edges, then this system will not perform vibrations after a push— the door will merely open or close by some angle which depends upon the force of the push, and then it will stop. A door with an oblique (skew) suspension, however, to which a weight is attached by means of a horizontal rod (Figure 316), acts differently. When this system is pushed, it will oscillate. Children may

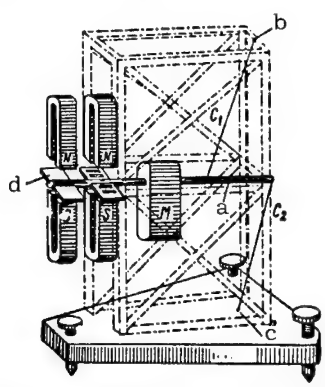


Figure 318. Diagram of Golitsyn's horizontal seismograph with a horizontal pendulum

sometimes be seen swinging on a door that has become loose. The period of vibration of such a system, which may be regarded as a horizontal pendulum, is greater as the angle between the door and the door jamb is less*. A seismograph with a horizontal pendulum for recording horizontal displacements of the earth was first constructed in 1906-1910 by Golitsyn. It

* The period of natural vibration of a horizontal pendulum with this type of suspension is

$$T=2\pi\sqrt{\frac{l}{g\sin i}},$$

where l is the length of the pendulum; g is the acceleration of gravity; and i is the angle between the vertical and the pendulum axis (see Figure 317). When i is small, $\sin i \sim i$, and

$$T=2\pi\sqrt{\frac{l}{gi}}.$$

proved to be so successful that it is still used by the best seismic stations both in the U.S.S.R. and abroad; and by means of this seismograph very distant earthquakes can be recorded. Figure 317 is a diagram of a horizontal pendulum; while Figure 318 represents schematically the Golitsyn horizontal seismograph. The pendulum consists of a brass rod a, suspended by two thin steel filaments C_1 and C_2; its axis of rotation passes through points b and a. A brass weight M (7.2 kg) is attached eccentrically to the rod, while at the end of the rod are attached an induction coil d and a copper vane, which is designed to increase the damping of the vibrations of the system. It was mentioned in Chapter I that when the damping of a system is great changes in the external forces applied to it are more faithfully reproduced. During the motion of the pendulum, eddy currents are generated in the copper vane; these tend to brake the motion of the vane and hence the pendulum as a whole.

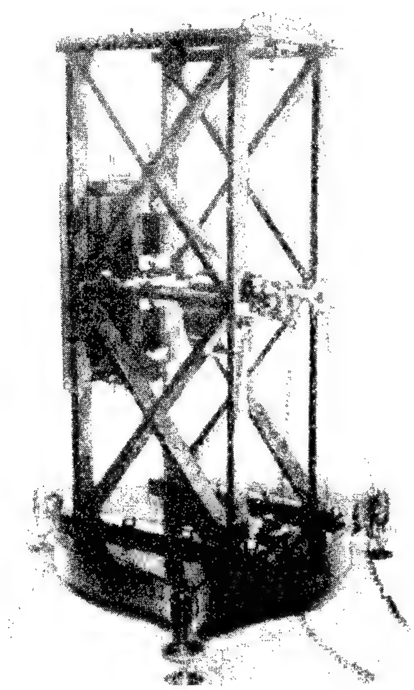


Figure 319. The Golitsyn horizontal seismograph

Four horseshoe magnets are set on slides fixed to the frame, which is in turn rigidly attached to the base of the seismograph. The clearance between the magnets can be adjusted by means of a micrometer screw, thus adjusting the sensitivity of the seismograph and the amount of damping. The natural period of vibration of the pendulum can be adjusted between 5 and 25 sec.

The passage of an elastic wave causes shifting of the ground, and simultaneous shifting of the seismograph frame. The resulting vibrations of the

pendulum generate an emf in the induction coil, and the mirror galvanometer shows a deflection. The vibrations are usually recorded on photographic paper stretched on a drum which is rotated by clockwork. The drum axis is shifted abruptly through a few millimeters after every complete revolution; this type of system provides continuous recording over a prolonged period (12 or 24 hours). Figure 319 is a photograph of the Golitsyn horizontal seismograph.

To record vertical ground movements, Golitsyn employed a horizontal pendulum of the type shown in Figure 320; the pendulum is maintained in the horizontal position by a spring. Figure 321 shows a photograph of the Golitsyn vertical seismograph, the natural period of vibration of which is about 10 sec.

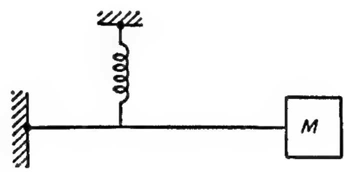


Figure 320. Diagram of a horizontal pendulum operating as a vertical seismograph

By means of Golitsyn seismographs, it is possible to obtain a clear record of the initial moment of arrival of seismic waves. However, due to the long natural period of vibration and to the different sensitivity of a seismograph to different wave lengths of the arriving signals, the subsequent record is distorted. The sensitivity of a Golitsyn seismograph

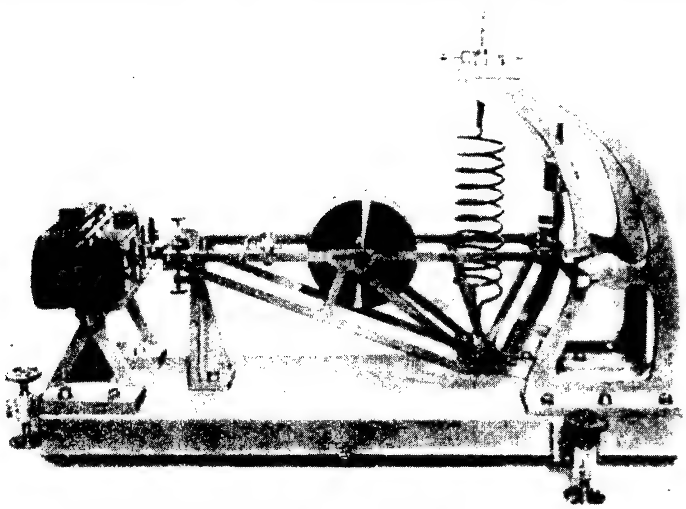


Figure 321. Golitsyn vertical seismograph

rapidly decreases with a decrease in the period of a seismic wave. Short waves with periods of a fraction of a second, however, are the most intense in nearby earthquakes; and these waves are just barely recordable with such seismographs.

Concussions due to the movement of vehicles or to heavy machinery operating near a seismograph are also almost unrecordable, since elastic waves arriving from such sources have short periods, a few tenths of a second or less. Nevertheless, in order to reduce the interference background, seismographs are placed in cellars, on large concrete foundations.

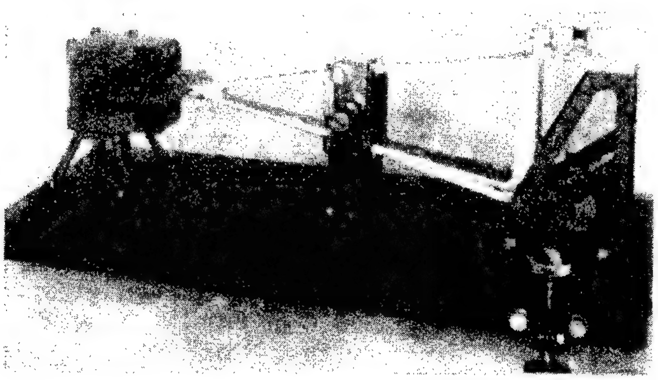


Figure 322. The Kirnos horizontal seismograph

In order to record nearby earthquakes, when waves of short period (0.1 to 0.5 sec) arrive at the observation point, the seismologist D. P. Kirnos designed the horizontal and vertical seismographs whose photographs are

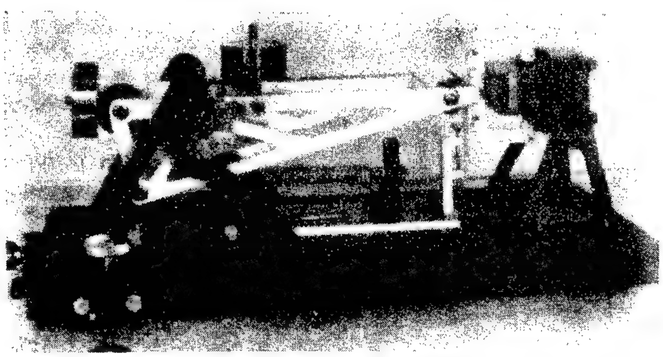


Figure 323. The Kirnos vertical seismograph

shown in Figures 322 and 323. These seismographs are based on the same principle as the Golitsyn seismographs, but they have been improved considerably in comparison with the latter. Such seismographs are included in the equipment of many seismic stations in the Soviet Union.

§ 3. Seismograms of earthquakes. Structure of the earth

Earthquakes generate both longitudinal (P waves) and transverse waves (S waves). Since the velocity of propagation of the former is nearly twice that of the latter, the longitudinal waves are the first to arrive at the recording station. Figure 324 represents schematically the propagation of elastic waves resulting from an earthquake. From the earthquake focus O,

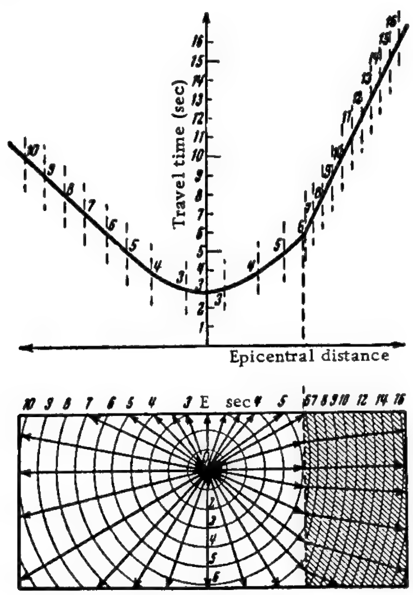


Figure 324. Propagation of elastic waves

When waves spreading from point 0 encounter a medium in which their velocity is changed (shaded area in figure), they are refracted. In the upper part of the figure the hodograph is shown

situated at a depth EO, spherical waves are propagated. Seismic rays (lines perpendicular to the wave surfaces) arrive at the earth's surface at different times. Point E is known as the epicenter of the earthquake; and the distance from the epicenter to the observation point (seismic station) is called the epicentral distance. The curve showing the time necessary for propagation of a wave to a given point on the earth's surface, as a function of the epicentral distance, is called the hodograph. As shown in Figure 324, when the waves encounter in their path a medium in which their velocity is less, they are refracted. At a point on the earth's surface where an interface between two media exists there is an abrupt change in the slope of the hodograph. The artificial case of two such media in which elastic waves have different velocities is considered in the figure, simply to demonstrate the hodograph. In reality, the layers in which elastic waves in the earth's crust have different velocities are generally horizontal, rather than vertical, layers.

In addition to ordinary longitudinal and transverse seismic waves, there are other types of elastic waves; for instance, in the preceding chapter

surface waves (Rayleigh waves) were discussed. Since the energy of surface waves (called L waves) is concentrated only in the surface layer, they travel great distances and have far greater amplitudes than P and S waves.

These three types of waves — longitudinal (P), transverse (S), and surface (L) — are the principal modes of vibration. However, several other types of waves may also arrive at the point of observation, since each of the above three types may undergo refraction and reflection during propagation, especially if the observation point is far from the earthquake. These reflections and refractions are due to the presence of different layers within the earth and to the earth's surface itself.

Figure 325 is a schematic diagram of the inner structure of the earth, according to modern theory. Under the crust of the earth, several tens of kilometers thick, lies the mantle of the earth, with a thickness of approximately 1200 km, a density (according to calculation) of 3.4, and a total volume of 500 billion km^3. Next is the so-called intermediate layer, with a density of 6.4, a thickness of about 2900 km, and a volume of 400 billion km^3. Finally, there is the iron or iron-nickel core, extending to the very center of the earth (to a depth of 6370 km); its density is 9.6 and its volume is 180 billion km^3.

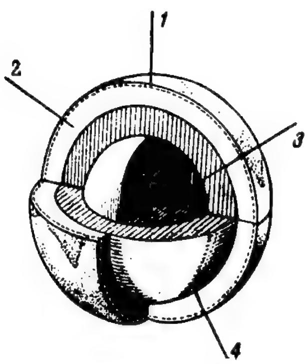


Figure 325. The internal structure of the earth

1) Crust; 2) mantle; 3) core; 4) intermediate layer

If the seismic station is situated near the epicenter of an earthquake, the earthquake waves arrive in the following sequence: P, S, L. At a greater epicentral distance Δ, the seismic station may be reached by waves reflected from the earth's surface. In addition to the P, S, and L waves, an experienced seismographer detects in the seismogram (the record of the elastic waves arriving at the seismograph) the arrival of waves that have undergone a series of reflections from the surface of the earth. The spreading out of waves from an earthquake focus is shown diagrammatically in Figure 326; outside the diagram of the earth, typical seismograms are shown. The upper seismogram is obtained by a station at a small epicentral distance; the lower one, by a station at a large epicentral distance.

P waves which have been reflected once from the surface of the earth have the symbol PP, and twice-reflected waves the symbol PPP; the same applies to transverse waves. As we already know, when waves arrive at the interface between two solids, a longitudinal wave may divide into a longitudinal and a transverse wave, or a transverse wave may divide into a transverse and a longitudinal wave. The PS wave in Figure 326 is a transverse wave formed by the reflection of a longitudinal wave; in the same figure are shown transverse and longitudinal waves which have been reflected from the earth's surface several times. In addition, longitudinal and transverse waves may be reflected from the core (P_cP, S_cS), and longitudinal waves may be refracted by the core (PKP).

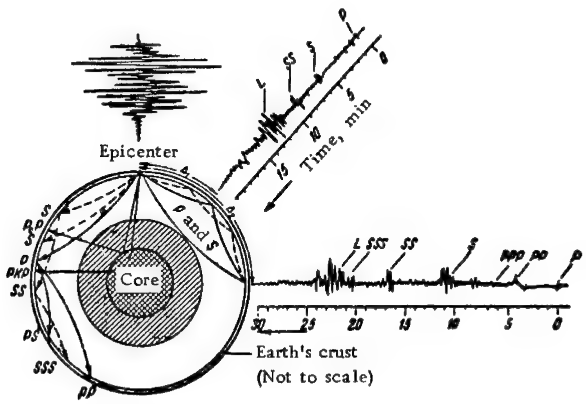


Figure 326. Seismograms of various types of elastic waves

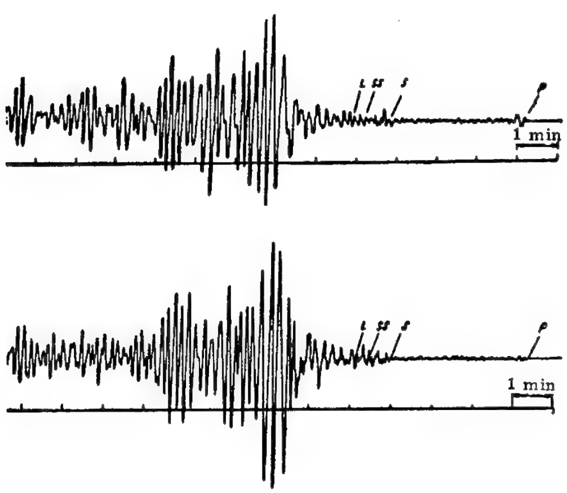


Figure 327. Seismograms of an earthquake (22 Sept. 1939) in Asia Minor ($\Delta = 2060$ km), recorded at the "Moskva" seismic station by Golitsyn horizontal seismographs. The upper seismogram was recorded by a seismograph oriented north-south; and the lower seismogram by a seismograph oriented east-west

Typical seismograms of a comparatively near earthquake are given in Figure 327, showing clearly the arrival of P and S waves, as well as of surface waves. Figure 328 is another sample of an earthquake recording. As mentioned previously, seismograms are recorded on photographic paper turning on a slowly revolving drum; to avoid overlapping the drum is shifted a small distance in the direction of its axis after every full rotation. Thus, in the absence of seismic waves, a seismogram is a series of parallel lines, as seen in Figure 328. This figure shows a portion of the seismogram

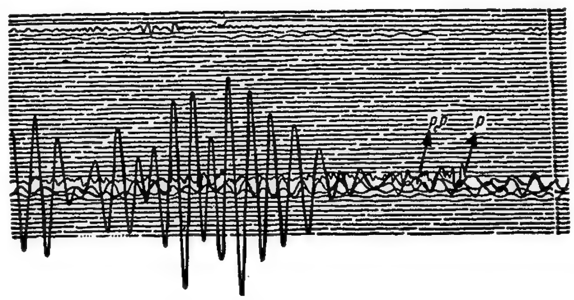


Figure 328. A portion of the seismogram of an earthquake in the Pacific (12 May 1948; Δ = 7500 km), recorded by a Golitsyn vertical seismograph at the "Moskva" seismic station

The small breaks in the lines are time markers (intervals of 1 min).

of a Pacific earthquake, recorded at the "Moskva" seismic station (Δ = 7500 km) on 12 May 1948, by a vertical seismograph. The resultant amplitude of vertical displacement of the ground at Moscow reached 100 microns. The seismogram shows clearly the first arrival of a P wave; the high-amplitude vibrations correspond to surface waves.

If the earth were uniform, with a constant velocity of propagation of P and S waves at different depths, the accurate location of the epicenter of a distant earthquake would present no difficulties; it would then be possible to locate an earthquake using observations at three stations.

Actually, even though the exact moment of the earthquake is unknown, the epicentral distance Δ_1 can be found from the data of one station by noting the difference in the arrival times of the P and S waves, the velocities of which must of course be known. From the point representing the seismic station, a circle of radius Δ_1 is drawn (Figure 329).

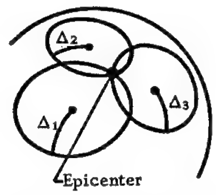


Figure 329. If the epicentral distances are known for three seismic stations, the earthquake focus is located at the point of intersection of three circles with radii equal to the corresponding Δ's

If another circle is drawn around the point representing a second seismic station (with epicentral distance Δ_2), then the epicenter is situated at a point of intersection of the two circles. Since there are two such points, a third circle of radius Δ_3, corresponding to a third seismic station, is required to locate the epicenter at a definite point (the point of intersection of the three circles).

It is not really necessary to have the results from three, or even two, seismic stations in order to locate an epicenter. The epicenter may be found by means of the seismograms obtained at a single station. Usually, there are three seismographs at a seismic station — two horizontal instruments, oriented at right angles to each other, and one vertical one. From the three seismograms obtained it is possible to determine the direction of the epicenter. It is also helpful to know that earthquakes occur mainly in seismically active regions, the locations of which are well-known.

However, in reality the task is much more complicated than we have indicated. Due to the nonuniform structure of the earth, seismic waves

have different velocities of propagation at different depths. Seismologists employ special methods of data processing to take into account the properties of elastic-wave propagation within the earth. Figure 330 shows the velocity of longitudinal and transverse waves as functions of the distance into the earth (the results of numerous seismic observations). The velocity of longitudinal waves is seen to increase until a depth of 2900 km. Then, at the edge of the earth's core, it decreases suddenly, but gradually increases again, reaching between 11 and 12 km/sec at the earth's center. The velocity of propagation of transverse waves also increases with depth. In examining these curves, it should be borne in mind that the velocities of longitudinal and transverse waves at zero depth actually refer to the "stripped earth" (the earth with its crust removed). At the true surface of the earth (granite layer) the velocity of P waves is about 5.5 km/sec and that of S waves is 3.3 km/sec; in the basalt layer the velocities are 6.3 and 3.7 km/sec, respectively; and in the hypogene rock of the mantle they are 7.9 and 4.4 km/sec.

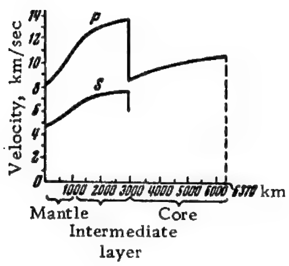


Figure 330. Change in velocity of longitudinal and transverse waves with depth into earth

Such high P and S wave velocities deep within the earth can be explained by the tremendous pressures and high temperatures existing there. At a depth of 2900 km (core boundary) the pressure is 1.5 million atm, and at the very center of the earth 3 million atm. Near the surface of the earth the temperature increases about 10-25°C for every kilometer down. If this rate of increase continued to the center of the earth, the temperature there would be 60,000°C. However, geophysical and astronomical data indicate that it is considerably lower (only a few thousand degrees); the temperature is apparently about 2000°C at a depth of 60 km. It has been found that the velocity of elastic waves is dependent on the density of the medium and on its elastic properties. For the immense pressures within the earth, the elastic moduli E and μ increase much more quickly than the density does.

Several conclusions about the internal structure of the earth result from the study of seismograms of deep-focus earthquakes.

Seismic data indicate that only longitudinal waves can pass through the core of the earth, which means that the core is a medium whose shear modulus μ is zero*. The physical properties of such a medium must be similar to those of a liquid (for liquids $\mu = 0$, and transverse elastic waves cannot propagate in them). However, gravitational considerations and studies of tidal motion indicate that the core of the earth must be solid. Thus, seismological and gravimetrical conclusions are found to contradict one another, and the cause for this remains unknown.

Seismology does more than just provide information about the internal structure of the earth. Its data form the basis for an extensive regional forecasting of earthquakes. Earthquake maps of the Soviet Union, utilizing geological data, make it possible to predict which regions will be struck by earthquakes of a certain intensity. This information is utilized in

* Or, if $\mu \neq 0$, then the transverse waves must be subject to high absorption, for some unknown reason.

taking special precautions to strengthen buildings and other structures in districts which are liable to strong earthquakes.

Seismic observations can be very useful in the Pacific basin, where earthquakes with epicenters in the sea (seaquakes) are very frequent. These result in huge waves [gravity waves] which are over 15 m high at the shore. Such waves, called "tsunami", are very destructive. By determining the epicenter and intensity of a seaquake, a seismic station can predict the "tsunami", because the velocity of propagation of elastic waves through the earth is incomparably greater than the velocity of the tsunami.

The seismic service of the U.S.S.R. comprises a great number of seismic stations. The principal task of seismology is earthquake forecast, i.e., the prediction of the time and site of an earthquake, as well as its intensity. This task, however, is as yet unfeasible.

§ 4. Seismic prospecting for useful minerals

As we have seen, the analysis of seismographic records makes it possible to draw many important conclusions concerning the structure of the earth's crust and of the deeper regions of the earth. The question naturally arises of the possibility of exploring the surface layers of the earth by producing "artificial earthquakes" by means of explosions and then observing the nature of the resulting elastic-wave propagation. Let us suppose that at a certain depth under the earth's surface there lies a layer (stratum) of rock with a density and a velocity of longitudinal and transverse waves which differ from those of the surface layer, i.e., a layer differing sharply from the surface in its elastic properties. The incidence of elastic waves upon the interface between the two layers results in reflected waves. Let us consider whether it is possible to determine the depth of the second layer by observing the reflected waves with a seismograph, in the case when the velocity of waves in the upper layer is known. It has been found that the thickness of the first layer may indeed by found, using echo methods, by exploding a small quantity of explosive near the earth's surface. The study of the structure of the surface layers of the earth, which is important in relation to prospecting for useful minerals, is the subject of the branch of geophysics called applied seismology (or, more frequently, seismic prospecting).

One would think that valuable information for the seismic probing of surface layers of the earth could be obtained by means of the pulse method (just as in hydroacoustical measurements of the sea depth, using sonic depth finders); for example, sound or ultrasound waves could be sent into the earth from some sort of generator and the reflections received. However, in practice this is found to be impossible, except over extremely short distances. This is true because the surface layers are mostly alluvial (thus they are very loose), with a very high absorption of elastic waves in the sound and ultrasound range*. Moreover, the sound waves are partially scattered and partially deflected due to the nonuniform structure of the surface layers of the earth (cracks and various inclusions in the soil).

* For ultrasonic absorption in sand, see Chapter IX, § 3.

Consequently, explosions continue to be the main practical source of elastic waves.

An explosion produces spherical longitudinal and transverse waves (P and S waves), spreading out in all directions from the site of the explosion. In an explosion close to the earth's surface, surface waves (L waves) are also produced. Such a source produces a whole spectrum of elastic vibrations, from very low to high frequencies. Waves of various frequencies are reflected or refracted at interfaces, and in these cases transformation from P to S or vice versa may occur; thus, the wave propagation presents a very complex picture. Moreover, the waves undergo diffraction and scattering at various nonuniformities of the medium.

Therefore, if the arriving elastic waves are recorded on one seismograph only, placed at some distance from the point of explosion, the seismogram does not enable us to draw any conclusions about the structure of the soil layers near the surface.

Refraction Method. In spite of the obvious feasibility of using artificial earthquakes to investigate the structure of the surface layers of the earth, for a long time no reliable data were obtained, because of the difficulties outlined above. It was only 20 or 25 years ago, after the emergence of powerful electronic methods of measurement and the development of a theory explaining elastic-wave propagation in solids, that it became possible to obtain valuable results in applied seismology. At present, there exist two principal methods of seismic prospecting: the method of refracted waves and the method of reflected waves. The former yields excellent results and is the principal method used at present.

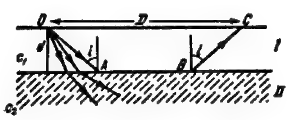


Figure 331. After an explosion at point O, the wave traveling along path OA undergoes total internal reflection and travels parallel to interface AB

Consider two rock layers (strata), an upper layer I and a lower layer II, and assume the velocity of elastic waves to be greater in the lower layer. This is very often the case in actual situations, since lower soil layers are denser and have greater elasticity than upper ones. At a seismograph C near an explosion site O (Figure 331), the first waves to arrive are those traveling along the straight line OC parallel to the surface of the earth. Now let the seismograph be gradually moved away. Some seismic rays from the explosion continue along the surface, and some penetrate to the interface, where they undergo reflection and refraction. Among these rays, there is always one ray AB which undergoes total internal reflection and then travels parallel to the interface. Let us now examine the situation in more detail.

Suppose the spherical wave from the explosion site O reaches the interface (point 1 in Figure 332) at a certain moment t_1. Partially reflected, the wave begins to propagate in the second medium, where its velocity is greater. During transition from the first to the second medium, the wave fronts reaching the interface at time t_2 (point 2) will change direction. At some time t_3 (point 3), the interface will be reached by a section of the wave front whose direction corresponds to the critical angle for total internal reflection; at this same moment, the front of the refracted wave,

moving in the second medium, will pass point 3 at right angles to the interface. Since the velocity of waves in the second medium is greater than in the first, it is evident that, beginning at this moment, the wave front in the second medium will begin to move ahead of the front of the incident wave (points 4, 5, etc.). In this way a disturbance will travel along the interface, with a velocity equal to that of wave propagation in the second medium.

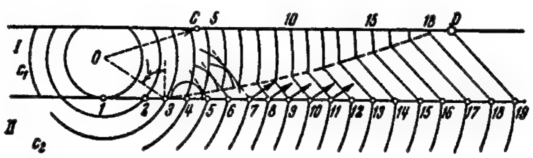


Figure 332. Formation of a lateral wave in the presence of two layers; the velocity of elastic waves in the second medium is greater than that in the first. The lateral wave arrives at point D before the direct wave proceeding along path OD

According to Huygens' principle, every point on the interface is a source of longitudinal and transverse waves which propagate in medium I; for simplicity, only longitudinal waves will be considered here. Elementary spherical wavelets may be constructed at every point on the interface, the the envelope of these wavelets forms the front of the wave propagating in the first medium and is called the lateral wave*. It is evident that this wave will arrive at point D on the earth's surface sooner than the direct (incident) wave. The lateral wave is similar in nature to the ballistic wave generated during supersonic flight of a projectile. Figures 333 and 334 are schlieren photographs of lateral waves. In the case shown in Figure 333, a source

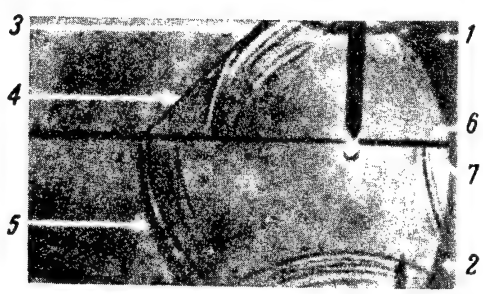


Figure 333. Waves generated by a spark at the interface between two liquids

The arrows denote: 1) xylene, 2) brine solution, 3) spherical wave in xylene, 4) lateral wave, 5) spherical wave in brine solution, 6) reflected lateral wave, 7) interface between liquids.

* In literature on theoretical and applied seismology this wave is often called a Mintrop wave, or a "head wave".

of spherical waves (a spark) is located near the interface between two liquids*. In the upper liquid, where the velocity of the waves is less, a lateral wave is formed (shown by arrow 4); the front of this wave lags behind the front of the wave in the lower liquid. Figure 334 is a photograph of the lateral waves generated in the upper and lower parts of a volume of water in which is placed an aluminum rod. When a spherical wave arrives at the rod, both longitudinal and transverse waves are generated in it, with velocities greater than that of longitudinal waves in the liquid. The two

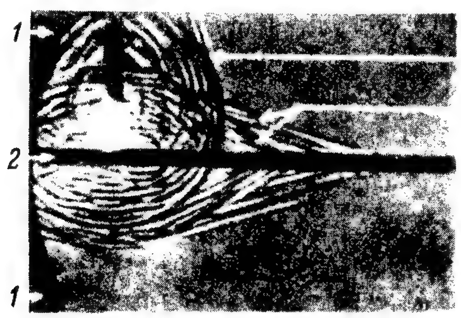


Figure 334. Waves generated by the incidence of spherical waves upon an aluminum rod placed in water

The arrows denote: 1) water, 2) aluminum rod, 3) spherical wave (from spark) spreading in water, 4) lateral waves produced by the longitudinal and transverse waves in the rod.

types of waves in the rod lead to the formation in the liquid of two lateral waves, which are evident in the photograph. Since the velocity of transverse waves in aluminum is less than that of longitudinal waves, the front of the lateral wave corresponding to the longitudinal waves forms a smaller angle with the rod than does the front of the lateral wave corresponding to the transverse waves.

Thus, to summarize, when the velocity of waves is greater in a second medium than in a first, a lateral wave is formed which, beginning at a certain distance (along the earth's surface) from an explosion site, will arrive at the seismograph before a direct wave. Figure 335 shows diagrammatically the wave fronts of direct and lateral waves for the case of three horizontal layers (strata); the velocity of waves is greater in the second layer than in the first, and greater in the third layer than in the second. In the upper part of the figure is a curve, for the given case of three layers, relating the travel time of a given type of wave to the distance between the points of excitation and recording of the concussions, i.e., the hodograph. As seen from the figure, both the direct and refracted (lateral) waves arrive simultaneously at point A on the earth's surface. Beyond point A, the refracted (lateral) waves will arrive first. The point of sudden bending on the hodograph corresponds to point A. At point B on the surface, refracted (lateral) waves from both the second and third layers will arrive

* It can be demonstrated that if plane waves instead of spherical waves arrive at the interface, then the lateral wave coincides with the reflected one.

simultaneously; while beyond point B the refracted waves from the third layer will arrive first. Accordingly, at B there is a second bend in the hodograph.

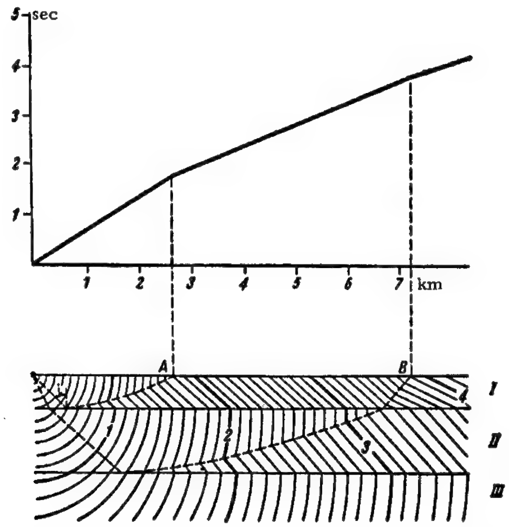


Figure 335. Formation of lateral waves in the presence of three layers. Hodograph of the refracted waves

It is always possible to construct a hodograph for refracted waves, provided the change with depth of the velocity of elastic waves is known, as well as the shape and position of the interfaces. This problem in applied seismology is known as the direct problem. The inverse problem consists in starting with the hodographs of the refracted waves, and then determining the change in the velocity of elastic waves with depth, as well as finding the shape and position of the interfaces. The refraction method makes possible solution of the inverse problem in several, practically important cases. Let us consider one very simple case.

The velocity of longitudinal waves in a lower layer (see Figure 331) is greater than in an upper. It is required to find the depth of the second layer. Let us assume the velocities of longitudinal waves to be given — c_1 in the upper layer and c_2 in the lower — as well as the distance D from the explosion site to point C, where the direct and refracted (lateral) waves arrive simultaneously. These data are quite sufficient to find the depth at which the second layer is located. If the direct waves (with velocity c_1) travel a distance D during time Δt_1, then $D = c_1 \Delta t_1$.

The travel time of the wave over the path OABC is also easily determined. Distances OA and BC are equal, and if the critical angle i for total internal reflection is known (in this case it is known, since velocities c_1 and c_2 are known, see Chapter IX, § 3), then the distance OA = BC can be expressed in terms of the depth d of the lower level*; the velocity of the wave while traversing these paths is c_1. It remains to find the travel time

* $OA = BC = \frac{d}{\cos i}$.

along the interface (over the path AB), where the wave travels with a velocity c_2. This time is determined without any special difficulty, by means of elementary algebraic and trigonometric considerations. The total travel time of the wave over OABC is the sum of the travel times along OA, AB, and BC; this is

$$\Delta t_2 = \frac{D}{c_2} + 2d\,\frac{\sqrt{c_2^2 - c_1^2}}{c_1 c_2}.$$

For $\Delta t_1 = \Delta t_2$, we have

$$\frac{D}{c_1} = \frac{D}{c_2} + 2d\,\frac{\sqrt{c_2^2 - c_1^2}}{c_1 c_2}, \text{ and so } d = \frac{D}{2}\sqrt{\frac{c_2 - c_1}{c_2 + c_1}}.$$

The position of the seismograph at which both the direct and refracted (lateral) waves arrive simultaneously (the distance D, at which $\Delta t_1 = \Delta t_2$) may be found experimentally. Velocities c_1 and c_2 can be found from the slopes of the straight portions of the hodograph; finally, d is found from the given formula. For example, if in the first layer $c_1 = 2000$ m/sec, and in the second layer $c_2 = 4000$ m/sec, and D = 200 m (found experimentally), then $d = \frac{200}{2}\sqrt{\frac{2000}{6000}} \approx 60$ m.

We have considered the simple case of two layers. For a greater number of layers, all parallel to the earth's surface, several hodographs will be determined in the same way as for two layers. Under natural conditions, of course, a more complex distribution of layers is encountered — they may be slanted or folded, or may possess veins, domes, faults, etc. In many cases, the shape of the lower strata can be quite well determined from the form of the hodographs. It is true, of course, that serious difficulties are sometimes encountered, and that investigations do not always succeed in determining correctly the inner structure of underground strata.

The refraction method not only gives information about the depth of layers with different acoustic impedance (or acoustic stiffness), but also makes it possible to form an opinion about the nature of the rock which makes up these layers, as indicated by the value of the velocity of longitudinal waves in the lower strata. The difference in acoustic stiffness cannot be less than 5-10% in order for the first arrivals of the refracted waves to be clearly discriminated.

In addition to the first arrivals, analysis of seismograms sometimes also takes account of second arrivals of refracted waves of mixed type, e.g., longitudinal-transverse waves. However, such arrivals are not always easy to isolate and to interpret correctly*.

Reflection Method. The detection of layer boundaries using reflected waves is very simple in principle. Its practical application, however, turns out to be quite complicated. The main difficulty lies in isolating the waves reflected from the interface from the other elastic waves of various kinds arriving at the seismographs. It is necessary to distinguish the arrivals of reflected waves from those of direct waves, refracted waves (first, and also later, arrivals), and surface waves (which have a lower velocity). The principal method of wave discrimination will now be described.

* G.A. Gamburtsev has developed a method of analysis (the "correlation method") which takes into account later arrivals as well.

Let us imagine several seismographs placed close together at a distance (not very great) from the explosion site. The direct waves and surface waves from the explosion, both traveling along the surface of the earth, will reach the seismographs at different moments, and so will have different phases; the difference in arrival time of these waves is determined by their velocity and by the distances between seismographs. At the same time, the reflected waves will reach all of the seismographs at nearly the same moment of time, since these waves travel almost vertically upward. Therefore, their "apparent" velocity, i.e., the velocity component of the wave front parallel to the earth's surface, is much greater than that of the direct, surface, and refracted waves. If vibrations from several seismographs are added together, then in the resultant record the reflected waves are reinforced, while other kinds of waves are reduced to a considerable degree. Figure 336 shows the results of placing eight seismographs at distances of 300 to 330 m from the explosion site. The reflected

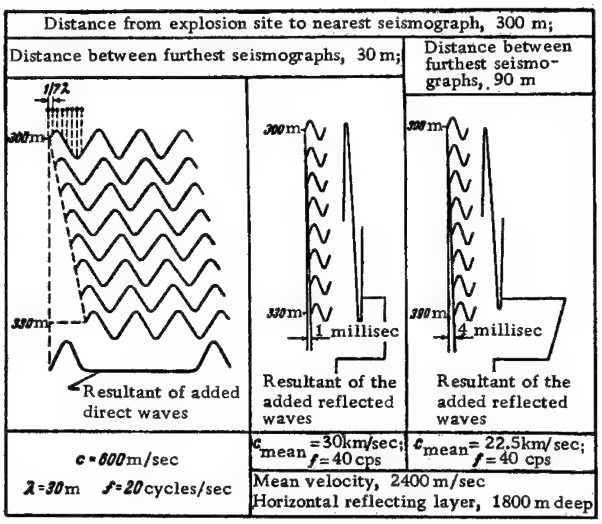


Figure 336. Use of a group of seismographs to isolate reflected waves

waves, with a frequency of 40 cycles/sec and an "apparent" velocity of 30 km/sec, reach all the seismographs nearly simultaneously; and so in the resultant record they are amplified by about a factor of eight. The "apparent" velocity of the direct and surface waves is about 600 m/sec, so that the difference in their arrival times at adjacent seismographs is 1/140 sec. A summation of these waves of different phase causes them to cancel one another almost completely (except the beginning and end of a wave — see Figure 336).

In addition to using a group of seismographs, the isolation [discrimination] of reflected waves is also facilitated by the use of vertical seismographs, inasmuch as the reflected waves arrive "from below". The amplitude of the surface waves is reduced by a proper choice of the explosion

site and size of explosive charge, as well as by setting off the explosion deep in the ground. The undesired waves are usually somewhat different in frequency from the reflected waves, so that electric filters may be very effectively used in seismic amplifiers (see below).

A few words should be added concerning the determination of the depth of the reflecting layer, using the reflection method. Let us consider once more the very simple case of two horizontal layers of different acoustic stiffness (Figure 337). D is the distance between the explosion site and the

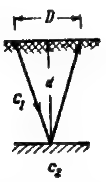


Figure 337. Determination of depth of a reflecting layer by the reflection method.

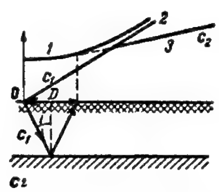


Figure 338. Hodographs of the reflected and refracted waves

seismographs, d is the thickness of the upper layer, and c_{long} is the velocity of longitudinal waves in this layer. Very simple reasoning indicates that the travel time of waves reflected from the interface is $\Delta t = \frac{1}{c_{\text{long}}}\sqrt{D^2 + 4d^2}$. If D is not large (i. e., if the reflected waves travel almost vertically upward), then $\Delta t \approx \frac{2d}{c_{\text{long}}}$.

Figure 338 shows the hodograph of the reflected waves for the case of two horizontal layers. For small angles of incidence (vertical sounding), hodograph 1 is nearly horizontal; with increasing D, this curve represents a branch of a hyperbola, climbing steeply upward and gradually approaching a straight line. At still greater distances between seismograph and explosion site, the tangent line 2, drawn from point O to curve 1, intersects the straight line 3, which is the hodograph of the refracted waves. If the velocity of longitudinal waves c_{long} in the first layer is known, Δt is found experimentally, and D is measured, then the above formulas give the depth d of the reflecting layer.

The reflection method, in contrast to the refraction method, can also be used when the velocity of elastic waves in a lower layer is less than in an upper one, provided there is a difference in the acoustic stiffnesses of the rocks forming the different strata. Among other reasons, this explains the very widespread application of the reflection method in petroleum prospecting, since the velocity of elastic waves in a petroleum layer is usually only a fraction of that in the overlying rock strata. The reflection method makes it possible to discover layer boundaries down to depths of several kilometers, while the refraction method is confined chiefly to small-depth measurements. However, correct interpretation of the seismograms obtained from deep shots is always made difficult by the presence of multiple reflections from the interfaces; while traveling downward from the earth's surface, waves are reflected from a series of layer boundaries, return to previous reflecting layers and to the earth's surface, are again reflected, and so on.

Along with its many advantages in comparison with the refraction method, the reflection method also has several disadvantages. For instance, this method cannot be used to determine the velocity of elastic waves in the reflecting layer itself, which we have seen to be possible using the other method. The reflection method, then, can only show the presence of a layer boundary, without yielding any information at all about the rock forming the reflecting layer.

§ 5. Seismic prospecting techniques

The refraction and reflection methods have proved to be useful in seismic prospecting for many useful minerals, especially in petroleum prospecting. Seismic prospecting leads to a reduction in expenditures by indicating the position and depth of possible oil-bearing layers. This work is conducted by prospecting teams in cooperation with geologists.

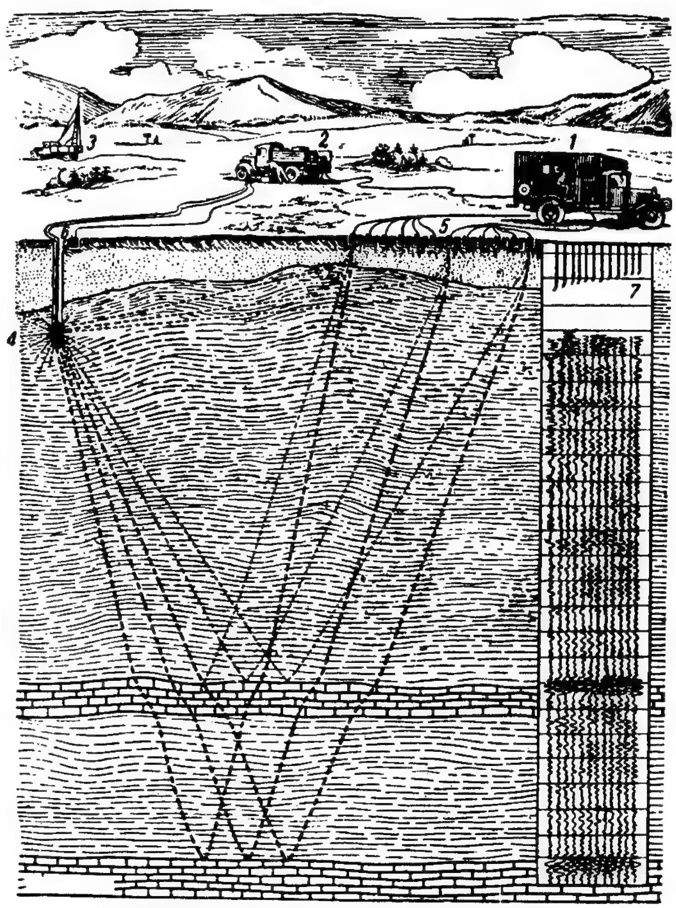


Figure 339. Seismic station using the reflection method

1) station, 2) blasting equipment, 3) drilling equipment, 4) explosion, 5) seismographs, 6) explosion-timing device, 7) seismogram

Seismic prospecting is valuable in engineering projects for determining the thickness and composition of the bedrock in the construction of dams, hydroelectric power-stations, bridges, etc.

At present, there is a definite technique for seismic prospecting. Figure 339 shows diagrammatically the setup of a seismic station using the reflection method. The station is a rather complicated mobile laboratory, with modern electronic equipment. A series of seismographs — up to a few dozen — are arranged in a definite order along the surface contour being worked, and cables connect these to amplifiers at the station. Usually, seismographs operating on the electrodynamical principle are used (Figure 340), and these instruments operate in essentially the same way as those used in "pure" seismology. The main difference is that prospecting seismographs respond to higher frequencies and can detect ground vibrations from a few cycles per second to several hundred. Such seismographs are not very sensitive. For this reason, the alternating voltage applied to the terminals of the moving coil of the seismograph (this coil is placed in the gap of a permanent magnet and oscillates during the passage of an elastic wave) is first amplified electronically. The amplifiers usually have quite high amplification (10^3 and above) and are equipped with filters which select certain frequency bands. In seismic prospecting, using either the refraction or the reflection method, comparatively narrow frequency bands are selected from the explosion spectrum — 10 to 15 cycles/sec, 15 to 25 cycles/sec, and so on. The specifications of the amplifiers (modern seismic stations may use as many as 24) are very strict; for example, all the amplifiers must possess exactly the same sensitivity for all frequencies*.

From the amplifier outputs the alternating voltage corresponding to the vibrations received by the seismographs is applied to a loop oscillograph, where it is usually recorded on a roll of photographic paper moving at constant speed.

An additional loop in the oscillograph records the moment of explosion, the signal of which is transmitted to the seismic station by wire or radio, and another loop provides time markers (usually by means of a tuning fork). A typical record of incoming elastic waves is shown in Figure 339 (right).

Figure 341 represents a seismogram obtained using the refraction method.

To obtain sufficiently well-defined records of refracted and reflected waves, it is necessary for the signal level to be higher than that of the interference background. In addition to undesirable types of waves, such as surface waves, the interference also includes microseisms. In the absence of explosion, and even under conditions of profound silence (far from towns with their traffic and machines) a seismograph still records background vibrations of the ground, or microseisms. Microseisms have

* Recently, attempts have been made to design seismic equipment somewhat differently. Instead of recording within narrow frequency bands in the explosion spectrum by means of filter-amplifiers, reception and recording are performed by wide-band equipment (seismographs and amplifiers passing frequencies, say, from 10 to several hundred cycles/sec); recording in all channels is made on wide magnetic tape. Processing the seismograms consists in selecting certain frequencies from the record by means of variable-filters; this means that from a single explosion a series of seismograms are obtained, corresponding to different frequencies, instead of the one seismogram described previously. This method cuts down greatly on the number of explosions needed and produces much more data for subsequent processing.

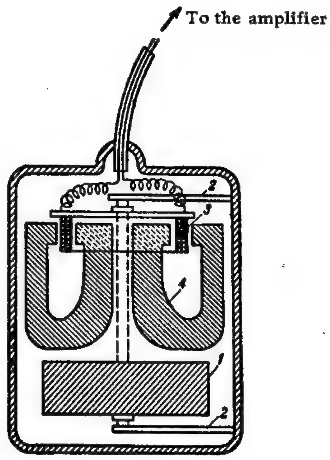


Figure 340. Electrodynamic seismograph

1) weight, 2) spring, 3) moving coil in gap of magnet, 4) magnet.

several causes, such as vibration transmitted to the ground by trees, blades of grass, etc., which are shaken by the wind. The interference level sets a limit to the sensitivity of the whole seismic measuring system, from the seismographs to the loop galvanometers.

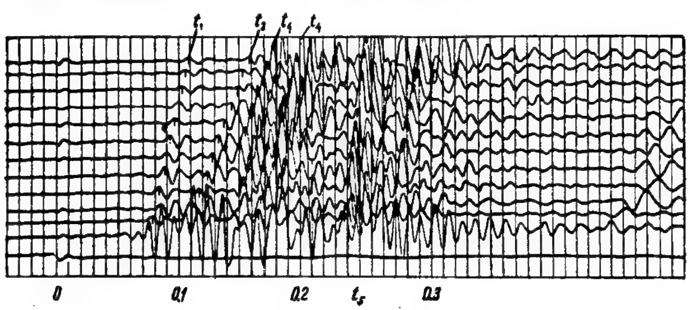


Figure 341. Seismogram obtained by the refraction method

The arrival of refracted waves at time t_1 corresponds to a boundary between limestone layers (depth = 2 m); that at t_2 to a boundary between sand-clay layers (h = 30 m); and that at t_3 to a deeper boundary between sand-clay layers (h = 260 m). Obtained by I. Berzon and A. Epinat'eva

The signal level is made higher than the interference level, first, by means of special filtering and by proper choice of the frequency responses of all the equipment in the seismic station, and second, by ensuring a powerful and properly carried-out explosion. In order for the explosion to be a sufficiently powerful source of elastic waves, it is usually set off in a layer which is denser than the upper alluvial layer. Explosions are also often set off in water (local reservoirs: lakes, rivers, swamps, or flooded ditches). However, in underwater explosions a very interesting physical phenomenon is encountered, which complicates analysis of the seismograms.

Figure 342 shows photographs (taken at intervals) of an explosion in water. It is seen that a gas bubble forming in an underwater explosion pulsates; when it reaches a certain volume, it contracts, and later expands. The cause of the pulsation is as follows. When the gas bubble produced by

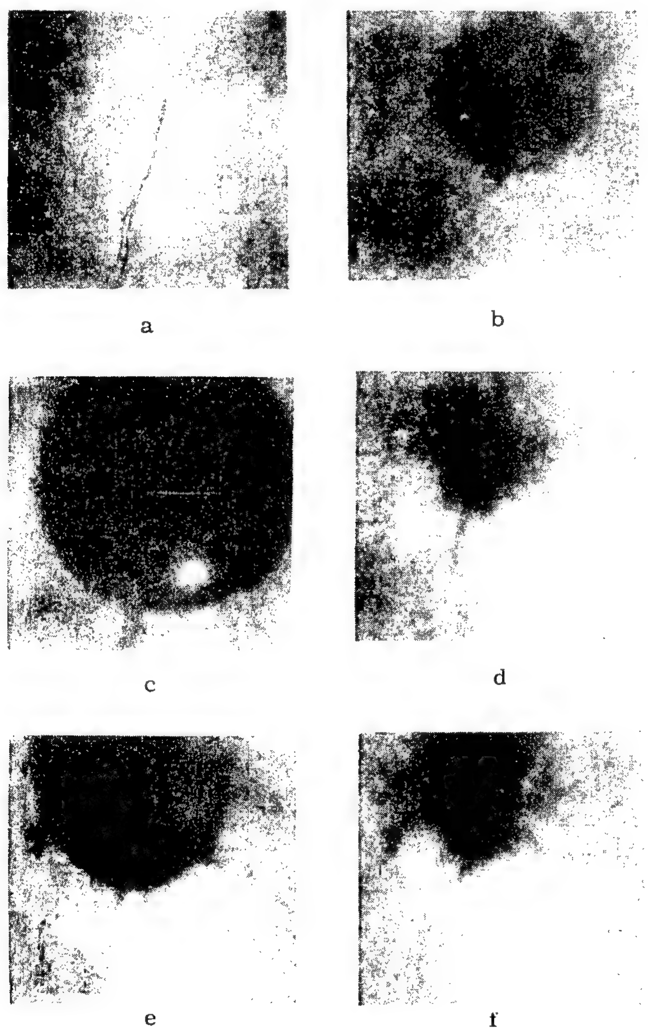


Figure 342. Photographs of an explosion in water, elapsed time as shown: a) 0.0000 sec, b) 0.0028 sec, c) 0.0098 sec, d) 0.0308 sec, e) 0.0420 sec, f) 0.0532 sec.

the explosion expands, it pushes the surrounding liquid out in all directions. But, just as a pendulum which has reached its equilibrium position does not stop, but is carried past by its inertia, so the bubble, after reaching a size where the gas pressure is balanced by the hydrostatic pressure, goes on expanding due to inertia. The expansion is progressively slowed down

until it stops altogether (as a pendulum stops when it reaches maximum deflection). Then, the bubble begins to contract under the action of hydrostatic pressure, and again passes through the equilibrium position. Because of the elasticity of the gas, inertia, and the hydrostatic pressure, the pulsation of the bubble is repeated. The radius of the gas bubble and its period of vibration depend upon the size of the explosive charge; in a deep explosion, when the depth is greater than the maximum radius of the bubble, there may be as many as ten pulsations*.

As a result of these gas-bubble pulsations during underwater explosions, not one, but two or more, pressure pulses may be produced, generating elastic waves. The repeated pulses produce additional wave arrivals on the seismogram, which may be misinterpreted during analysis of the seismogram, unless bubble formation has been taken into account. (The effect of secondary pulsations can be weakened by performing the explosion near the surface of the water.)

It has also been attempted to obtain directed radiation of elastic waves of low (seismic) frequencies, by means of groups of explosions. In many cases good results were obtained with explosions in the air.

A modern seismic station possesses high sensitivity and is capable of recording explosions of very small quantities of explosive material (but of high explosive power, such as ammonal or TNT).

By using seismographs and amplifiers of high sensitivity, it is possible to record reflections from layers which are several kilometers deep. However, as already pointed out, in the analysis of these seismograms it is necessary to take into account multiple reflections from the earth's surface and from intermediate layers.

In many cases, seismic prospecting must be conducted underwater, such as in studies of the structure of the sea bottom for the subsequent drilling of oil wells several hundred or more meters under the sea. During undersea explosions, reverberation appears, a phenomenon which has been discussed in a preceding chapter. Reverberation greatly complicates analysis of the seismogram**.

It should not be assumed that seismic prospecting is limited to the very simple cases (parallel layers) discussed above. There are a large number of different methods of studying underground contours and of seismogram analysis, and these make it possible to obtain information about sloping layers, faults, various inclusions, etc.

However, all these methods are based mainly upon the geometrical (ray) interpretation, without taking into account the wave nature of elastic vibrations of the ground. It is not always clear whether such methods are permissible, especially since in many cases the information obtained refers to layers 50 to 100 m below the surface, while the lengths of the longitudinal waves, the first arrivals of which are utilized, are of this same order of

* Cole, R. Underwater Explosions. –Princeton University Press. 1948 [Russian translation published by Inostrannaya literatura. 1950].

** Studies of the propagation of explosion sounds in shallow seas, important in seismology and hydroacoustics, have recently developed greatly. The wave theory of the related phenomena has also been quite well developed (subject to certain simplifying assumptions). See the collections of articles: Rasprostranenie zvuka v okeane (Propagation of Sound in the Ocean). –Inostrannaya literatura. 1951; Voprosy seismicheskoi razvedki (Problems in Seismic Prospecting). –Inostrannaya literatura. 1953 [both translated into Russian from other languages].

magnitude. Nevertheless, the data obtained in many cases agree quite well with the results of drilling. The complete explanation of these factors is a very important task of applied seismology.

The development of a wave theory for the propagation of a given type of elastic wave in the presence of layer boundaries is very complicated. It is made even more complicated, even for isotropic, uniform media, by the several types of elastic waves involved (longitudinal, transverse, and surface waves), and also by the transformation of waves from one mode to another. The diffraction of elastic waves, traveling in a solid, at a solid sphere of different rigidity is not yet fully understood, while the analogous problem for sound waves in air and liquids, as well as for electromagnetic waves, has a strict solution. Hence, a basic task of the theory of propagation of elastic waves in the presence of interfaces is the development of an approximate theory based on wave concepts; in addition, the limits of applicability of the geometrical (ray) interpretation (geometrical seismology) must be determined.

The difficulty of correct seismogram analysis in the case of complicated profiles (faults, inclusions, layers of irregular shape, etc.) necessitates development of new ways of solving the problems of seismic prospecting. One such innovation is the irradiation of models with ultrasonic waves*.

The method consists in sending ultrasonic waves into comparatively small typical models of geological structures, with densities and velocities of elastic-wave propagation which simulate real possible cases. The ratio of model dimensions to the wave length of the elastic vibrations used should be approximately the same as the ratio in full-scale seismic prospecting.

A block diagram of the modeling is given in Figure 343. By means of an instrument called a seismoscope, similar in design to an ultrasonic flaw detector, ultrasonic pulses are sent into the model under investigation. The frequency of pulse repetition varies from a few cycles/sec to some tens of cycles/sec. Sharp voltage pulses (a few microseconds in duration) are applied to the piezoelectric Rochelle salt source, causing the piezoelectric transducer to emit a series of damped oscillations at its natural frequency (an artificial explosion). The ultrasonic frequencies employed are between 70 and 200 kc. The piezoelectric source is small and serves as a nondirectional source of ultrasonic waves. At some distance from the piezoelectric transducer, a piezoelectric receiver is placed near the model. The voltage generated between its electrodes on arrival of the elastic waves is amplified and applied to the vertical plates of an electronic oscilloscope. The sweep of the oscilloscope is synchronized with the application of the electrical pulse to the piezoelectric transducer. In this way (as in the case described in Chapter V, § 3), there is on the oscilloscope a stationary pattern showing the moment of sending and also the waves arriving at the piezoelectric receiver. If the receiver is moved through equal intervals along the surface upon which it rests (the "profile"), and if the pattern on the oscilloscope is photographed each time, the same pattern is obtained as on the tape of the loop oscillograph

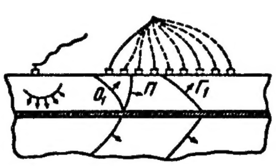


Figure 343. Block diagram of ultrasonic modeling

* This method is being developed by Yu.V. Riznichenko and his co-workers.

with the refraction method, using several seismographs. The model in Figure 343 is a horizontal brass sheet 0.4 cm thick placed 13 cm deep in a water bath. The receiver is moved over the water surface in intervals of 2 cm. The oscillograms obtained are shown in Figure 344, with the moment of "explosion" marked at the beginning of the recording*. The direct wave (Π), the wave reflected from the brass sheet (O_1), and the lateral wave (Γ_1) due to the thin layer of brass, in which the velocity of elastic waves is higher than in water, are all evident in the photograph.

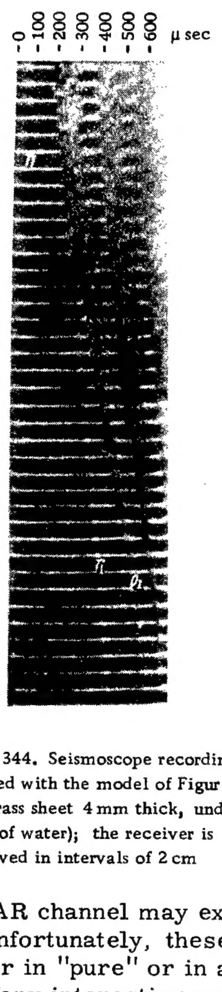


Figure 344. Seismoscope recordings obtained with the model of Figure 343 (brass sheet 4 mm thick, under 13 cm of water); the receiver is moved in intervals of 2 cm

The use of modeling and ultrasound may be quite valuable in solving various problems in applied seismology. However, in the application of this method new problems are encountered — primarily that of deciding whether the scaling of a given model makes it similar to the medium dealt with in seismic prospecting (especially difficult is an accurate modeling of wave damping).

When the propagation of sound waves in the atmosphere and the sea was discussed in the preceding chapter, it was noted that in many cases peculiar sound-conducting channels [SOFAR channels] are formed in these media, within which the sound may travel a considerable distance. A similar situation is bound to exist during the propagation of elastic waves in the earth's crust, since the latter is composed of layers of different acoustic stiffness and different thickness. It is not impossible that even within a stratum of a given type of rock the velocity of propagation of elastic waves may change with the depth in such a way that a SOFAR channel may exist, with a minimum velocity along its axis.

Unfortunately, these problems have still not been properly studied, either in "pure" or in applied seismology.

Many interesting and important problems, however, have found solution, or will be solved using applied seismology, and their number increases with further development of this fascinating branch of geophysics. Recently, great attention has been paid to research using high-frequency elastic waves — up to several hundred or even a thousand cycles/sec. Attempts have been made, especially in dense rock, of using not pulse methods, but tonal methods, i.e., continuous elastic-wave radiation into the ground by means of electromechanical converters; the final signals are picked up by sensitive seismographs.

* Photographs by Yu.V. Riznichenko, B.N. Ivakin, and V.R. Bugrov.

BIBLIOGRAPHY

Popular Science Texts Dealing with Subjects Discussed in this Book.

1. BARKHAUSEN, G. Vvedenie v uchenie o kolebaniyakh (Introduction to the Theory of Vibrations). — ONTI - Energoizdat. 1934.
2. BONCHKOVSKII, V.F. Zemletryaseniya i metody ikh izucheniya (Earthquakes and Methods for Studying Them). -Izdatel'stvo AN SSSR*. 1950.
3. CARLIN, B. Ultrasonics. -McGraw-Hill, New York. 1949. Russian translation. 1950.
4. FLEMING, J. Waves on Water, in Air, and in Liquids. Russian translation. 1926.
5. LAZAREV, P.P. Uspekhi geofiziki (Advances in Geophysics). -In: Series "Noveishie techeniya nauchnoi mysli", No. 17, GIZ. 1929.
6. SHULEIKIN, V.V. Ocherki po fizike morya (Studies of Marine Physics). -Izdatel'stvo AN SSSR. 1949.
7. TINDALL, D. Sound. Russian translation. 1922.
8. WATSON, F.R. Architectural Acoustics. Russian translation. 1948.
9. WOOD, A. Sound Waves and their Applications. Russian translation. 1934.

Scientific Texts on Elastic-Wave Propagation in Gases, Liquids, and Solids

1. BERGMANN, L. Ultrasound and its Applications in Science and Industry. Russian translation. 1956.
2. BLOKHINTSEV, D.I. Akustika neodnorodnoi dvizhushcheisya sredy (The Acoustics of an Inhomogeneous Moving Medium). -Gostekhizdat. 1946 [English translation by R.T. Beyer and D. Mintzer, Brown University, Providence, R.I.].
3. DAVIS, A. Modern Acoustics. Russian translation. 1938.
4. FURDUEV, V.V. Electroakustika (Electroacoustics). -Gostekhizdat. 1948.
5. GAMBURTSEV, G.A. Seismicheskie metody razvedki (Seismic Methods of Prospecting) Part I, 1937; Part II, 1938. -ONTI.
6. GORELIK, G.S. Kolebaniya i volny (Vibrations and Waves). -In: "Vvedenie v akustiku, radiofiziku i optiku", Gostekhizdat. 1959.
7. KHARKEVICH, A.A. Spektry i analiz (Spectra and Their Analysis). -Gostekhizdat. 1957.
8. KOLSKY, H. Stress Waves in Solids. -Clarendon Press, Oxford. 1953. Russian translation. 1955.
9. LANDAU, L.D. and E.M. LIFSHITS. Mekhanika sploshnykh sred (Mechanics of Continuous Media). — Gostekhizdat. 1953.
10. RAYLEIGH, Lord. Theory of Sound, Vols. I and II. -Dover, New-York. 1945. Russian translation. 1955.
11. RZHEVKIN, S.N. Kolebaniya i volny (Vibrations and Waves). -In: "Kurs fiziki", Vol. I, edited by Academician N.D. Papaleksi, Gostekhizdat. 1948.
12. SAVARENSKII, E.F. and D.P. KIRNOS. Elementy seismologii i seismometrii (Elements of Seismology and Seismometry). -Gostekhizdat. 1949.
13. SKUDRCZIK, E. Principles of Acoustics. Russian translation. 1958.
14. PRANDTL, L. Fluid Mechanics. Russian translation. 1949.

* [Akademiya Nauk SSSR — Academy of Sciences of the U.S.S.R.]